Mathematical treatment for Soliton-soliton interactions in mono-mode optical Fiber

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Abstract

Mutual interaction of two solitons with equal amplitude ratio (K=1.0) of initial pulse separation (τ_0=3.5pw) propagate in mono-mode optical fiber (30,90)km is studied analytically.

The examination of propagation of two solitons with unequal amplitude ratio(K=1.1) initially separated by (τ_0=3.5pw) is studied also.

Results show that solitons of equal amplitude coalesce into one pulse at $\Psi=\pi$ and eventually separate to the initial state at $\Psi=2\pi$ and so on.

Launching solitons with unequal amplitude i.e (K=1.1) is the simplest way to reduce soliton interaction in order to maintain high bandwidth (10Gbits/s) in communication system. Also the same study is done numerically using Split Step Fourier Method.

Introduction:

Optical solitons **[1,2]** are desirable for extra –high bit rate transmission systems where the effect of fiber dispersion can be balanced by the nonlinear Kerr effect. However, an undesirable effect of the nonlinearity of the refractive index is to cause mutual interaction **[3-5]** between pulses if they are launched close together. In1981,karpman and solovev first considered the two –soliton interaction in their study of the nonlinear schrödinger equation (NLS) by means of single –soliton perturbation theory.

Separating the neighboring pulses to avoid such interaction, results in the degradation of the system bandwidth **[6]** depending on the of initial pulse separation (τ_0) .

Several ways for reducing the effect of the interaction has been proposed. For example Chu and Desem have suggested the use of Gaussian shaped pulses instead of solitons [7]. This has the advantage that the mutual interaction between pluses is reduced and consequently increases the available bandwidth significantly. It has also shown that launching the pulses with an initial phase difference **[8-10]**, or utilizes the higher-order dispersion of the fiber **[11,12]** can lead to a reduction in the interaction. Another method which is considered to be the most stable and simplest method for reduction soliton interaction by launch the solitons with unequal amplitude and equal phases **[13,14]**. This will result in a stable oscillatory system.

Theoretical Background:

The soliton propagation in a mono-mode lossless fiber is described by the nonlinear schrödinger equation

$$i\frac{\partial q}{\partial z}+\frac{1}{2}\frac{\partial q^2}{\partial \tau^2}+|q|^2 q=0\,\ldots\ldots\ldots\ldots\ldots(1)$$

In which fiber loss and third order dispersion are ignored. Z and τ are normalized distance and time, respectively, q is the envelope of the light pulse. For the case of two solitons, the exact solution of eq. (1) can be obtained by the inverse scattering method of Zakharov and shabat **[15]**. The general solution can be simplified as follows**[14]**

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$$q(\tau,z)= Q\{\eta_1 \operatorname{sech}\eta_1(\tau+\gamma_0)e^{i\eta_1^2 z/2} + \eta_2 \operatorname{sech}\eta_2(\tau-\gamma_0)e^{i\eta_2^2 z/2}\} \quad \ldots\ldots\ldots\ldots(2)$$

$$Q=\left\{\frac{(\eta_2^2-\eta_1^2)}{(\eta_2^2+\eta_1^2)-2\eta_1\eta_2\begin{bmatrix}\tanh a_1 \tanh a_2\\ -\sec ha_1 \sec ha_2 \cos\Psi\end{bmatrix}}\right\} \quad \ldots\ldots\ldots\ldots(3)$$

$$a_{1,2}=\eta_{1,2}(\tau\pm\gamma_0)$$

$$\Psi=\frac{(\eta_2^2-\eta_1^2)z}{2} \quad \ldots\ldots\ldots\ldots(4)$$

The two sech functions in the parenthesis of Eqn.2 describe the propagation of the two solitons if mutual interaction is absent. In this case the separation between the solitons is maintained at a constant distance $2\gamma_0$ while the width and amplitude of each soliton is determined by η_1 or η_2 .The two pulses described by eqn.2 undergo an interaction which is periodic in z through $\cos\Psi$ in Q. The mutual interaction is described by the function Q in eq. (3). Let us examine two cases in more detail:

(a) Solitons with equal amplitudes:

In this case the width and amplitude of each soliton is determined by η_1 or η_2.The eigenvalues are given by [14]

$$\eta_{1,2}\cong\left|1+\frac{2\tau_0}{\sinh(2\tau_0)}\pm\sec h(\tau_0)\right| \ldots(5)$$

$\gamma_0=0$

Two pulses initially separated by τ_0 , where τ_0 is the initial soliton separation in unit of the effective width of solitons(pw), then coalesce into one pulse π degree later ,then they separate and revert to two soliton with separation τ_0 at $\Psi=2\pi$ and so on .

(b) Solitons with unequal amplitudes:

In this case, the amplitude of two solitons is written as [14]

$$\eta_{1,2}\cong\left(\frac{K+1}{2}+\frac{2\tau_0 K^{1/2}}{\sinh(2\tau_0 K^{1/2})}+(K^{1/2}-1)\sec h(K\tau_0)\right)\pm\left(\frac{K-1}{2}+\sec h(K\tau_0)\right) \quad \ldots\ldots\ldots(6)$$

$$\gamma_0\cong\tau_0-\left(1-\frac{2\sec h(K\tau_0)}{K}\right)\left(\frac{1+K}{2K}\right)\ln\left(1-\frac{1+K}{1-K}\right) \quad \ldots\ldots(7)$$

$\tau_0=3.5$ and $K=1.1$, K being the ratio of the pulse amplitudes.

Numerically the solution of eq.(1) can be obtained by using Split Step Fourier Method (SSFM) **[16,17,18]**. The initial condition of the form is given by **[14]**

$$q(0,\tau)=\sec h(\tau-\tau_0)+K\sec h[K(\tau-\tau_0)]\exp(j\theta) \quad \ldots\ldots(8)$$

Results and conclusions:

Using MATLAB environment to represent eq.(4) and eq.(3),where MATLAB has a facility to solve equations with complex variables. Fig.(1) shows that the mutual interaction function Q close to 16 for the case of solitons with equal amplitudes ($\eta_1=\eta_2$) i.e K =1,while Q close to 12 for the case of solitons with unequal amplitudes ($\eta_1\neq\eta_2$) i.e K=1.1 as shown in fig.(4),this means that the mutual interaction Q is reduced by launching solitons with unequal amplitudes . The trajectories of first – order bright solitons with initial separation (τ_0 =3.5 pw) and amplitudes ratio (K=1,1.1) are shown in figure(2,3,5,6) in fiber of (40,90)km length respectively. Initially the separation between neighboring pulses in the soliton transmission decreases as the pulses propagate inside the fiber, then the pulses will collapse with other forming oscillating system at $\psi=\pi$ at the periodic collapse length L_{osc} , then they eventually separates from each other and the separation return to the initial state with separation τ_0 at $\psi = 2\pi$. This behavior will repeat periodically along the fiber. Numerically figs(7,8)shows soliton interaction of two pulses of equal amplitudes and with τ_0=(2,3.5)pw, while fig(9) shows soliton interaction of unequal amplitudes pulses

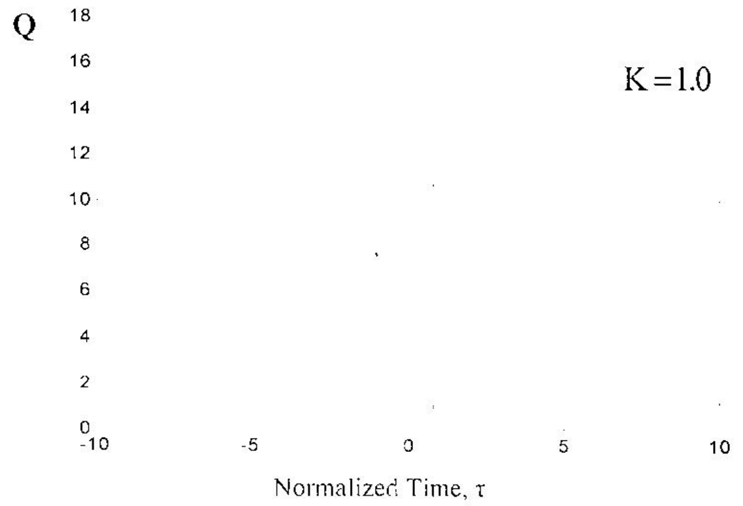

Fig. (1): Soliton interaction Q as a function of τ for equal amplitude solitons. Initial separation τ_0=3.5pw

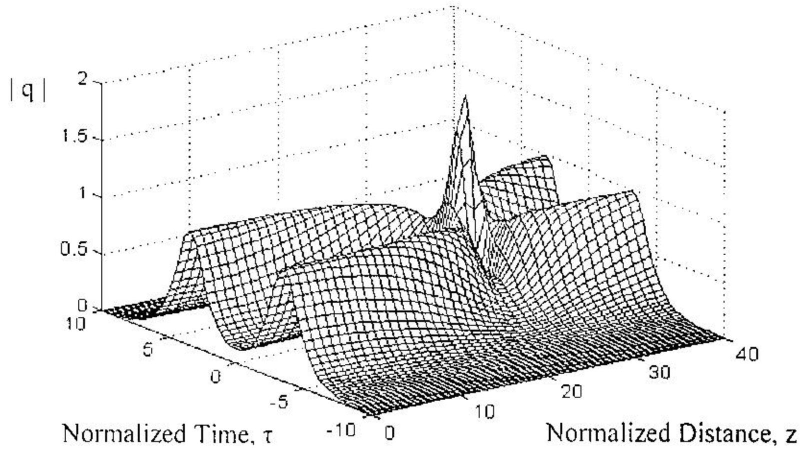

Fig.(2): Soliton interaction with two equal amplitude pulses, initial pulse separation τ_0=3.5pw in mono-mode fiber 30km

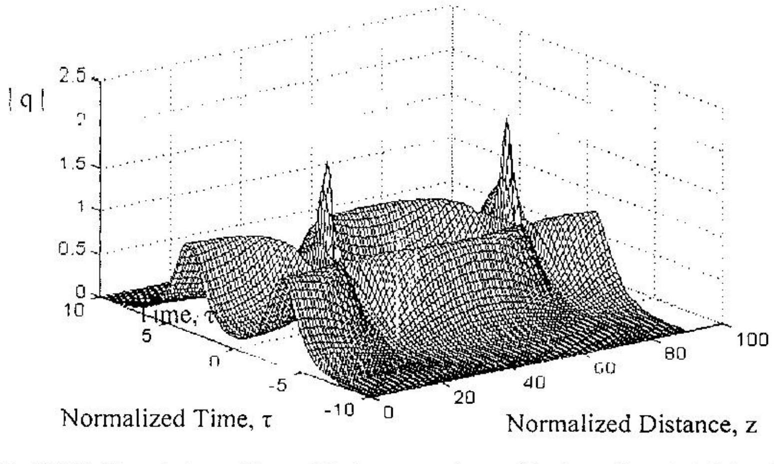

Fig.(3): Soliton interaction with two equal amplitude pulses, initial pulse separation τ_0=3.5pw in mono-mode fiber 90km.

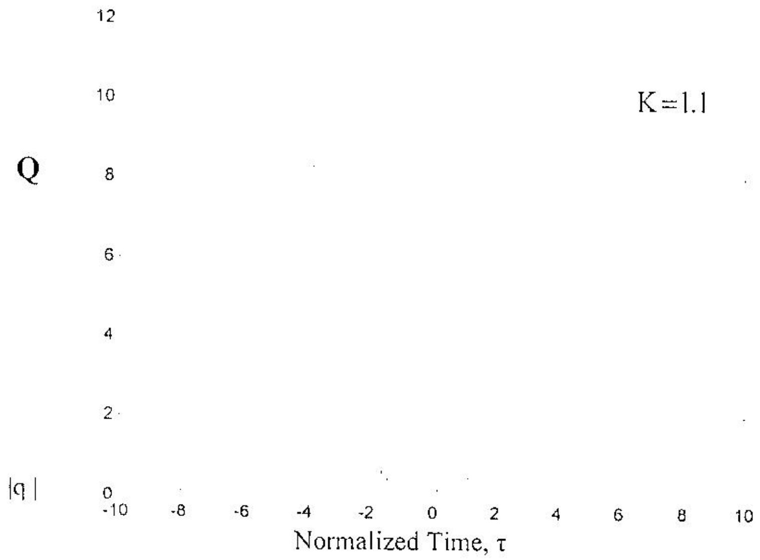

Fig. (4) :Soliton interaction function Q as a function of τ for equal amplitude solitons. Initial pulse separation τ_0=3.5pw.

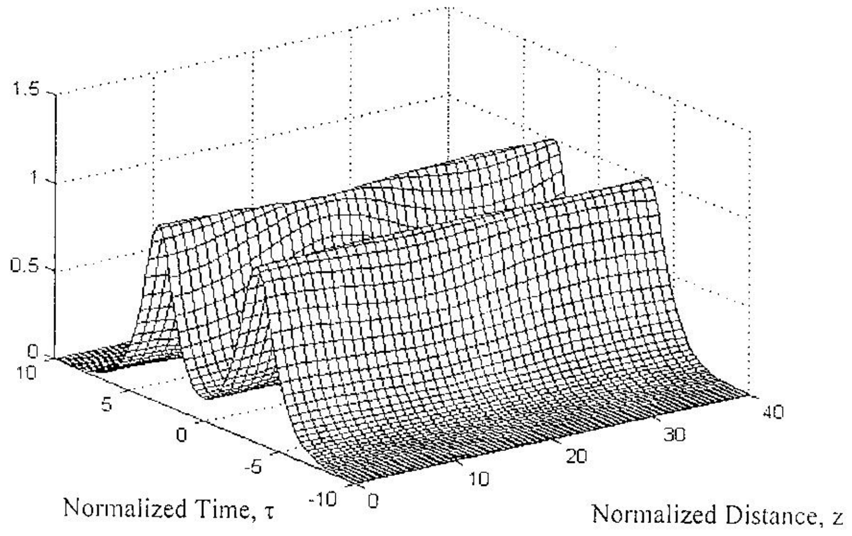

Fig. (5): Soliton interaction with two unequal amplitude pulses, initial pulse separation τ_0=3.5pw in mono-mode fiber 30km.

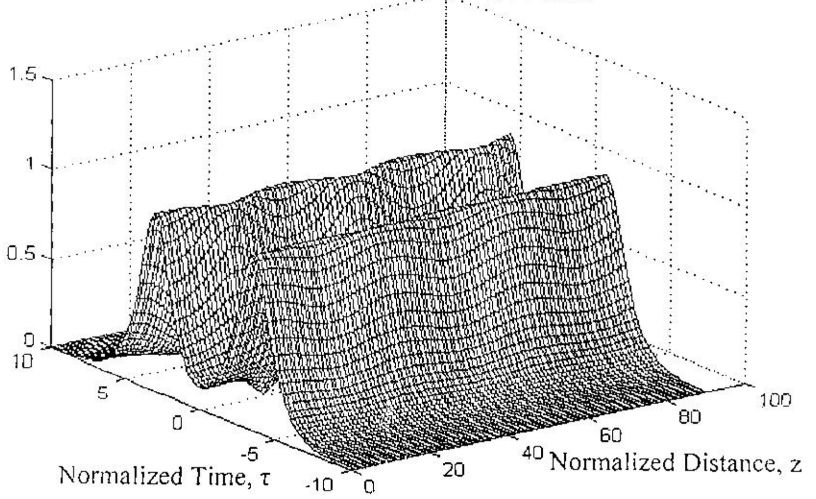

Fig. (6): Soliton interaction with two unequal amplitude pulses, initial pulse separation τ_0=3.5pw in mono-mode fiber 90km.

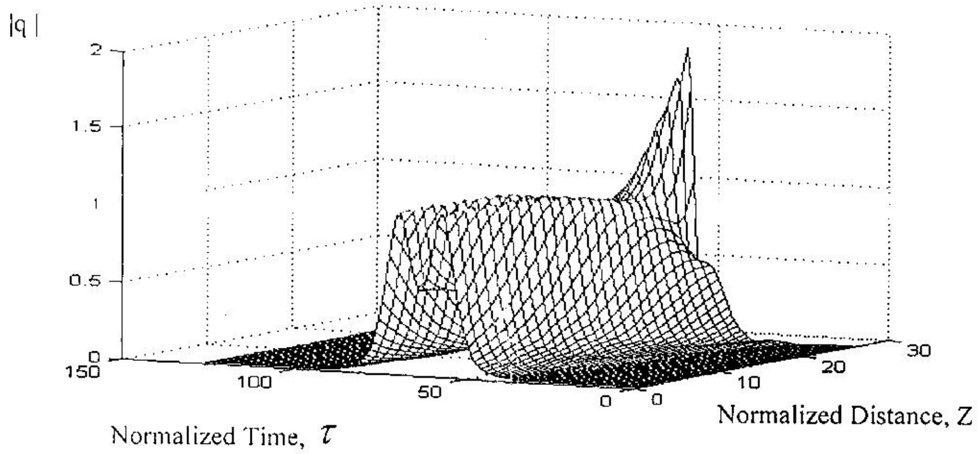

Fig. (7): Soliton interaction with two equal amplitude pulses, initial pulse separation τ_0=2.0pw in mono-mode fiber 30km.

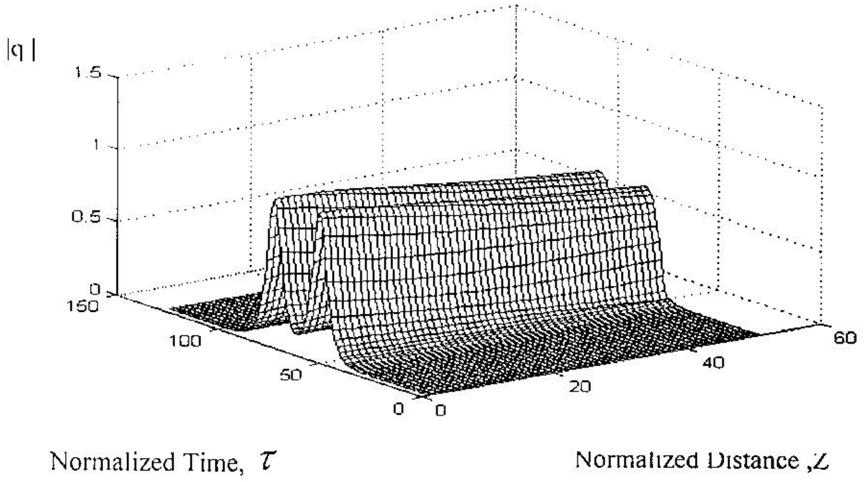

Fig.(8): Soliton interaction with two equal amplitude pulses, initial pulse separation τ_0=3.5pw in mono-mode fiber 30km.

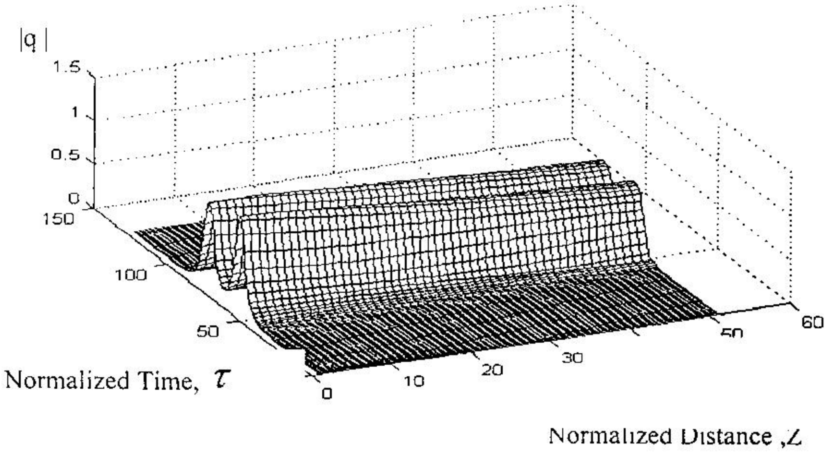

Fig.(9):Soliton interaction with two unequal amplitude pulses K =1.1, initial pulse separation τ_0=3.5pw in mono-mode fiber 30km.

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معالجة رياضية لتفاعل سوليتون – سوليتون في ليف بصري أحادي النمط

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الخلاصة :

درس رياضيا تفاعل سوليتون – سوليتون بنسبة سعات متساوية (K=1.0) في ليف بصري أحادي النمط بحيث أن المسافة الابتدائية بين النبضات السوليتونية المنتشرة τ_0=3.5 pw وبطول (30,90)km.

درس أيضا انتشار سوليتون – سوليتون بنسبة سعات غير متساوية (K=1.1) بحيث أن المسافة الابتدائية بين النبضات السوليتونية τ_0=3.5 pw.

أظهرت النتائج بان السولتيونات ذات السعات المتساوية تندمج مع بعضها مكونة نبضة واحدة عند $\Psi=\pi$ ثم تنفصل عن بعضها لترجع للحالة الابتدائية عند $\Psi=2\pi$ وتستمر هكذا.

إن افضل الطرق لتقليل التفاعل بين السولتيونات عن طريق إدخال النبضات السوليتونية بسعات مختلفة أي بنسبة سعات (K=1.1) من أجل الحصول على أعلى عرض نطاق ترددي 10Gbit/s في منظومة الاتصالات.

نفس الدراسة أجريت تحليليا باستخدام طريقة Split Step Fourier Method (SSFM).