Elastic Longitudinal Electron Scattering Form Factors of ^{11}B

*Nadia . M. Adeeb

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Abstract

Elastic longitudinal electron scattering form factors have been studied for $^{11}B(J^{\pi}=3/2^{+}, T=1/2)$ 0.0 MeV in the frame work of shell model. The multipoles are indicated as C0 and C2 . Using the Cohen-Kurath (CK) interaction to give the 1p-shell model space wave functions.

Core-polarization effects (CP) ; (the effects from outside the 1p-shell model space) are included through the first --order perturbation theory. To get a good agreement with the experimental data an extended model (The admixture of 2p-shell wave functions with the 1p-shell) are used in our calculations.

Introductin

Electron scattering has proven itself as one of the most effective methods of studying the properties of the energy levels of atomic nuclei [1]; so it is a powerful tool for studying nuclear structure.

Elastic electron scattering has been mainly employed to measure the charge radius of nuclei and their nuclear surface thickness.[2].

Calculations have been performed on the shell-model in order to obtain systematic explanation of the lower-laying levels in the nuclei , such as ^{11}B nucleus [3,4].

The study of electron scattering has provided important information about the electromagnetic currents inside the nuclei [5].

The 1p-shell has been a testing ground for nuclear models[6], but inspite of it's success in describing the static properties of nuclei ,it failed to describe electron scattering data at high momentum transfer[4]. So an extension of the space(which is restricted to the 1p-shell model space $(0+2)\hbar\omega$) was used to describe the electron scattering form factors.

Coulomb form factors of C2 transitions in several selected p-shell $(^{6}Li, ^{7}Li, ^{10}B, ^{12}C, ^{13}C)$ are discussed taking into account core-polarization effects [7]. Inelastic form factors of ^{6}Li has been studied by Radhi [8]. Elastic magnetic form factors has been discussed with core-polarization effects for ^{11}B by Adeeb.[9].

Also Salman[10] studied longitudinal form factors for $(^{6}Li, ^{7}Li, ^{10}B, ^{12}C)$ by using Tassi-model. Kharrofah[11] calculated longitudinal elastic and inelastic form factors for$(^{6}Li, ^{7}Li, ^{9}Be)$ by using core-polarization effects. These studying gave a good agreement with the experimental data.

The effect from outside the model space (CP) has to be considered in this work for ^{11}B as a core of (^{4}He) and seven nucleons over the 1p-shell to determine the success of the shell model wave functions in describing electron scattering data.

The core-polarization effect

* Department of Physics , College of Science ,University of Baghdad ,Baghdad ,IRAQ.

combines the shell model wave functions and highly excited states using first –order perturbation theory . The single particle states has been described by using Harmonic Oscillator(HO) model with the parameter (b) chosen to reproduce the root mean square (rms) charge radius, which is for ^{11}B ; b=1.611 fm , [12].

The calculations of 1p-shell model space wave functions depend on two – body interaction of (CK) [13] and the (CP) are calculated with the Modified Surface Delta Interaction(MSDI), [14]. Our calculations depends on Dr.Radhi program (Cp- program) which was written in Fortran Language.

THEORY

The cross-section for the scattering of an electron through an angle (θ) with the excitation of the nucleus of charge (Z)from the initial state $|J_i\rangle$ to the final state $|J_f\rangle$ can be written in the first Plane Wave Born Approximation (PWBA) [15].

$$\frac{d\sigma}{d\Omega}=\sigma_M\eta^{-1}\left\{\left(\frac{q_\mu}{q}\right)^4 F_L^2(q)+\left[\frac{1}{2}\left(\frac{q_\mu}{q}\right)^2+\tan^2\left(\frac{\theta}{2}\right)\right]F_T^2(q)\right\} \text{--}(1)$$

where (σ_M) is the cross-section of scattering which is given by:

$$\sigma_M=\left[\frac{Z\alpha\hbar c\cos\frac{\theta}{2}}{2E\sin^2\frac{\theta}{2}}\right]^2 \text{ --------------- } (2)$$

where ($\alpha = e^2/\hbar c$) is the fine structure ,

(η) is the recoil correction factor for nuclear charge (Ze) .

q is the three momentum transfer, q_μ is the four momentum transfer

$q_\mu = q^2 - \omega^2 = 4E_i E_f \sin(\theta/2) \geq 0$

$\omega = E_f - E_i$: is the difference between the scattered electron energy and the incident one.

The nuclear structure information is contained in the longitudinal form factor $F_L(q)$,which representing the scattering from the nucleon charge density and $F_T(q)$ is the transverse form factor which represents the scattering from the current density [16].

$$F_L^2(q)=\frac{4\Pi}{Z^2}\frac{1}{(2J_i+1)}\sum_{J\geq 0}\left|\left\langle J_f\left\|\hat{T}_J^{coul}(q)\right\|J_i\right\rangle\right|^2 \text{---}(3)$$

$$F_T^2(q)=\frac{4\Pi}{Z^2}\frac{1}{(2J_i+1)}\sum_{J\geq 1}\left\{\left|\left\langle J_f\left\|\hat{T}_J^{mag}(q)\right\|J_i\right\rangle\right|^2+\left|\left\langle J_f\left\|\hat{T}_J^{ele}(q)\right\|J_i\right\rangle\right|^2\right\} \quad (4)$$

In this work we calculated the longitudinal form factor using taken eq.(3) where $\hat{T}_J^{coul}$ is the coulomb operator and C_J is the multipole operators related to the charge density operator $\hat{\rho}(\vec{r})$ [15].

$$\hat{T}_{JM}^{coul}(q)=\int d\vec{r}\, j_J(qr)Y_{JM}(\hat{r})\hat{\rho}(\vec{r}) \text{ -(5)}$$

where $j_J(qr)$ is the spherical Bessel function and Y_{JM} is spherical harmonic.

The angular momentum selection rule is :

$|J_i - J_f| \leq J \leq J_i + J_f$

The reduced matrix element is expressed as the sum of the product of the elements of the one –body density matrix elements times the single particle matrix elements[9].

$$\left\langle J_f\left\|\hat{T}_{J\eta_z}^{\eta}\right\|J_i\right\rangle=\sum_{a,b} OBDM^{J\eta_z}(J_i,J_f,a,b)\left\langle a\left\|\hat{T}_{J\eta_z}^{\eta}\right\|b\right\rangle$$

--(6)

where $\Gamma_i = J_i T_i$, $\Gamma_f = J_{fi} T_{fi}$, and $\wedge = JT$ are used to denote the quantum numbers in space and isospace coordinates, where $a = jt$, $b = j't$ are the single particle isospins.

The longitudinal (coulomb)form factors describe the nuclear volume distribution of charges so it comes from the interaction of electrons with the charge distribution of nucleus ,where $\eta = L$, and the (OBDM) are obtained from the work of Lee and Kurath [13].

The effect of core-polarization is calculated by using the first order perturbation theory with harmonic oscillator wave functions.

The single particle matrix element is the sum of the Model-Space (MS) contribution in (1p-shell)space and the core-polarization (CP) contribution which is given by:

$$\left\langle a \left\| \hat{T}_\wedge \right\| b \right\rangle = \left\langle a \left\| \hat{T}_\wedge \right\| b \right\rangle_{MS} + \left\langle a \left\| \delta \hat{T}_\wedge \right\| b \right\rangle_{CP} \quad ---(7)$$

The (CP) contribution is written as [12]:

$$\left\langle a \left\| \delta \hat{T}_\wedge \right\| b \right\rangle = \sum_{a_1 a_2 \Gamma} \frac{(-1)^{b+a_2+\Gamma}}{e_b - e_a - e_{a_1} + e_{a_2}} (2\Gamma+1) \begin{Bmatrix} a & b & \wedge \\ a_2 & a_1 & \Gamma \end{Bmatrix} \sqrt{(1+\delta_{a_1 a})(1+\delta_{a_2 b})} \left\langle a a_1 \middle| V_{rev} \middle| b a_2 \right\rangle \left\langle a_2 \left\| \hat{T}_\wedge \right\| a_1 \right\rangle$$

+ terms with a_1 and a_2 exchanged--(8)

The index a_1 runs over particle states and a_2 over hole states where (e) is the single particle energy. For the residual interaction the Modified Surface Delta Interaction (MSDI) Adopt with the strength parameter.

The calculations of form factors sometimes do not agree with the experimental data for that reason an extended model (The admixture of 2p-shell wave functions with that of 1p-shell) was used to get good agreement.

RESULTS , DISCUSSION AND CONCLUSIONS

The nucleus ^{11}B is considered as an inert (^{4}He) core plus seven nucleons distributed over ($1p_{3/2}$, $1p_{1/2}$) shell with ($b_{rms} = 1.611 fm$) .

The elastic longitudinal form factor of ^{11}B for the transition to the $J^{\Pi} = 3/2^{+}, T = 1/2$ (0.0MeV) is of mixed multi-polarity of (C0+C2) as shown in fig.(1) with the results of 1p-shell model calculations without (CP) as (solid curve) in comparison with the experimental data from Ref.[2]

In fig .(2) the contribution of (CP) was given as (Cross-symbol)and the 1p-shell model is drawn as (dashed curve), while the total form factors which consist of (1P+CP) is shown as(solid curve) .

In this figure the data are well described ,where the(CP) effects brings the calculations up near to the experimental data in the range of $q = 0 \rightarrow 1.2 fm^{-1}$ of momentum transfer and at the $q = 1.6 \rightarrow 2 fm^{-1}$, at the high momentum transfer the theoretical results falls too rapidly compared to the measured values.

In fig.(3) the (solid curve) represents the admixture of the model space (1p) with (CP) and the extended model of (2p)

Which gives (CP+1p+2p) ,where the (long dashed curve)

Shows the (1p+CP) only and the (short dashed curve) shows the model space of (1p) .

The contribution of (2p) brings the calculated data close to the experimental one in the first region of $q = 0 \rightarrow 1.3 fm^{-1}$ and at

$q = 2 \to 3\,fm^{-1}$, but the calculated data underestimate the experimental data at $q = 1.3 \to 1.9\,fm^{-1}$.

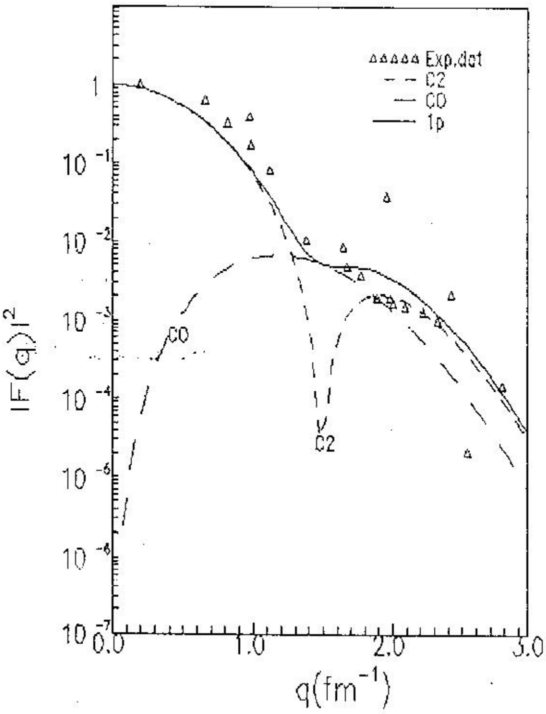

Fig (1): Elastic Longitudinal form factors from ^{11}B without core-polarization effects

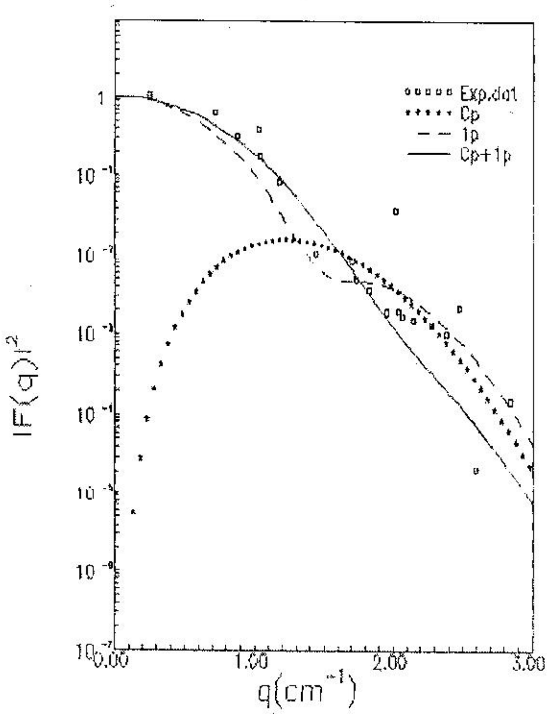

Fig (2): Elastic Longitudinal form factors from ^{11}B with core-polarization effects

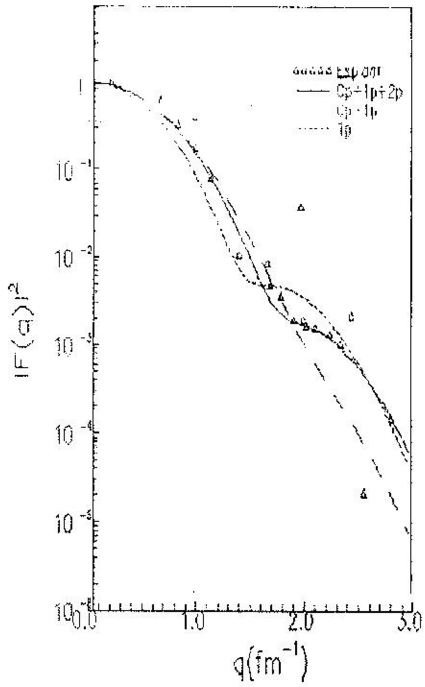

Fig (3) Elastic Longitudinal ^{11}B with core-polarization effects and using an extended model space (2p).

The most important conclusions of the present work can be briefly summarized as follow:

1- Core-polarization effects introduced by first order perturbation theory gives reasonable description of the data especially in the first region of (q) where (q ≤1.2).
2- Longitudinal elastic electron scattering form factors are fairly well reproduced using the extended model space (2p) for ^{11}B. In general , an improvement in data fitting is achieved compared to the results obtained using the restricted model space.

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عوامل التشكل للاستطارة الالكترونية الطولية المرنة من نواة البورون B''

نادية محمد اديب*

* د/ جامعة بغداد/ كلية العلوم/ قسم الفيزياء

الخلاصة

تمت دراسة الاستطارة الالكترونية الطوليـــة المرنـــة لعوامـــل التشـــكل لنـــواة البـــورون ^{11}B $(J^{\pi}=\frac{3}{2}^{+},T=\frac{1}{2})$ (٠،٠) م.أ. ف في إطار العمل بأنموذج القشرة ، وتم تحديد متعدد الأقطاب بالعوامل C0 , C2. باستخدام تفاعل (CK) من (كوهين كوراث) وذلك للحصول على الدوال الموجية لأنمـــوذج القشرة-1p . وقد أدخلت تأثيرات استقطاب القلب (التأثيرات من خارج أنمـــوذج القشـــرة – 1p) فـــي حسابات المرتبة الأولى لنظرية الاضطراب وللحصول على نتائج توافق القـــيم العمليـــة اســـتخدمنا فـــي حساباتنا الأنموذج الموسع (خليط الدوال الموجية لأنموذج القشرة – 1p مع القشرة – 2p).