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Generalized Left Derivations with Identities on Near-Rings

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Abstract:

In this paper, new concepts which are called: left derivations and generalized left derivations in near-rings have been defined. Furthermore, the commutativity of the 3-prime near-ring which involves some algebraic identities on generalized left derivation has been studied.

Keywords: Derivations, Generalized left derivations, Left derivations, 3-Prime near-ring s, Semigroup ideals.

Introduction:

Let $\mathcal{N}$ be a right near-ring dwith commutative center $\mathcal{Z}(\mathcal{N})$, recall that $\mathcal{N}$ is 3-prime whenever $n\mathcal{N}m = \{0\}$ implies that either $n = 0$ or $m = 0$. Right distribute law of $\mathcal{N}$ involves $0n = 0$ for each $n \in \mathcal{N}$, but when $n0 = 0$ for each $n \in \mathcal{N}$, then $\mathcal{N}$ will be named zero symmetric. If $\mathcal{A} \subseteq \mathcal{N}$ satisfies $\mathcal{N}\mathcal{A} \subseteq \mathcal{A}$ and $A\,\mathcal{N} \subseteq \mathcal{A}$, then $\mathcal{A}$ will be called a semigroup ideal of $\mathcal{N}$. The symbols $[n, m] = nm - mn$ and $n \circ m = nm + mn$ stand for Lie product and Jordan product, respectively. For more about near ring, see Pilz[1].

In 1994 Wang[2] defined the concept of derivation in near-rings and studied the commutativity of the near-rings that satisfies certain algebraic identities involving derivation, from there authors began to study different types of derivations,[3] in near-rings, such as semiderivation,[4] generalized derivations,[5-8] n-derivation,[9] $\alpha - n$derivation,[10] generalized n-derivation,[11] homoderivation,[12-14] right n-derivation,[15] generalized right n-derivation,[16] (where another reference can be found). Majeed[15] presented the notion of right derivation in near-rings. Furthermore, the author[16] defined the generalized right derivation in near-rings.

After revising the literature, you will find that Bresar[17] started studying left derivations in rings. Moreover, Auday[18] and Ikram[19] investigated the study of left derivation and its generalization on the ring. But this type of mapping has not been defined and studied in the near- rings so far. So it is ordinary to study this kind of derivation which is named left derivation in near ring.

In fact, this work has been devoted to defining the notion of left derivation and generalized left derivation on near-ring, as well as to investigating the commutativity of near-ring with left derivations and generalized left derivations.

Remark: In this article $\mathcal{N}$ is 3-prime right near-ring and $\mathcal{A}$ is semigroup ideal of $\mathcal{N}$ unless otherwise noted. Also, the article refers to the "commutative ring" by $\mathcal{C}.\mathcal{R}$.

Definitions and Preliminaries:

Definition 1: An additive mapping d from $\mathcal{N}$ into itself is called left derivation if $d(nm) = nd(m) + md(n)$ for every $n, m \in \mathcal{N}$.

Moreover, An additive mapping G from $\mathcal{N}$ into itself is a generalized left derivation connected with d if $G(nm) = nd(m) + mG(n)$ for each $n, m \in \mathcal{N}$.

Example 1: Let $\mathcal{K}$ be a zero symmetric right near-ring and

$$\mathcal{N} = \left\{\begin{pmatrix} 0 & 0 & 0 \\ u & 0 & v \\ w & 0 & 0 \end{pmatrix} : u, v, w, 0 \in \mathcal{K}\right\},$$

$d.G: \mathcal{N} \longrightarrow \mathcal{N}$

$$d\left(\begin{pmatrix} 0 & 0 & 0 \\ u & 0 & v \\ w & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & v \\ 0 & 0 & 0 \end{pmatrix},$$

$$G\left(\begin{pmatrix} 0 & 0 & 0 \\ u & 0 & v \\ w & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & 0 \\ w & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

One can easily see that $\mathcal{N}$ is an aright near-ring with matrix addition and multiplication and d is a left derivation of $\mathcal{N}$ which is neither right derivation nor derivation and G is generalized left derivation connected with d.
With the above notations, we have the following lemmas:

Lemma 1: [4]

(i) If $n \in \mathcal{Z}(\mathcal{N}) / \{0\}$, s.t $2n \in \mathcal{Z}(\mathcal{N})$, then $\mathcal{N}$ is abelian.

(ii) If $\mathfrak{a} \in \mathcal{N}$ and $n \in \mathcal{Z}(\mathcal{N}) / \{0\}$, s.t $\mathfrak{n}\mathfrak{a} \in \mathcal{Z}(\mathcal{N})$ or $an \in \mathcal{Z}(\mathcal{N})$, then $\mathfrak{a} \in \mathcal{Z}(N)$.

Lemma 2: [4]

(i) If $\mathcal{A} \neq \{0\}$ s.t $\mathcal{A}\mathfrak{n} = \{0\}$ or $n\mathcal{A} = \{0\}$, then $n = 0$.

(ii) If $n\mathcal{A}m = \{0\}$, then $n = 0$ or $m = 0$.

(iii) If $n \in \mathcal{N}$ which centralizes $\mathcal{A}$, then $n \in \mathcal{Z}(\mathcal{N})$.

Lemma 3: [4] If $\mathcal{Z}(\mathcal{N})$ includes a nonzero semigroup ideal, then $\mathcal{N}$ is a $\mathcal{C.R}$.

Lemma 4: If G is a generalized left derivation on $\mathcal{N}$ connected with d of $\mathcal{N}$. If $G(\mathcal{A}) = \{0\}$, then $G(\mathcal{N}) = d(\mathcal{N}) = \{0\}$.

Proof: By assumption, $0 = G(an) = ad(n)$ for each $a \in \mathcal{A}$, $n \in \mathcal{N}$. It follows that $\mathcal{A}d(\mathcal{N}) = \{0\}$ and using Lemma 2(i) implies $d(\mathcal{N}) = \{0\}$. Now, $0 = G(na) = aG(n)$ for each $a \in \mathcal{A}$, $n \in \mathcal{N}$, and this means $\mathcal{A}G(\mathcal{N}) = \{0\}$ and again Lemma 2(i) implies $G(\mathcal{N}) = \{0\}$.
Corollary 1 is consequence of Lemma 4.

Corollary 1: Let d be a left derivation on $\mathcal{N}$. If $d(\mathcal{A}) = \{0\}$, then $d(\mathcal{N}) = \{0\}$.

Lemma 5: If $\mathcal{N}$ admits a left derivation $d \neq 0$, then $\mathcal{N}$ is zero-symmetric.

Proof: It is evident that $d(0) = d(00) = 0d(0) + 0d(0) = 0$, therefore $0 = d(0n) = 0d(n) + nd(0) = n0$, which means that $n0 = 0$ for each $n \in \mathcal{N}$.

The hypothesis that $\mathcal{N}$ is zero symmetric has not be required in the remainder of this work, if $\mathcal{N}$ admits a left derivation, which shows the importance of Lemma 4.

Results:

Theorem 1: If G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$ such that $G([\mathfrak{a}, \mathfrak{b}]) = 0$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, then $\mathcal{N}$ is a $\mathcal{C.R}$.

Proof: From assumption: $0 = G([\mathfrak{a}, \mathfrak{b}\mathfrak{a}]) = G([\mathfrak{a}, \mathfrak{b}]\mathfrak{a}) = [\mathfrak{a}, \mathfrak{b}]d(\mathfrak{a}) + \mathfrak{a}G([\mathfrak{a}, \mathfrak{b}]) = [\mathfrak{a}, \mathfrak{b}]d(\mathfrak{a})$ for each $\mathfrak{a}, \mathfrak{b} \in A$, which leads to
$\mathfrak{a}bd(\mathfrak{a}) = b\mathfrak{a}d(\mathfrak{a})$ for each $\mathfrak{a}, \mathfrak{b} \in A$. 1

Replacing b by nb, where $n \in \mathcal{N}$, in Eq.1 and using it to get $\mathfrak{a}nbd(\mathfrak{a}) = nb\mathfrak{a}d(\mathfrak{a}) = n\mathfrak{a}bd(\mathfrak{a})$ for each $\mathfrak{a}, \mathfrak{b} \in A, n \in \mathcal{N}$, that is, $[a, n]bd(a) = 0$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}, n \in \mathcal{N}$. Hence $[a, n]\mathcal{A}d(a) = \{0\}$ for each $a \in \mathcal{A}, n \in \mathcal{N}$, and using Lemma 2(ii) lastly getting
$a \in \mathcal{Z}(\mathcal{N})$ or $d(\mathfrak{a}) = 0$ for each $\mathfrak{a} \in \mathcal{A}$. 2

If there is $a_0 \in \mathcal{A}$ and $a_0 \in \mathcal{Z}(\mathcal{N})$, by hypothesis, $0 = G([a, ba_0]) = G([\mathfrak{a}, \mathfrak{b}]a_0) = [\mathfrak{a}, \mathfrak{b}]d(\mathfrak{a}_0) + \mathfrak{a}_0 G([\mathfrak{a}, \mathfrak{b}]) = [\mathfrak{a}, \mathfrak{b}]d(a_0)$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$. Therefore
$\mathfrak{a}bd(\mathfrak{a}_0) = b\mathfrak{a}d(a_0)$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$. 3

Replacing $\mathfrak{b}$ by $\mathfrak{n}\mathfrak{b}$, where $\mathfrak{n} \in \mathcal{N}$, in Eq.3 and using it, concluding that $\mathfrak{a}\mathfrak{n}\mathfrak{b}d(\mathfrak{a}_0) = \mathfrak{n}\mathfrak{b}\mathfrak{a}d(\mathfrak{a}_0) = \mathfrak{n}\mathfrak{a}\mathfrak{b}d(\mathfrak{a}_0)$ for each $\mathfrak{a}, \mathfrak{b} \in A, \mathfrak{n} \in \mathcal{N}$, that is, $[\mathfrak{a}, \mathfrak{n}]\mathfrak{b}d(\mathfrak{a}_0) = 0$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$. Hence $[\mathfrak{a}, \mathfrak{n}]\mathcal{A}d(\mathfrak{a}_0) = 0$ for each $\mathfrak{a} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$, and using Lemma 2(ii) forces $\mathcal{A} \subseteq \mathcal{Z}(\mathcal{N})$ or $d(\mathfrak{a}_0) = 0$, hence Eq.2 becomes $\mathcal{A} \subseteq Z(\mathcal{N})$ or $d(A) = \{0\}$, if $d(A) = \{0\}$, so $d = 0$ by Corollary 1: a contradiction. Hence $\mathcal{A} \subseteq \mathcal{Z}(\mathcal{N})$ and the required have been achieved by Lemma 3.

Corollary 2: If G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$ and $G([n, m]) = 0$ for each $n, m \in \mathcal{N}$, then $\mathcal{N}$ is a $\mathcal{C.R}$.
Note that: if $(\mathcal{N}, +)$ is abelian, then $d[n, m] = 0$ for each $n, m \in \mathcal{N}$ (according to the definition of d) therefore the next corollaries are direct results of Theorem 1.

Corollary 3: If $d \neq 0$ is a left derivation on $\mathcal{N}$, then the next affirmations are equivalence:

(i) $(\mathcal{N}, +)$ is $\mathfrak{abelian}$;

(ii) $d([\mathfrak{a}, \mathfrak{b}]) = 0$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;

(iii) $\mathcal{N}$ is a $\mathcal{C.R}$.

Corollary 4: If $d \neq 0$ is a left derivation on $\mathcal{N}$, then the next affirmations are equivalence:

(i) $(\mathcal{N}, +)$ is abelian;

(ii) $d[n, m] = 0$ for each $n, m \in \mathcal{N}$;

(iii) $\mathcal{N}$ is a $\mathcal{C.R}$.

Theorem 2: If G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$, then the next affirmations equivalence

(i) $G(\mathfrak{a} \diamond \mathfrak{b}) = 0$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;

(ii) $\mathcal{N}$ is a $\mathcal{C.R}$ and $char(\mathcal{N}) = 2$.

Proof: Assume that: $G(\mathfrak{a} \diamond \mathfrak{b}) = 0$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, so $0 = G(a \diamond \mathfrak{b}\mathfrak{a}) = G((\mathfrak{a} \diamond \mathfrak{b})\mathfrak{a}) = (\mathfrak{a} \diamond \mathfrak{b})d(a) + \mathfrak{a}G(\mathfrak{a} \diamond \mathfrak{b}) = (\mathfrak{a} \diamond \mathfrak{b})d(\mathfrak{a})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, which leads to
$\mathfrak{a}\mathfrak{b}d(\mathfrak{a}) = -\mathfrak{b}\mathfrak{a}d(\mathfrak{a})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$ 4

Replacing $\mathfrak{b}$ by $n\mathfrak{b}$, where $n \in \mathcal{N}$, in Eq.4 and using it, concluding that $[-\mathfrak{a}, n]\mathcal{A}d(\mathfrak{a}) = \{0\}$ for each $\mathfrak{a} \in \mathcal{A}, n \in \mathcal{N}$, and using Lemma 2(ii) forces
$-\mathfrak{a} \in \mathcal{Z}(\mathcal{N})$ or $d(\mathfrak{a}) = 0$ for each $\mathfrak{a} \in \mathcal{A}$. 5

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If there is $\mathfrak{a}_0 \in \mathcal{A}$ and $-\mathfrak{a}_0 \in Z(\mathcal{N})$, by hypothesis, $0 = G(\mathfrak{a} \diamond \mathfrak{b}(-\mathfrak{a}_0)) = G((\mathfrak{a} \diamond \mathfrak{b})(-\mathfrak{a}_0)) = (\mathfrak{a} \diamond \mathfrak{b})d(-\mathfrak{a}_0) + (-\mathfrak{a}_0)G(\mathfrak{a} \diamond \mathfrak{b}) = (\mathfrak{a} \diamond \mathfrak{b})d(-\mathfrak{a}_0)$. Therefore

$$\mathfrak{a}\mathfrak{b}d(-\mathfrak{a}_0) = -\mathfrak{b}\mathfrak{a}d(-\mathfrak{a}_0) \text{ for each } \mathfrak{a}, \mathfrak{b} \in \mathcal{A}. \quad 6$$

Replacing $\mathfrak{b}$ by $n\mathfrak{b}$, where $n \in \mathcal{N}$, in Eq.6 and using it, concluding that $[-\mathfrak{a}, n]\mathcal{A}d(-\mathfrak{a}_0) = \{0\}$ for each $\mathfrak{a} \in \mathcal{A}, n \in \mathcal{N}$, and using Lemma 2(ii) gets $-\mathcal{A} \subseteq \mathcal{Z}(\mathcal{N})$ or $d(\mathfrak{a}_0) = 0$. Thus Eq.5 becomes $-\mathcal{A} \subseteq \mathcal{Z}(\mathcal{N})$ or $d(\mathcal{A}) = \{0\}$, if $d(\mathcal{A}) = \{0\}$, it follows that $d = 0$ by Corollary 1: a contradiction which acquires that $-\mathcal{A} \subseteq \mathcal{Z}(\mathcal{N})$ and using Lemma 3 implies that $\mathcal{N}$ is $\mathcal{C}.\mathcal{R}$. Returning to the hypothesis, $0 = G(\mathfrak{a} \diamond \mathfrak{b}c) = G((\mathfrak{a} \diamond \mathfrak{b})\mathfrak{c}) = (\mathfrak{a} \diamond \mathfrak{b})d(\mathfrak{c}) + \mathfrak{c}G(\mathfrak{a} \circ \mathfrak{b}) = (\mathfrak{a} \circ \mathfrak{b})d(c) = (2\mathfrak{a})\mathfrak{b}d(\mathfrak{c})$ for each $\mathfrak{a}, \mathfrak{b}, \mathfrak{c} \in \mathcal{A}$, that is $(2\mathfrak{a})\mathcal{A}d(\mathfrak{c}) = \{0\}$ for each $\mathfrak{a}, \mathfrak{c} \in A$, by Lemma 2(ii) and Corollary 1, getting $2a = 0$ for each $a \in A$, which implies $2\mathfrak{n}\mathfrak{a} = 0$ for each $\mathfrak{a} \in \mathcal{A}$, $n \in \mathcal{N}$. i.e. $(2\mathfrak{n})\mathcal{A} = \{0\}$ for each $\mathfrak{n} \in \mathcal{N}$ and thus $\mathfrak{c}h\mathfrak{a}\mathfrak{r}(\mathcal{N}) = 2$ by Lemma 2(i).
For ease, the opposite direction can be demonstrated.

Corollary 5: If G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$, then the next affirmations are equivalence:
(i) $G(n \diamond m) = 0$ for each $n, m \in \mathcal{N}$;
(ii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$ and $\mathfrak{c}h\mathfrak{a}\mathfrak{r}(\mathcal{N}) = 2$.

Corollary 6: Let $0 \neq d$ be a left derivation $\mathcal{N}$, then the next affirmations are equivalence:
(i) $d(a \diamond b) = 0$ for each $a, b \in \mathcal{A}$;
(ii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$ and $char(\mathcal{N}) = 2$.

Corollary 7: If $d \neq 0$ is a left derivation on $\mathcal{N}$, then the next affirmations are equivalence:
(i) $d(n \diamond m) = 0$ for each $n, m \in \mathcal{N}$;
(ii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$ and $char(\mathcal{N}) = 2$.

Theorem 3: If G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$ and $G(\mathcal{A}) \subseteq \mathcal{Z}(\mathcal{N})$, then $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Proof. By assumption, $G(a) \in \mathcal{Z}(\mathcal{N})$ for each $a \in \mathcal{A}$. If $G(\mathfrak{a}) = 0$ for each $\mathfrak{a} \in \mathcal{A}$, it follows $d(\mathcal{N}) = \{0\}$ by Lemma 4 and this is a contradiction. Hence, there is $\mathfrak{a}_0 \in \mathcal{A}$ and $0 \neq G(\mathfrak{a}_0) \in \mathcal{Z}(\mathcal{N})$, besides $G(\mathfrak{a}_0) + G(\mathfrak{a}_0) = G(\mathfrak{a}_0 + \mathfrak{a}_0) \in \mathcal{Z}(\mathcal{N})$. Thus $(\mathcal{N}, +)$ is abelian by Lemma 1(i), so $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$ via Corollary 4.

Corollary 8: If G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$ and $G(\mathcal{N}) \subseteq \mathcal{Z}(\mathcal{N})$. Then $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 9: If $d \neq 0$ is a left derivation on $\mathcal{N}$ and $d(\mathcal{A}) \subseteq Z(\mathcal{N})$, then $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 10: If $d \neq 0$ is a left derivation on $\mathcal{N}$ and $d(\mathcal{N}) \subseteq \mathcal{Z}(\mathcal{N})$, then $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Theorem 4: If G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$, then the next affirmations are equivalence:
(i) $[G(\mathfrak{a}), \mathfrak{b}] \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;
(ii) $[\mathfrak{b}, G(\mathfrak{a})] \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;
(iii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Proof: It is evident that (iii) $\Longrightarrow$ (i) and (iii) $\Longrightarrow$ (ii).
(i) $\Longrightarrow$ (iii) Suppose that $[G(\mathfrak{a}), \mathfrak{b}] \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, it follows $[G(\mathfrak{a}), \mathfrak{b}G(\mathfrak{a})] = [G(\mathfrak{a}), \mathfrak{b}]G(\mathfrak{a}) \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$. Lemma 1(ii) assures that either $G(\mathfrak{a}) \in \mathcal{Z}(\mathcal{N})$ or $[G(\mathfrak{a}), \mathfrak{b}] = 0$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, and using Lemma 2(iii) implies that $G(a) \subseteq \mathcal{Z}(\mathcal{N})$, hence $\mathcal{N}$ is $\mathcal{C}.\mathcal{R}$ by Theorem 3.
In the same way (ii) $\Longrightarrow$ (iii) can be proved.

Corollary 11: If G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$, then the next affirmations are equivalence:
(i) $[G(\mathfrak{n}), \mathfrak{m}] \in \mathcal{Z}(\mathcal{N})$ for every $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$;
(ii) $[\mathfrak{m}, G(\mathfrak{n})] \in \mathcal{Z}(\mathcal{N})$ for every $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$;
(iii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 12: If $d \neq 0$ is a left derivation on $\mathcal{N}$, then the next affirmations are equivalence:
(i) $[d(\mathfrak{a}), \mathfrak{b}] \in \mathcal{Z}(\mathcal{N})$ for any $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;
(ii) $[\mathfrak{b}, d(\mathfrak{a})] \in \mathcal{Z}(\mathcal{N})$ for any $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;
(iii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 13: If $d \neq 0$ is a left derivation on $\mathcal{N}$, then the next affirmations are equivalence:
(i) $[d(\mathfrak{n}), \mathfrak{m}] \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$;
(ii) $[\mathfrak{m}, d(\mathfrak{n})] \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$;
(iii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Theorem 5: If $\mathcal{N}$ is two torsion-free and G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$, then the next affirmations are equivalence:
(i) $G(\mathfrak{a}) \diamond \mathfrak{b} \in \mathcal{Z}(\mathcal{N})$for all $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;
(ii) $\mathfrak{b} \diamond G(\mathfrak{a}) \in \mathcal{Z}(\mathcal{N})$for all $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;
(iii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Proof: It is evident that (iii) $\Longrightarrow$ (i) and (iii) $\Longrightarrow$ (ii).
(ii) $\Longrightarrow$ (iii) Suppose that $G(\mathfrak{a}) \diamond \mathfrak{b} \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, it follows $\left(G(\mathfrak{a}) \diamond \mathfrak{b}G(\mathfrak{a})\right) = (G(\mathfrak{a}) \diamond \mathfrak{b})G(\mathfrak{a}) \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$. Lemma 1(ii) assures that

$$G(a) \in \mathcal{Z}(\mathcal{N}) \text{ or } G(a) \diamond \mathfrak{b} = 0 \text{ for each } \mathfrak{a}, \mathfrak{b} \in \mathcal{A}. \quad 7$$

If there is $a_0 \in \mathcal{A}$ such that $G(a_0) \in \mathcal{Z}(\mathcal{N})$, then our hypothesis forces $G(a_0) \diamond c = G(a_0)(2c) \in \mathcal{Z}(\mathcal{N})$, Lemma 1(ii) forces $G(\mathfrak{a}_0) = 0$ or $2c \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{c} \in \mathcal{A}$. If $2\mathfrak{c} \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{c} \in \mathcal{A}$, then $2cn \in Z(\mathcal{N})$ for each $c \in \mathcal{A}, n \in \mathcal{N}$, and Lemma 1(ii) forces $2c = 0$ or $n \in \mathcal{Z}(\mathcal{N})$ for each $c \in \mathcal{A}, n \in \mathcal{N}$. Thus $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$ because it is two torsion free and $\mathcal{A} \neq \{0\}$.

So Eq.7 becomes $G(\mathfrak{a}) \circ \mathfrak{b} = 0$ for each $a, \mathfrak{b} \in \mathcal{A}$ or $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$. If $G(a) \circ \mathfrak{b} = 0$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, that is

$G(a)\mathfrak{b} = -\mathfrak{b}G(\mathfrak{a})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$. 8

Replacing $\mathfrak{b}$ by $\mathfrak{n}\mathfrak{b}$ in Eq.8 and using it to get $[G(-\mathfrak{a}), \mathfrak{n}]\mathcal{A} = \{0\}$ for each $\mathfrak{a} \in \mathcal{A}$, $\mathfrak{n} \in \mathcal{N}$. Therefore $G(-\mathcal{A}) \subseteq \mathcal{Z}(\mathcal{N})$ by Lemma 2(i) and hence $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$ by Theorem 3.

In the same way (ii) $\Longrightarrow$ (iii) can be proved.

Corollary 14: If $\mathcal{N}$ is two torsion free and G is a generalized left derivation on $\mathcal{N}$ connected with $d \neq 0$, then the next affirmations are equivalence":

(i) $G(\mathfrak{n}) \circ \mathfrak{m} \in \mathcal{Z}(\mathcal{N})$ for all $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$;
(ii) $\mathfrak{m} \circ G(\mathfrak{n}) \in \mathcal{Z}(\mathcal{N})$ for all $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$;
(iii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 15: If $\mathcal{N}$ is two orsion free and $d \neq 0$ is a left derivation on $\mathcal{N}$, then the next affirmations are equivalence:

(i) $d(\mathfrak{a}) \circ \mathfrak{b} \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;
(ii) $\mathfrak{b} \circ d(\mathfrak{a}) \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$;
(iii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 16 If $\mathcal{N}$ is two torsion free and $d \neq 0$ is a left derivation on $\mathcal{N}$, then the next affirmations are equivalence:

(i) $d(\mathfrak{n}) \circ \mathfrak{m} \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$;
(ii) $\mathfrak{m} \circ d(\mathfrak{n}) \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$;
(iii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Theorem 6: There is no generalized left derivation on $\mathcal{N}$ which connected with $d \neq 0$ and satisfies one of the next identities:

(i) $G([\mathfrak{a}, \mathfrak{b}]) = d(\mathfrak{a})b$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$.
(ii) $G([\mathfrak{a}, \mathfrak{b}]) = d(b)\mathfrak{a}$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$.

Proof: Suppose that

$G([\mathfrak{a}, b]) = d(\mathfrak{a})b$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$. 9

Puttinga $\mathfrak{a}$ for $\mathfrak{b}$ in Eq.9 to get

$d(\mathfrak{a})\mathfrak{a} = 0$ for each $\mathfrak{a} \in \mathcal{A}$. 10

Replace $\mathfrak{a}$ by $\mathfrak{a}b$ in Eq.9 to get

$G([\mathfrak{a}, \mathfrak{b}]\mathfrak{b}) = d(\mathfrak{a}\mathfrak{b})\mathfrak{b}$ for all $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$. 11

Develop Eq.11 to get $[\mathfrak{a}, \mathfrak{b}]d(\mathfrak{b}) + \mathfrak{b}G([\mathfrak{a}, \mathfrak{b}]) = \mathfrak{a}d(\mathfrak{b})\mathfrak{b} + bd(\mathfrak{a})\mathfrak{b}$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, then using hypothesis and Eq.10 implies $[\mathfrak{a}, \mathfrak{b}]d(b) = 0$ for all $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$. That is

$\mathfrak{a}\mathfrak{b}d(\mathfrak{a}) = \mathfrak{b}\mathfrak{a}d(\mathfrak{a})$ for each $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$. 12

Replacing $\mathfrak{b}$ by $\mathfrak{n}\mathfrak{b}$, where $\mathfrak{n} \in \mathcal{N}$ in Eq.12 and using it concluding that $[\mathfrak{a}, \mathfrak{n}]\mathcal{A}d(\mathfrak{a}) = \{0\}$ for each $\mathfrak{a} \in \mathcal{A}, n \in \mathcal{N}$, and using Lemma 2(ii) implies

$\mathfrak{a} \in \mathcal{Z}(\mathcal{N})$ or $d(a) = 0$ for each $\mathfrak{a} \in \mathcal{A}$. 13

If there is $\mathfrak{a}_0 \in \mathcal{A}$ such that $\mathfrak{a}_0 \in Z(\mathcal{N})$, replacing $\mathfrak{a}$ by $\mathfrak{a}_0$ in Eq[9] to get $d(\mathfrak{a}_0)\mathfrak{b} = 0$ for each $\mathfrak{b} \in \mathcal{A}$. Therefore $d(\mathfrak{a}_0) = 0$ according to Lemma 2(i), so Eq.13 becomes $d(A) = \{0\}$ and hence $d = 0$ by Corollary 2, a contradiction.

In the same way, (ii) can be proved.

Corollary 17: There is no generalized left derivation on $\mathcal{N}$ which is connected with $d \neq 0$ and satisfies one of the next identities:

(i) $G([\mathfrak{n}, \mathfrak{m}]) = d(\mathfrak{n})\mathfrak{m}$ for each $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$.
(ii) $G([\mathfrak{n}, \mathfrak{m}]) = d(\mathfrak{m})\mathfrak{n}$ for each $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$.

Corollary 18: There is no left derivation $d \neq 0$ on $\mathcal{N}$ which satisfies one of the next identities:

(i) $d([\mathfrak{a}, b]) = d(a)b$ for each $\mathfrak{a}, b \in \mathcal{A}$.
(ii) $d([\mathfrak{a}, b]) = d(b)a$ for each $\mathfrak{a}, b \in \mathcal{A}$.

Corollary 19: $\mathcal{N}$ consents no left derivation $d \neq 0$, which satisfies one of the next identities:

(i) $d([\mathfrak{n}, \mathfrak{m}]) = d(n)\mathfrak{m}$ for each $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$.
(ii) $d([\mathfrak{n}, \mathfrak{m}]) = d(\mathfrak{m})n$ for each $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$.

Theorem 7: If $d \neq 0$ is a left derivation on $\mathcal{N}$, then the next affirmations are equivalence:

(i) $d([\mathfrak{n}, \mathfrak{a}]) = [\mathfrak{n}, \mathfrak{a}]$ for each $\mathfrak{a} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$;
(ii) $d([\mathfrak{a}, \mathfrak{n}]) = [\mathfrak{a}, \mathfrak{n}]$ for each $\mathfrak{a} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$;
(iii) $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Proof: Just need to prove that (i) $\Rightarrow$ (iii) and (ii) $\Rightarrow$ (iii).

(ii) $\Rightarrow$ (iii) It is obvious that $[n, \mathfrak{a}n] = d([n, an]) = d([n, a]n) = [n, a]d(n) + nd([n, a]) = [n, a]d(n) + n[n, a]$ for each $a \in A$, $n \in \mathcal{N}$, which leads to $[n, \mathfrak{a}]n = [n, a]d(n) + n[n, \mathfrak{a}]$ for each $\mathfrak{a} \in \mathcal{A}$, $n \in \mathcal{N}$. Putting $[n, b]$ in place of n in the last equation and using the hypothesis getting $[n, b]\big[[n, b], \mathfrak{a}\big] = 0$ for each $\mathfrak{a}, b \in \mathcal{A}$, $n \in \mathcal{N}$, which implies that

$$\begin{aligned} 0 &= d\big([n, b]\big[[n, b], \mathfrak{a}\big]\big) \\ &= [n, b]d\big(\big[[n, b], \mathfrak{a}\big]\big) + \big[[n, b], \mathfrak{a}\big]d([n, b]) \\ &= [n, b]\big[[n, b], \mathfrak{a}\big] + \big[[n, b], \mathfrak{a}\big][n, b] \\ &= \big[[n, b], \mathfrak{a}\big][n, b] \end{aligned}$$ for each $\mathfrak{a}, b \in \mathcal{A}, n \in \mathcal{N}$.

Therefore,

$$[n, b]\mathfrak{a}[n, b] = \mathfrak{a}[n, b][n, b]$$
for each $\mathfrak{a}, b \in \mathcal{A}, n \in \mathcal{N}$. 14

Replace $\mathfrak{a}$ by $\mathfrak{m}\mathfrak{a}$, where $\mathfrak{m} \in \mathcal{N}$, in Eq.14, and use it, to get $[n, b]\mathfrak{m}\mathfrak{a}[\mathfrak{n}, b] = \mathfrak{m}\mathfrak{a}[\mathfrak{n}, b][\mathfrak{n}, b] = \mathfrak{m}[\mathfrak{n}, b]\mathfrak{a}[\mathfrak{n}, b]$ for each $\mathfrak{a}, b \in \mathcal{A}$, $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$. It follows that $\big[[\mathfrak{n}, b], \mathfrak{m}\big]\mathcal{A}[\mathfrak{n}, b] = \{0\}$ for each $b \in \mathcal{A}$, $\mathfrak{n}, \mathfrak{m} \in \mathcal{N}$. So $[\mathfrak{n}, b] \in Z(\mathcal{N})$ for each $b \in A$, $n \in \mathcal{N}$ by Lemma 2(ii). Now, $[n, bn] = [n, b]n \in Z(\mathcal{N})$ for each $b \in \mathcal{A}$, $x \in \mathcal{N}$ and use Lemma 1(ii), to obtain $\mathcal{A} \subseteq \mathcal{Z}(\mathcal{N})$, hence $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$ by Lemma 4.

(ii) $\Rightarrow$ (iii) can be proved in the same way above.

Corollary 20: If $d \neq 0$ is a left derivation on $\mathcal{N}$ and $d([\mathfrak{n},\mathfrak{m}]) = [\mathfrak{n},\mathfrak{m}]$ for each $\mathfrak{n},\mathfrak{m} \in \mathcal{N}$, then $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Theorem 8: If $\mathcal{N}$ is two torsion-free and $d \neq 0$ is a left derivation on $\mathcal{N}$, then $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$ if it has one of the next conditions:

(i) $d(\mathfrak{n} \diamond \mathfrak{a}) = \mathfrak{n} \diamond \mathfrak{a}$ for each $\mathfrak{a} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$.

(ii) $d(\mathfrak{a} \diamond \mathfrak{n}) = \mathfrak{a} \diamond \mathfrak{n}$ for each $\mathfrak{a} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$.

Proof: Suppose that $d(\mathfrak{n} \diamond \mathfrak{a}) = n \diamond \mathfrak{a}$ for each $\mathfrak{a} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$. Thus $d(\mathfrak{n} \diamond \mathfrak{a}\mathfrak{n}) = \mathfrak{n} \diamond \mathfrak{a}\mathfrak{n}$ for each $\mathfrak{a} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$, which can be reduced to $(\mathfrak{n} \diamond \mathfrak{a})d(\mathfrak{n}) + \mathfrak{n}(\mathfrak{n} \diamond \mathfrak{a}) = (\mathfrak{n} \diamond \mathfrak{a})\mathfrak{n}$ for each $\mathfrak{a} \in \mathcal{A}, n \in \mathcal{N}$. Now, putting $\mathfrak{n} \diamond \mathfrak{b}$ instead of n in the last equation implies $(n \diamond \mathfrak{b})((n \diamond \mathfrak{b}) \diamond \mathfrak{a}) = 0$ for each $\mathfrak{a},\mathfrak{b} \in \mathcal{A}, n \in \mathcal{N}$.

Therefore,

$$\begin{aligned} 0 &= d\Big((\mathfrak{n} \diamond \mathfrak{b})\big((\mathfrak{n} \diamond \mathfrak{b}) \diamond \mathfrak{a}\big)\Big) \\ &= (\mathfrak{n} \diamond \mathfrak{b})d\big((\mathfrak{n} \diamond \mathfrak{b}) \diamond \mathfrak{a}\big) + \big((\mathfrak{n} \diamond \mathfrak{b}) \diamond \mathfrak{a}\big)d(\mathfrak{n} \diamond \mathfrak{b}) \\ &= (\mathfrak{n} \diamond \mathfrak{b})\big((\mathfrak{n} \diamond \mathfrak{b}) \diamond a\big) + \big((\mathfrak{n} \diamond \mathfrak{b}) \diamond \mathfrak{a}\big)(\mathfrak{n} \diamond \mathfrak{b}) \\ &= \big((\mathfrak{n} \diamond \mathfrak{b}) \diamond \mathfrak{a}\big)(\mathfrak{n} \diamond \mathfrak{b}) \text{ for each } \mathfrak{a},\mathfrak{b} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}. \end{aligned}$$

Hence,

$$(\mathfrak{n} \diamond \mathfrak{b})\mathfrak{a}(\mathfrak{n} \diamond \mathfrak{b}) = -\mathfrak{a}(\mathfrak{n} \diamond \mathfrak{b})(\mathfrak{n} \diamond b) \quad \text{for each } \mathfrak{a},\mathfrak{b} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}. \qquad 15$$

Replacing $\mathfrak{a}$ by $\mathfrak{m}\mathfrak{a}$, where $\mathfrak{m} \in \mathcal{N}$, in Eq.15 and using it, to arrive at $[-(\mathfrak{n} \diamond \mathfrak{b}),\mathfrak{m}]\mathcal{A}(-(\mathfrak{n} \diamond \mathfrak{b})) = \{0\}$ for each $\mathfrak{b} \in \mathcal{A}, \mathfrak{m},\mathfrak{n} \in \mathcal{N}$, thus using Lemma 2(ii) to get

$$-(n \diamond \mathfrak{b}) \in \mathcal{Z}(\mathcal{N}) \text{ for each } \mathfrak{b} \in \mathcal{A}, n \in \mathcal{N}. \qquad 16$$

Now, replacing $\mathfrak{b}$ by bn in Eq.16 to get $-(n \diamond \mathfrak{b}n) = -(n \diamond \mathfrak{b})n \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{b} \in \mathcal{A}, n \in \mathcal{N}$, then Lemma 1(ii) assures that

$$n \diamond \mathfrak{b} = 0 \text{ or } n \in \mathcal{Z}(\mathcal{N}) \text{ for each } \mathfrak{b} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}. \qquad 17$$

If there exists $n_0 \in \mathcal{N}$ such that $\mathfrak{n}_0 \diamond \mathfrak{b} = 0$ for each $\mathfrak{b} \in \mathcal{A}$, it follows that $\mathfrak{n}_0\mathfrak{b} = -\mathfrak{b}\mathfrak{n}_0$ for each $\mathfrak{b} \in \mathcal{A}$, so $\mathfrak{n}_0\mathfrak{n}\mathfrak{b} = -\mathfrak{n}\mathfrak{b}\mathfrak{n}_0$ for each $\mathfrak{b} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$ that is $[-\mathfrak{n}_0,\mathfrak{n}]\mathcal{A} = \{0\}$ for each $\mathfrak{n} \in \mathcal{N}$, it follows that $-\mathfrak{n}_0 \in \mathcal{Z}(\mathcal{N})$ by Lemma 2(i), substituting $-\mathfrak{n}_0$ for $\mathfrak{n}$ in Eq.16 forces $(-2\mathfrak{b})(-\mathfrak{n}_0) \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{b} \in \mathcal{A}$ and using Lemma 1(ii) getting $\mathfrak{n}_0 = 0$ or $-2\mathfrak{b} \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{b} \in \mathcal{A}$. Hence Eq.17 becomes

$-2\mathfrak{b} \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{b} \in \mathcal{A}$ or $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

If $-2\mathfrak{b} \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{b} \in \mathcal{A}$, then $-2\mathfrak{b}\mathfrak{n} \in \mathcal{Z}(\mathcal{N})$ for each $\mathfrak{b} \in \mathcal{A}, \mathfrak{n} \in \mathcal{N}$, two torsion freeness of $\mathcal{N}$ and Lemma 1(ii) forces that $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$.

Similarly (ii) can be proved.

Corollary 21: If $\mathcal{N}$ is two torsion-free and $d \neq 0$ is a left derivation on $\mathcal{N}$, then $\mathcal{N}$ is a $\mathcal{C}.\mathcal{R}$ if $d(\mathfrak{n} \circ \mathfrak{m}) = \mathfrak{n} \circ \mathfrak{m}$ for each $\mathfrak{n},\mathfrak{m} \in \mathcal{N}$.

Conclusion:

The author arrives at very interesting results about the commutativity of near-ring by using semigroup ideals and generalized left derivations involving some algebraic identities. Our results generalize many results on left derivations.

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- Conflicts of Interest: None.

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تعميم الاشتقاقات اليسارية مع متطابقات عل الحلقات المقتربة

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الخلاصة:

في هذا العمل قدمنا تعاريف لمفاهيم جديدة والتي تسمى : الاشتقاقات اليسارية وتعميم الاشتقاقات اليسارية في الحلقات المقتربة الاولية الثلاثية. اضافة الى ذلك قمنا بدراسة ابدالية الحلقات المقتربة الاولية الثلاثية عند وجود بعض المتطابقات الجبرية على تعميم الاشتقاقات اليسارية.

الكلمات المفتاحية: الاشتقاقات، تعميم الاشتقاقات اليسارية، الاشتقاقات اليسارية، الحلقات المقتربة الاولية الثلاثية، مثاليات شبه الزمرة.