Study the Nuclear Structure of Some Cobalt Isotopes

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Abstract:

The nuclear structure of some cobalt (Co) isotopes with mass number A=56-60 has been studied depending on the effect of some physical properties such as the electromagnetic properties effects, such as, elastic longitudinal form factors, electric quadrupole moments, and magnetic dipole moments. The fp model space is used to present calculations using GXFP1 interaction by adopting the single particle wave functions of the harmonic oscillator. For all isotopes under consideration, the 40Ca nucleus is regarded as an inert core in fp model space, while valence nucleons are moving through 1f7/2, 2p3/2, 1f5/2, and 2p1/2 orbits. The effects of core-polarization are obtained by the first order core polarization through a microscopic theory. In addition, the core polarization was added using the effective charge and effective g factors to calculate quadrupole and magnetic moments, respectively. The results obtained are compared to experimental data that is accessible.

Keywords: Cobalt isotopes, Elastic longitudinal form factors, Fp model space, Magnetic dipole moments, Quadrupole moments.

Introduction:

The nucleus description by the shell model is based on the configurations of individual particles, and all nuclear structure information is dependent on them. In this model the features of a nucleus containing several nucleons or holes outside a closed shell are described in an initial approximation by an inert nucleus and some nucleons that can move in a given space, which interact with the nucleus and each other by a "residual" reaction 1,2.

In addition to stable nuclei, recent experiments have demonstrated exotic nuclei as one of the most nuclear physics critical topics, revealing a wide range of new phenomena. Where nuclear physics is expanding rapidly due to the massive increase in the rare isotopes available that contain different neutron-proton ratios than stable nuclei 3.

Electron scattering is influenced by nuclear charge dispersion, also the scattering of electrons from nuclear electromagnetic current distributions reveals detailed details about nuclear convection in the ground state and magnetization currently available distributions 4-6.

The quadrupole and magnetic moments are frequently used as a test to see if the model space is adequate and the parameterizations are suitable, so one of the most noteworthy features of knowing the structure of exotic nuclei is knowledge of the electromagnetic properties, such as the electric quadrupole and magnetic moments of nuclei 7,8.

In the current work, the single particle wave functions of the harmonic oscillator (HO) for the shell model will adopt to calculate the magnetic (μ), quadrupole (Q) moments and elastic longitudinal form factors for 56,57,58,59,60Co isotopes. The wave function for the GXFP1 interaction 9 in the fp shell model space was obtained by using the OXBASH shell model program 10, which calculates the one-body density matrix (OBDM) elements in the spin-isospin formalism. The core-polarization (CP) effects are included through effective charge using the Bohr-Mottelson formula 11, in addition to obtaining the effective charge from a fit to spectroscopic data such as modified surface delta interaction theory (MSDI) 12.
Theoretical framework

The nuclear matrix element of the electromagnetic transition operator ($\hat{O}$) between the initial and final states is equal to the sum of the components of the one-body density matrix (OBDM) multiplied the single-particle matrix elements:

$$\langle \Lambda_f | \hat{O} | \Lambda_i \rangle = \sum_{j_fj_i} OBDM (j_f,j_i,\Lambda_f,\Lambda_i)^{JT} \times$$

$$\langle j_f t | \hat{O}_{j_t} | j_i \rangle ,$$

where states $|\Lambda_i\rangle$ and $|\Lambda_f\rangle$ define the beginning and final states shell model-space wave functions.

Through a microscopic theory, one considers the core nucleons and the cut-out region of space, and then higher-energy wave functions and configurations as first-order perturbations, which are referred to as core polarization effects. The electromagnetic operator $\hat{O}_A^n$ 's are stated as a contribution from the model space (MS) and core polarization (CP), as shown below:

$$\langle \Lambda_f | \hat{O} | \Lambda_i \rangle_{CP} = \langle \Lambda_f | \hat{O}_A^n \hat{O}_A^n | \Lambda_i \rangle_{MS} +$$

$$\langle \Lambda_f | \delta \hat{O}_A^n \hat{O}_A^n | \Lambda_i \rangle_{CP} .$$

The CP effects are computed to use the MSDI residual effective interaction.

$$\langle \Lambda_f | \delta \hat{O}_A^n \hat{O}_A^n | \Lambda_i \rangle_{CP} = \sum_{\alpha,\beta} \hat{X} \Lambda_f | \alpha, \beta \rangle \langle \alpha | \delta T_{\Lambda} | \beta \rangle ,$$

where $Q$ is the operator for projection that projects the model space onto it. Between the beginning (i) and final (f) states, the electron scattering form factor including momentum transfer $q$, is given by:

$$|F(Q,q)|^2 = \frac{4\pi}{Z^2(2J_f+1)} |\langle f | \tilde{T}(\eta, q) | i \rangle| F_{cm}(q) F_{fs}(q)|^2 ,$$

where $\tilde{T}$ is electron scattering operator between initial and final state and $\eta$ is selecting the longitudinal or transverse magnetic form factors. $F_{fs} = [1 + \left(\frac{q}{4.33}\right)^2]^{-2}$ is the finite size nucleon form factors and $F_{cm} = e^{q^2 b^2/4A}$ is an adjustment for the shell model’s deficit of translational invariance. Whereas $A$ is the mass number, and $b$ is the size parameter of the harmonic oscillator (HO) obtained from a global formula:

$$b = \frac{hc}{\sqrt{M_p c^2 h \omega}}.$$
Table 1. Calculated $Q$ moments for $^{56,57,58,59,60}$Co isotopes using GXFPI interaction is compared to experimental data provided \(^18\). CP is used to calculate the effective charges using MSDI theory and with B.M formula.

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>$f^T$</th>
<th>$b$ (fm)</th>
<th>$Q_{bare}$ (e fm(^2))</th>
<th>$e_p, e_n$ C.P effective charge</th>
<th>$Q_{Theory}$ (e fm(^2))</th>
<th>$e_p, e_n$ B.M effective charge</th>
<th>$Q_{B.M.}$ (e fm(^2))</th>
<th>$Q_{exp.}$ (e fm(^2))</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{56}$Co</td>
<td>$^+$ 1</td>
<td>2.032</td>
<td>10.61</td>
<td>1.290,0.290</td>
<td>15.83</td>
<td>1.161,0.780</td>
<td>18.08</td>
<td>+25.0(9)</td>
<td></td>
</tr>
<tr>
<td>$^{57}$Co</td>
<td>$^-$ 3</td>
<td>2.036</td>
<td>12.05</td>
<td>1.332,0.332</td>
<td>23.13</td>
<td>1.152,0.761</td>
<td>29.21</td>
<td>+54.0(10)</td>
<td></td>
</tr>
<tr>
<td>$^{58}$Co</td>
<td>$^+$ 2</td>
<td>2.042</td>
<td>8.35</td>
<td>1.340,0.340</td>
<td>16.23</td>
<td>1.144,0.742</td>
<td>20.53</td>
<td>+23.0(3)</td>
<td></td>
</tr>
<tr>
<td>$^{59}$Co</td>
<td>$^-$ 5</td>
<td>2.046</td>
<td>9.320</td>
<td>1.333,0.333</td>
<td>16.98</td>
<td>1.135,0.725</td>
<td>20.51</td>
<td>+42.0(3)</td>
<td></td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>$^+$ 3</td>
<td>2.051</td>
<td>13.95</td>
<td>1.309,0.309</td>
<td>22.17</td>
<td>1.128,0.708</td>
<td>24.69</td>
<td>+46.0(6)</td>
<td></td>
</tr>
</tbody>
</table>

The results of $Q$ for $^{56}$Co with CP and BM effective charges, are 15.83 e fm\(^2\) and 18.08 e fm\(^2\), accordingly. The value of $Q$ with CP agrees with the experimental value +25.0(9) e fm\(^2\) with the error.

For $^{57}$Co isotope, the theoretical quadrupole moment with CP effective charges $e_p=1.332$ e, $e_n=0.332$e is equal to 23.13 e fm\(^2\), and the $Q_{B.M.}$ with effective charges $e_p=1.152$ e, $e_n=0.761$e is 29.21 e fm\(^2\) well underestimate the experimental value +54.0(10)e fm\(^2\) this is due to increase in the number of occupation nucleons outside the core. The calculated $Q$ moments for $^{58}$Co with MSDI theory and B.M effective charges are 16.23 and 20.53 e fm\(^2\), respectively. With a prolate deformation, the value of $Q$ using the Bohr-Mottelson formula is close to the measured value +23.0(3) e fm\(^2\) of Ref \(^18\) within the error.

The $Q$ moments values computed with CP using MSDI theory for $^{59}$Co and $^{60}$Co are 16.98 and 22.17 e fm\(^2\), respectively, as well as by employing B.M effective charges are 20.51 and 24.69 e fm\(^2\), respectively, which increases the discrepancy with the experimental +42.0(3) and +46.0(6) e fm\(^2\) with large prolate deformation. The uncoupled neutron is to blame for this mismatch, which explains the enormous prolate deformation.

The calculations $Q$ moments with the experimental values are displayed in Fig.1. All calculation results underestimate the experimental values except at neutron numbers equal 29 and 31 the $Q$ moments agree with the experimental value inside the experimental uncertainty.

Figure 1. Quadrupole moment comparison among the experimental data \(^18\) (green circles) and the calculated value for bare (black diamonds), CP value (red circles), with BM effective charges (blue triangles).

2. Magnetic dipole moments ($\mu$)

The magnetic dipole moments and the g-factor are calculated for Cobalt isotopes studied in the present results. The nucleon g factors that are both free and effective are used to calculate the magnetic moments. The g factors for orbital and spin free nucleons are: $g_1^p=1.06$, $g_1^n=5.055$, $g_2^p=0$, $g_2^n=-3.826$ \(^{12}\), while the effective single-nucleon g1 factors are equal $g_1^p=1.06$, $g_1^n=5.055$, $g_2^p=0$, $g_2^n=-3.19$ and g2 factors are equal $g_2^p=1.15$, $g_2^n=4.748$, $g_3^p=-0.15$ $g_3^n=-3.252$ \(^{19,20}\). In comparison to the existing experimental data \(^{21}\), Table. 2 displays the estimated results for the moments and collective model expectations of $g = \frac{Z}{A}$.

The calculated value of $\mu$ for $^{56}$Co with $g_{free}$ factor 3.319 nm and with the effective nucleon g1 and g2 factors are 3.369 and 2.775nm, respectively, are underestimates the measured value +4.720(10) nm with a positive sign indicates the direction to the z-components are same.
The calculated magnetic moments for $^{51}$Co and $^{58}$Co with free and with effective g factor have the same behaviors and underestimate the measured value +4.720(10) nm and +4.044(8), respectively. Although the code of the shell model is set for spectroscopic features, it isn’t usually optimized for the nuclear charge, convection, or magnetization current densities 19.

Using effective g1 and g2 factor the value of $\mu$ for calculated $^{59}$Co isotopes are close to each other and become overestimated the experimental value. The effective charge decreases but does not eliminate the gap between theory and experiment.

The calculated value of $\mu$ for $^{60}$Co with gfree factor 4.774nm and with the effective nucleon g1 and g2 factors are 4.480 and 3.959 nm, respectively, are overestimates the measured value +3.799(8)nm with a positive sign indicates the direction to the z-components are same.

Table 2. Calculated the $\mu$ moments for $^{56,57,58,59,60}$Co isotopes using GXFP1 interaction in compared to experimental data provided 21.

| Isotopes | $J^T$ | $\mu_{\text{bare}}$ (nm) | $\mu_{g1}$ (nm) | $\mu_{g2}$ (nm) | $\mu_{\text{exp}}$ (nm) | $g_{\text{cal}}=|\frac{\mu_{\text{cal}}}{\mu_{\text{T}}}|$ | $g_{\text{exp}}$ | $g_{\text{~Z / A}}$ |
|----------|-------|--------------------------|------------------|------------------|------------------------|----------------|----------------|----------------|
| $^{58}$Co$_{29}$ | 4$^+$ | 3.319 | 3.369 | 3.111 | +3.85(1) | 1.07 | 0.96 | 0.48 |
| $^{57}$Co$_{30}$ | 7$^-$ | 3.21 | 1.43 | 2.775 | +4.720(10) | 1.59 | 1.35 | 0.47 |
| $^{56}$Co$_{31}$ | 2$^+$ | 3.052 | 2.865 | 2.689 | +4.044(8) | 3.12 | 2.02 | 0.47 |
| $^{55}$Co$_{32}$ | 7$^-$ | 4.964 | 4.922 | 4.974 | +4.615(25) | 1.66 | 1.32 | 0.46 |
| $^{60}$Co$_{33}$ | 5$^+$ | 4.774 | 4.889 | 3.959 | +3.799(8) | 1.23 | 0.76 | 0.45 |

Figure 2. Comparison in magnetic dipole moments among the experimental data 21 (green circles) and the calculated with bare g factors value (black diamonds) and with effective g factors charges g1 (red circles) and g2 (blue triangles).

Fig. 2, represents the comparison of magnetic dipole moments with the neutron numbers. The computed values have the same behaviors for all N values. Fig.3, represents the relationship between the neutron numbers and g-factors. The calculated g-factor has the same forms and overestimates the experimental values the all neutron numbers.

3. Elastic electron scattering

One of the most important methods for determining the electromagnetic properties of a nuclear structure is calculating the form factors 22. The elastic longitudinal electron scattering form factors for stable isotope $^{59}$Co with fp-shell model with mixed configuration is adopted using GXFP1 interactions with the core of $^{40}$Ca plus (A− 20) residual nucleons divided over 1f$_{7/2}$, 2p$_{3/2}$, 1f$_{5/2}$ and 2p$_{1/2}$ orbits. The reason for choosing to calculate the form factors of $^{59}$Co isotope is because of the availability of experimental data.

Fig. 4, shows the individual multipole contributions C0, C2, C4, C6 and the solid black curves representing the total longitudinal form
factors in model space (MS) only which is attributed to the C0 multipole. Since the diffraction minimum for MS is located at momentum transfer $q = 1.0$ and 1.7 fm$^{-1}$ which underestimates the experimental data$^{23}$.

The results are shown in Fig.5, by adding the Core polarization effect using the MSDI theory (red solid curves) and with B.M formula (blue solid curves), the results have given a good description in general and close to total form factors in MS (black solid curves) at all momentum transfers. With the addition of CP effects, there is a noticeable improvement in the form factor using B.M formula for the factors for the second lope, for $q \geq 1.7$ fm$^{-1}$.

**Figure 4. Elastic longitudinal form factors for $^{59}$Co isotope with the contribution of the different multipolarities. The experimental data are taken from Ref. $^{23}$**

**Figure 5. Comparison among the total elastic longitudinal form factors with MS only (black curves) and the MS+CP form factors using the MSDI theory and B.M form. The experimental data are taken from Ref. $^{23}$**

**Conclusions:**

In this work, we have investigated the nuclear structure of some Co isotopes by some electromagnetic properties which were calculated using GXFPI interaction in fp model space. By implementing this model, the high ratio of this including the CP by using microscopic theory and effective charges using B.M formula improves the values of the electric quadrupole moments and is help to interpret the experimental data of form factors. The Q moment is improved by CP, and the experimental values are more accurately described. The effective charges calculated using CP are lower than the usual charges. In general, calculations considering the dispersion property of loosely bound particles demonstrate the lower effect charges in neutron rich nuclei. Because the majority of nucleon spins and orbital momenta pair off, the contribution is nil, the nuclear magnetism as a result of several valence nucleons, CP with MSDI theory and B.M formula has does not effect on the value of a magnetic moment, but effective g-factors, on the other hand, eliminate this disparity for these isotopes.

**Authors' declaration:**

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

**Authors' contributions statement:**

B. S. H. and B. K. R. are in visualizing and designing the study, obtaining data, in addition to analyzing and interpreting the results and writing the manuscript.

**References:**

دراسة التركيب النووي لبعض نظائر الكوبلت

 باسم خلف رجه

قسم الفيزياء، كلية العلوم للبنات، جامعة بغداد، بغداد، العراق.

الخلاصة:

تم دراسة التركيب النووي لبعض نظائر الكوبلت للعدد الكتلي = 50-60 أعداماً على تأثير بعض الخواص الفيزيائية مثل تأثيرات الخواص الكهرومغناطيسية، مثل عوامل التشكل الطولية المرن، وعزم رباعي القطب الكبلي الناري وعزم ثنائي القطب المغناطيسي. تم استخدام نموذج GXFP1 وتفاعل fp تفاعلات الموجة الجسيمية المفردة للعوامل التكافؤية في الخصائص المتبقي. تم استخدام نموذج FPQ1، حيث نظرت تكاملات القيااس عبر مدارين 1f5/2، 2p1/2 و 2p3/2، 1f7/2 تأثيرات استقطاب القلب باستخدام الشحنة الفعالة وعوامل g الفعالة. تم مقارنة النتائج المحاسبية مع القيم العملية المتوقعة.

الكلمات المفتاحية: نظائر الكوبلت، عوامل التشكل الطولية المرن، نموذج الفضاء fp، عزم ثنائي القطب المغناطيسي، العزم الرباعي.