

# Landau damping of dust acoustic solitary waves in nonextensive dusty plasma

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## Abstract

Dust acoustic (DA) solitary waves have been investigated under the influence of Landau damping in space dusty plasma with  $q$ -nonextensive velocity distributed of ions. The effect of the Landau damping, and nonextensive parameter  $q$  of the ions on DA solitary structures has been illustrated by a numerical solution of the Landau damping modified KdV equation. By applying the reductive perturbation technique (RPT) Korteweg-de Vries (KdV) an equation with an additional Landau damping term for our model has been derived. This study showed that the density of dusty particles plays an essential role in appearance or disappearance of DA solitary waves in dusty plasmas, and the nonextensive character of the ions has a noteworthy influence on the Landau damping phenomenon and formed nonlinear structures; the study provides a way to illustrate the physical mechanism of nonlinear propagation of DA solitary waves under Landau damping in the nonextensive distributed plasma in many fields, such as planetary areas, magnetospheres, and space plasma environments.

**Keywords:** Dusty plasma, Landau damping, Nonextensivity, Reductive perturbation technique, Solitary waves.

## Introduction

Recently, dusty plasma has attracted the attention of researchers because of its industrial and laboratory applications, as well as its natural presence in various spaces and astronomical environments (cometary tails, planetary rings, the interstellar medium, Earth's magnetosphere and upper atmosphere, D and lower E regions, etc.)<sup>1-3</sup>. Dusty plasma consists of plasma components (electrons and ions), in addition to negatively-charged dust grains due to the interaction of electrons with the surface of the grains. The radii of the dust particles range between  $0.05 - 10\mu\text{m}$  and atomic numbers between  $(10^3 - 10^5)^{4-6}$ . The presence of dust, which has a large mass and charge relative to the plasma components, contributes to the emergence of new low-frequency nonlinear

modes in the dust plasma. As in the case of dust ion-acoustic (DIA) nonlinear mode<sup>7-9</sup>, and dust acoustic (DA) nonlinear mode<sup>10-12</sup>.

In statistical mechanics and thermodynamics, systems characterized by the property of nonextensivity are systems for which the entropy of the whole is different from the sum of the entropies of the respective parts<sup>13</sup>. In other words, the entropy of the sum (A+B) of two independent systems A and B is equal to  $S_q^{(A+B)} = S_q^{(A)} + S_q^{(B)} + (1 - q)S_q^{(A)}S_q^{(B)}$ .  $q$  Parameter underpins the generalized entropy of Tsallis, it measures the amount of its nonextensivity (the degree of its correlation) and it is linked to the underlying dynamics of the system. The generalized entropy of the whole system is

greater than the sum of the entropies of its parts if  $q < 1$  (superextensivity), whereas the generalized entropy of the system is smaller than the sum of the entropies of its parts if  $q > 1$  (subextensivity).  $q$ -Entropy has been used to explain various astrophysical phenomena, such as stellar polytropes, solar neutrino problem, and peculiar velocity distribution of galaxy clusters<sup>14-16</sup>. It should be noted that  $q$ -distribution is not normalizable for  $q < -1$ . The advantage of employing  $q$ -distribution lies in the fact that the Maxwellian distribution is a special case of the  $q$  function in the limit of  $q \rightarrow 1$ . Furthermore,  $q$ -distribution for  $-1 \geq q \geq 1$ , corresponding to the family of kappa distributions is obtained from the positive definite part  $12 \geq k \leq \infty$ . The  $k$ -distribution can be obtained from the  $q$ -distribution by the expression  $= \frac{1}{1-q}$ .

The Landau damping of oscillations and waves propagating in the collision-less plasma is caused by resonant wave-particle interactions, where an exchange of energy takes place between the waves and the particles of the plasma. In this case, the phase velocity of the wave is equal to the velocity of the particles (resonance state) and the energy exchange is of the greatest value. Linear Landau damping for Langmuir waves was first predicted five decades ago, and since then the effect of Landau damping on many linear and nonlinear modes in plasmas has been investigated<sup>17</sup>.

The nonlinear theory of ion-acoustic waves modified by Landau damping due to electrons was proposed by Ott and Sudan<sup>18</sup>, neglecting the particle's trapping effects on the assumption that the particle trapping time is much longer than that of Landau damping. They derived a Korteweg-de Vries (KdV) equation with a source term that models the lowest-order effects of resonant particles.

Based on wave-particle response interactions used to modify the shape of the solution a detailed study was proposed, as it was observed, these interactions in the plasma lead to the acceleration of charged particles as a result of the transfer of wave energy to these particles<sup>19</sup>. Recently, experimental results have predicted the formation of acoustic ion shock waves due to the dissipation caused by Landau damping<sup>20</sup>.

Arnab Barman and A. P. Misra investigated the effect of Landau damping of two types of ions (positive and negative) on electrostatic solitary waves in dusty multicomponent plasma<sup>21</sup>. Cold negatively charged dust is described by fluid equations, and two types of ions (negative and positive) were also described by Vlasov's kinetic equations. They employed the reductive perturbation method to derive an equation KdV with a Landau damping term. They found that in some space and laboratory plasma systems, the effects of Landau damping (and nonlinearity) are more pronounced than the effects of finite Debye length (dispersive), which makes the (KdV) DA solitary wave theory inapplicable to DAWs in dusty pair-ion plasmas<sup>21</sup>. The effect of linear Landau ion damping of low-frequency, weakly nonlinear, and weakly dispersive nonlinear waves has been described by Arnab Sikdar and Manoranjan Khan. They found that the Landau damping causes the amplitude of the wave to decrease over time and the change in the dust charge enhances the damping rate<sup>22</sup>. Yashika Ghai et al. investigated the effect of wave-particle response interactions on nonlinear dust acoustic structures (Dust acoustic (DA) solitary and shock structures) in dusty plasma containing two different types of nonthermal ions<sup>23</sup>. Using the reductive perturbation method, they derived the KdV equation with Landau damping, and they also obtained the analytical solution for Landau damping modified KdV-equation. The results of the study of time evolution showed that the initial shock wave turns into an oscillatory shock at later times as a result of the balance between the nonlinearity, the dispersion, and the dissipation caused by the Landau damping<sup>23</sup>. Yashika Ghai and N. S. Saini studied the Landau damping of dusty acoustic solitary waves in a dusty plasma containing two types of superthermal ions obeying a kappa distribution<sup>24</sup>. Their study focused on investigating the phenomenon of wave-particle interaction and highlighting the effect of Landau damping caused by cold and heavy ions on the properties of Dust acoustic waves in a specific region of the Earth's magnetic tail. They found that the super thermally of the ion species affects the Landau damping phenomenon formed in the studied region. No work has been reported in the study of Landau damping of dusty acoustic solitary waves in a dusty plasma

containing  $q$ -nonextensive velocity distributed ions and Boltzmann distributed electrons. In this work, we present an analytical numerical study of the effect of the nonextensive parameter  $q$  of ions on

**Materials and Methods**

The propagation of dusty acoustic solitary waves in a dusty plasma containing  $q$ -nonextensive velocity distributed ions and Boltzmann distributed electrons are governed by the following normalized Eqs<sup>3-25</sup>:

$$\frac{\partial n}{\partial t} + \frac{\partial(n\vartheta)}{\partial x} = 0 \quad 1$$

$$\frac{\partial \vartheta}{\partial t} + \vartheta \frac{\partial \vartheta}{\partial x} = \frac{\partial \Phi}{\partial x} \quad 2$$

$$\frac{\partial^2 \Phi}{\partial x^2} = \mu_e n_e + n - \mu_i n_i \quad 3$$

Where  $n$  is the dust grain number density,  $\vartheta$  is the dust fluid velocity,  $\Phi$  is the electrostatic potential,  $n_e$  is the electron number density,  $n_i$  is the ion number density. The following normalization<sup>23-24</sup>:

$$n \rightarrow \frac{n}{n_0}, \quad \Phi \rightarrow \frac{e\phi}{K_B T_i}, \quad \vartheta \rightarrow \frac{\vartheta}{C_d}, \quad x \rightarrow \frac{x}{\lambda_D}, \quad t \rightarrow t\omega_{pd}$$

Where  $\lambda_D = \left(\frac{K_B T_i}{n_{d0} Z_d e^2}\right)^{1/2}$  is the dust Debye length

with,  $C_d = \left(\frac{Z_d K_B T_i}{m_d}\right)^{1/2}$  is the dust acoustic speed,

$\omega_{pd} = \left(\frac{n_{d0} Z_d^2 e^2}{m_d}\right)^{1/2}$  is the dust plasma frequency,

$K_B$ ,  $n_{d0}$ ,  $e$ ,  $m_d$  are Boltzmann constant, the unperturbed dust grain number density, the electron charge, and dust grain mass, respectively.

The condition of quasi-neutrality at equilibrium is given by the following equation  $n_{i0} = n_{e0} + n_0 Z_d$ , where  $n_{i0}$ ,  $n_{e0}$ ,  $n_0$  are the unperturbed ion,

electron, and dust number densities,  $Z_d$  is the number of electrons residing on the dust grains. To condition of quasi-neutrality we used the following equation  $\mu_e + \mu_i = 1$ , where  $\mu_e = \frac{n_{e0}}{n_0 Z_d}$ ,  $\mu_i =$

$$\frac{n_{i0}}{n_0 Z_d} \quad 23-24$$

The normalized kinetic Vlasov equation for ions is given as<sup>18</sup>:

$$M \frac{\partial f_i}{\partial t} + v \frac{\partial f_i}{\partial x} - \frac{\partial \Phi}{\partial x} \frac{\partial f_i}{\partial v} = 0 \quad 4$$

And their number densities are given by:

the propagation of dusty acoustic solitary waves under the influence of Landau damping in dusty Plasma.

$$n_i = \int_{-\infty}^{+\infty} f_i dV \quad 5$$

Where  $M = \sqrt{\frac{Z_d m_i}{m_d}}$  represents the inertial effects due to ions, especially, Landau damping by ions. The ion velocity  $v$  is normalized by ion thermal velocity  $v_{ti} = \sqrt{\frac{K_B T_i}{m_i}}$ .

We derived KdV equation from Eqs [(1-3)] and [(6-7)] by adding an additional Landau damping term and employing the reductive perturbation technique. The independent variables are stretched as<sup>23</sup>:

$$\left. \begin{aligned} \xi &= \varepsilon^{1/2} (x - v_{ph} t) & ; & \quad \tau = \varepsilon^{3/2} t \\ \frac{\partial}{\partial x} &= \varepsilon^{1/2} \frac{\partial}{\partial \xi} & ; & \quad \frac{\partial}{\partial t} = -v_{ph} \varepsilon^{1/2} \frac{\partial}{\partial \xi} + \varepsilon^{3/2} \frac{\partial}{\partial \tau} \end{aligned} \right\} 6$$

The dependent physical quantities are expanded with respect to  $\varepsilon$  about the equilibrium state as:

$$\left. \begin{aligned} n &= 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \dots \\ \vartheta &= \varepsilon \vartheta_1 + \varepsilon^2 \vartheta_2 + \dots \\ \Phi &= \varepsilon \Phi_1 + \varepsilon^2 \Phi_2 + \dots \\ n_i &= 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \dots \\ f_i &= f_i^{(0)} + \varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)} + \dots \end{aligned} \right\} 7$$

Where  $f_i^{(0)}$  represents the equilibrium distribution function for ions. The ions are assumed to obey  $q$ -distribution whose normalized distribution function has the form<sup>15</sup>:

$$f_i^{(0)} = A_q (1 - (q-1)v^2)^{\frac{1}{q-1}} \quad 8$$

Where:

$$A_q = \begin{cases} \frac{(1-q)^{\frac{3}{2}} \Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q} - \frac{3}{2}\right)} & -1 > q < 1 \\ \frac{(3q-1)(q-1)^{3/2} \Gamma\left(\frac{1}{q-1} + \frac{3}{2}\right)}{2 \Gamma\left(\frac{1}{q-1}\right)} & q > 1 \end{cases}$$

According to Boltzmann distribution, the electron number density can be written in a normalized form as:

$$n_e = \mu_e \exp(\sigma_i \emptyset) \quad 9$$

Where  $\sigma_i$  temperature ratio of ion to electron,  $\varepsilon$  is a small parameter proportional to the strength of perturbation. Substituting Eqs 6-8 into Eqs 1-3 and 4-5 and taking the terms in different powers of  $\varepsilon$ , we obtain in the lowest order of  $\varepsilon^{23}$ :

$$-v_{ph} \frac{\partial n_1}{\partial \xi} + \frac{\partial \vartheta_1}{\partial \xi} = 0 \quad 10$$

$$-v_{ph} \frac{\partial \vartheta_1}{\partial \xi} = \frac{\partial \Phi_1}{\partial \xi} \quad 11$$

$$\mu_e \sigma_i \emptyset_1 - \mu_i n_{i1} + n_1 = 0 \quad 12$$

$$n_{i1} = \int_{-\infty}^{+\infty} f_i^{(1)} dv \quad 13$$

$$v \frac{\partial f_i^{(1)}}{\partial \xi} - \frac{\partial f_i^{(0)}}{\partial v} \frac{\partial \Phi_1}{\partial \xi} = 0 \quad 14$$

From the Eq 14, we get:

$$\frac{\partial f_i^{(1)}}{\partial \xi} = \frac{\partial f_i^{(0)}}{\partial v} \frac{\partial \Phi_1}{\partial \xi} + \lambda(\xi, \tau) \delta(v) \quad 15$$

Where  $\delta(v)$  is Dirac's delta function and  $\lambda(\xi, \tau)$  is an arbitrary function of  $\xi$  and  $\tau$ . This equation does not have a unique solution because the above

solution for  $\frac{\partial f_i^{(1)}}{\partial \xi}$  involves the arbitrary function

$\lambda(\xi, \tau)$ . So to get the unique solution, the problem was presented as an initial value problem<sup>18</sup>. We

include an extra higher-order term  $\gamma_1 \varepsilon^{\frac{7}{2}} \frac{\partial f_i^{(1)}}{\partial \tau}$  taken from the third order of  $\varepsilon$  in Eq 4 after the expressions 6 and 7 are substituted. We get the following Eq

$$\gamma_1 \varepsilon^2 \frac{\partial f_i^{(1)}}{\partial \tau} + v \frac{\partial f_i^{(1)}}{\partial \xi} - \frac{\partial f_i^{(0)}}{\partial v} \frac{\partial \Phi_1}{\partial \xi} = 0 \quad 16$$

Then  $f_i^{(1)}$  is uniquely determined by taking  $f_i^{(1)} = \lim_{\varepsilon \rightarrow 0} f_{i\varepsilon}^{(1)}$ . Next, taking the Fourier transform of equation 16 concerning  $\xi$  and  $\tau$ :

$$\hat{f}(\omega, k) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(\xi, \tau) \exp i(k\xi - \omega\tau) d\xi d\tau \quad 17$$

we get:

$$\hat{f}_{i\varepsilon}^{(1)} = -\frac{k \frac{\partial f_i^{(0)}}{\partial v}}{k v - \omega \gamma_1 \varepsilon^2} \hat{\Phi}_1 \quad 18$$

To avoid the singularity appearing in Eq 18, we replace  $\omega$  with  $\omega + i\eta$ , where  $\eta > 0$  is a small parameter, to obtain<sup>18</sup>:

$$\hat{f}_{i\varepsilon}^{(1)} = -\frac{k \frac{\partial f_i^{(0)}}{\partial v}}{(k v - \omega \gamma_1 \varepsilon^2) - i\eta \gamma_1 \varepsilon^2} \hat{\Phi}_1 \quad 19$$

By proceeding to the limit  $\varepsilon \rightarrow 0$  and using the Plemelj's formula:

$$\lim_{\varepsilon \rightarrow 0} \frac{1}{k v - \omega \gamma_1 \varepsilon^2} = P\left(\frac{1}{kv}\right) + i\pi \delta(kv) \quad 20$$

$P$  and  $\delta$ , respectively, and by denoting the Cauchy principal value and the Dirac delta function, we obtain:

$$\hat{f}_i^{(1)} = -2 \frac{\partial f_i^{(0)}}{\partial v^2} \hat{\Phi}_1 \quad 21$$

By using the properties  $xP\left(\frac{1}{x}\right) = 1$  and  $x\delta(x) = 0$ .

Now, taking the inverse Fourier transform of Eq 23, we obtain:

$$f_i^{(1)} = -2 \frac{\partial f_i^{(0)}}{\partial v^2} \Phi_1 \quad 22$$

Next, From Eqs 24 and 15, we obtain the number density for ions:

$$n_{i1} = -\beta \Phi_1 \quad 23$$

$$\text{Where } \beta = \frac{\sqrt{2} (1-q) \Gamma\left(\frac{q-3}{2q-2}\right)}{\Gamma\left(\frac{q+1}{2-2q}\right)}$$

By substituting Eqs 23 and 10 – 11 with 12, we obtain the wave phase velocity as follows<sup>23</sup>:

$$v_{ph} = \sqrt{\frac{1}{(\mu_e \sigma_i + \mu_i \beta)}} \quad 24$$

Similarly, by using the next higher order of  $\varepsilon$  in Eqs 1-3 and 6-7 we obtain:

$$-v_{ph} \frac{\partial n_2}{\partial \xi} + \frac{\partial n_1}{\partial \tau} + \frac{\partial \vartheta_2}{\partial \xi} + n_1 \frac{\partial \vartheta_1}{\partial \xi} + \vartheta_1 \frac{\partial n_1}{\partial \xi} = 0 \quad 25$$

$$-v_{ph} \frac{\partial \vartheta_2}{\partial \xi} + \frac{\partial \vartheta_1}{\partial \tau} + \vartheta_1 \frac{\partial \vartheta_1}{\partial \xi} = \frac{\partial \Phi_2}{\partial \xi} \quad 26$$

$$\frac{\partial^2 \Phi_1}{\partial \xi^2} = \mu_e \sigma_i \Phi_2 + \left(\frac{\mu_e \sigma_i^2}{2}\right) \emptyset_1^2 - \mu_i n_{i2} + n_2 \quad 27$$

$$n_{i2} = \int_{-\infty}^{+\infty} f_i^{(2)} dv \quad 28$$

$$v \frac{\partial f_i^{(2)}}{\partial \xi} - \frac{\partial f_i^{(0)}}{\partial v} \frac{\partial \Phi_2}{\partial \xi} = \gamma_1 v_{ph} C_{q1} \frac{\partial f_i^{(0)}}{\partial v^2} + \frac{\partial \Phi_1}{\partial \xi} \frac{\partial f_i^{(1)}}{\partial v} \quad 29$$

By solution of system Eqs 25-29, we obtain the following Korteweg- de Vries (KdV) equation with an additional Landau damping term for the first-order perturbed electrostatic potential  $\Phi_1$  as follows<sup>23</sup>:

$$\frac{\partial \Phi_1}{\partial \tau} - A \Phi_1 \frac{\partial \Phi_1}{\partial \xi} + B \frac{\partial^3 \Phi_1}{\partial \xi^3} + C P \int_{-\infty}^{+\infty} \frac{\partial \Phi_1}{\partial \xi'} \frac{d\xi'}{\xi - \xi'} = 0 \quad 30$$

where the nonlinear coefficient  $A$ , the dispersion coefficient  $B$  and Landau damping coefficient  $C$  are given by<sup>23</sup>:

$$\left. \begin{aligned} A &= \frac{v_{ph}^3}{2} \left[ \frac{1}{v_{ph}^4} + \mu_e \sigma_i^2 - \mu_i \right] \\ B &= \frac{v_{ph}^3}{2} \\ C &= \frac{v_{ph}^4 \mu_i \gamma_1}{\sqrt{2\pi}} \frac{\Gamma\left(\frac{1}{1-q}\right) \sqrt{1-q}}{\Gamma\left(\frac{1}{1-q} - \frac{1}{2}\right)} \end{aligned} \right\} \quad 31$$

## Results and Discussion

Numerical analysis has been performed to study the effect of nonextensive parameter  $q$  of ions and other plasma parameters on DA solitary structures under the influence of Landau damping. It is important to indicate the numerical data used in this study obtained from references<sup>28-30</sup>.

Fig. 1 and 2 show the changes in the Landau damping coefficient with the nonextensive parameter  $q$ . The Landau damping decreases with increasing of the parameter  $q$ . In addition, one can observe also Fig.1 and 2 hat for a constant value of the parameter  $q$  the value of the Landau damping coefficient decreases with the increase of the two ratios  $\sigma_i$  and  $\mu_i$ .

Now, following the procedures, the solution of the KdV equation with the Landau damping term, is given as follows<sup>21-24</sup>:

$$\Phi_1 = \phi(\tau) \operatorname{sech}^2 \left[ \frac{\left( \xi - \frac{A}{3} \int_0^\tau \phi(\tau) d\tau \right)}{W} \right] \quad 32$$

Where  $W = \sqrt{\frac{12B}{\phi(\tau)A}}$  is the width,  $\phi(\tau) =$

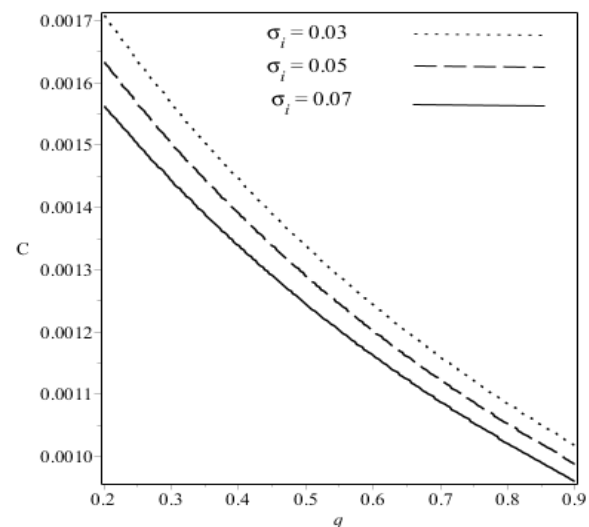
$\phi_0 \left(1 + \frac{\tau}{\tau_0}\right)^{-2}$  is the amplitude,  $\tau_0 = \frac{1.37}{c} \sqrt{\frac{3B}{A\phi_0}}$  and  $\phi_0 = \frac{3U_0}{A}$ ,  $U_0$  is the solitary wave velocity.

The DA solitary wave energy can be expressed in terms of the first-order perturbed velocity as follows<sup>26-27</sup>:

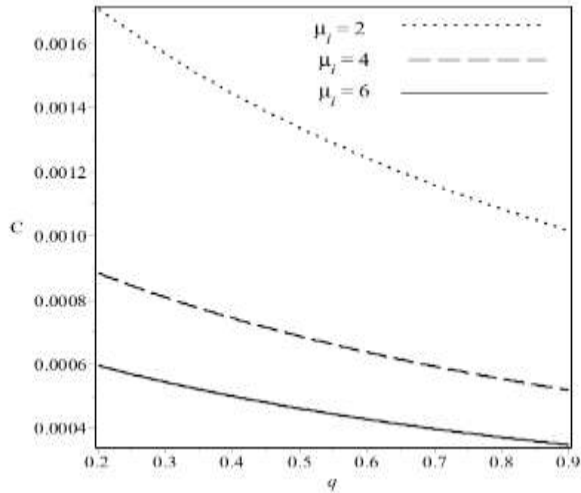
$$\begin{aligned} v_1 &= -v_{ph}(\mu_e \sigma_i + \mu_i \beta) \Phi_1 \\ &= -v_{ph}(\mu_e \sigma_i \\ &\quad + \mu_i \beta) \phi(\tau) \operatorname{sech}^2 \left[ \frac{\eta}{W} \right] \end{aligned}$$

Where  $\eta = \xi - \frac{A}{3} \int_0^\tau \phi(\tau) d\tau$

$$E = \int_{-\infty}^{+\infty} (v_1)^2 d\eta = \frac{4W^2}{3v_{ph}^2} \phi^2(\tau) \quad 33$$



**Figure 1. Shows variation of the Landau damping coefficient with nonextensive parameter  $q$  of ions for different values of temperature ratio of ion to electron  $\sigma_i = 0.07$  (solid line),  $\sigma_i = 0.05$  (dashed line),  $\sigma_i = 0.03$  (dot line), Other parameters being  $\gamma_1 = 0.01$ ,  $\mu_i = \mu_e = 2$ .**



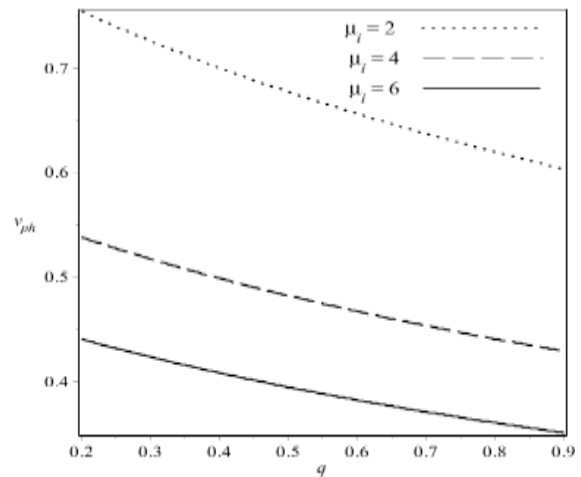
**Figure 2.** Variation of the Landau damping coefficient with nonextensive parameter  $q$  of ions for different values of density ratio of ion to dust  $\mu_i = 6$  (solid line),  $\mu_i = 4$  (dashed line),  $\mu_i = 2$  (dot line), Other parameters being  $\gamma_1 = 0.01$ ,  $\mu_e = 2$ ,  $\sigma_i = 0.03$ .

It appears that for a constant value of the parameter  $q$  the value of the Landau damping coefficient decreases with the increase of the two ratios  $\sigma_i$  and  $\mu_i$ , and the graphs converge to each other with the increase of the value  $q$ . The value of the Landau damping coefficient is almost zero for  $\sigma_i = 0.07$  in Fig. 1 and  $\mu_i = 6$  in Fig 2. This indicates that the collective effects of the dusty plasma system do not play a significant role in this case. Moreover, the significant effect of the ratio  $\mu_i$  in Fig 2 is observed compared with the ratio  $\sigma_i$  in Fig 1, where fig. 1 shows a slight shift of curve  $C(q)$  with the change of the ratio  $\sigma_i$ , while the shift of curve  $C(q)$  in Fig 2 is large for different values of the ratio  $\mu_i$ . The increase in the value of the coefficient  $C$  is related to the rise of the value of the ratio  $\mu_i$ , which agrees with the fact that the density of dust is close to the density of ions, the dust charge is considered a dynamic variable that modifies the collective properties of the dusty plasma. This may be due to the nature of the dusty acoustic wave propagating in the plasma resulting from the compression and rarefaction of the dust grains<sup>21</sup>.

This allow us to conclude that the effect of the compression and rarefaction of plasma particles on energy exchange between plasma

particles and DA solitary wave could be the reason behind increment in both  $\sigma_i$  and  $\mu_i$  when  $q \rightarrow 0.9$ .

Figs 3 and 4 show the variation of the phase velocity  $v_{ph}$  with the nonextensive parameter  $q$  for different values of the temperature ratio of ion to electron and the unperturbed density ratio of ion to dust, respectively. The phase velocity of the DA solitary waves decreases with increasing the nonextensive parameter  $q$  of ions (decreasing nonextensivity of ions). In other words, the DA solitary waves propagate faster if there are more nonextensive ions present.



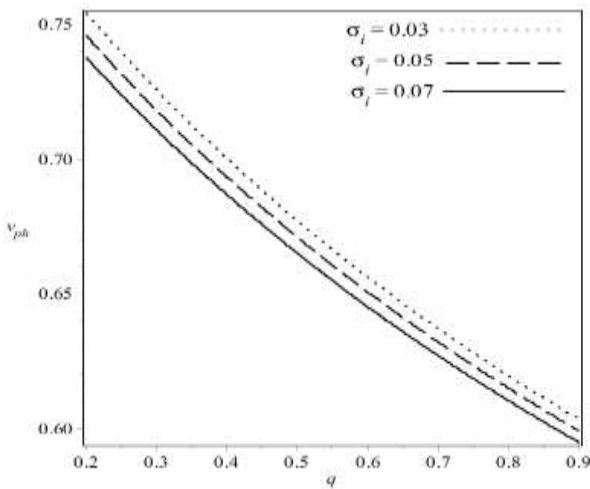
**Figure 3.** Variation of phase velocity with nonextensive parameter  $q$  of ions for different values of density ratio of ion to dust  $\mu_i = 6$  (solid line),  $\mu_i = 4$  (dashed line),  $\mu_i = 2$  (dot line), Other parameters being  $\gamma_1 = 0.01$ ,  $\mu_e = 2$ ,  $\sigma_i = 0.03$ .

One can observed also from figure 3 that the phase velocity of DA solitary waves decreases with an increase in ion to dust density ratio, and for  $\mu_i \geq 6$  the phase velocity get to zero, when  $q \rightarrow 0.9$  as illustrated in relation 26 These result is very useful in determining dust density in plasma, in which DA soliton Waves disappeared.

Fig 4 depicts the decrease phase velocity of DA solitary waves with an increase in ion to electron temperature ratio. Physically, a decrease in the phase velocity leads to an increase in the number of particles that have a velocity close to the phase velocity, such a phenomenon enhances the resonance state and increases the extraction of wave energy. The comparison between Figs 3 and 4

shows that the effect of the ratio  $\mu_i$  is greater than the effect of ratio  $\sigma_i$  on the wave phase velocity  $v_{ph}(q)$ . Furthermore, the curves converge to each other for large values of  $q$  (i.e.  $q \rightarrow 0.9$ ). It is also noted that the wave phase velocity is reduced to zero for large values of the two ratios  $\sigma_i$  and  $\mu_i$ . This result indicates that the DA solitary wave disappears near these limit values and appears in a region where  $0.2 < q < 0.9$ .

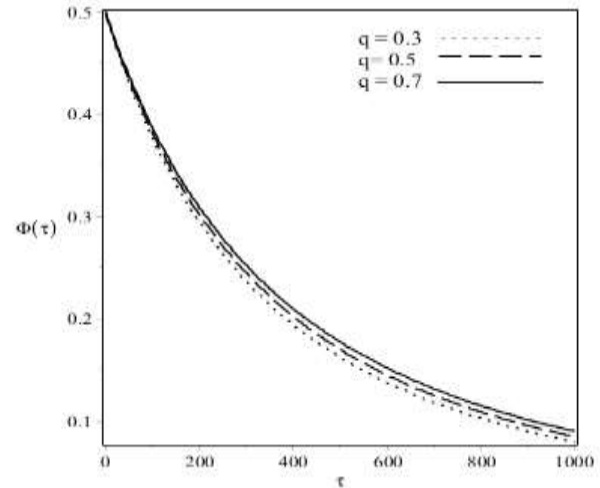
At this point we can conclude that the distribution is insufficient to describe long-range interactions in a dusty, collisionless and non-magnetized plasma in case of instability of velocity and their position during wave- Particle interaction. In case of such instability, it might be possible to describe Plasma medium in steady states, in which the distribution function is applicable outside the state of thermal equilibrium, where it can be encountered in space and astronomical plasmas.



**Figure 4. Variation of phase velocity with nonextensive parameter  $q$  of ions for different values of temperature ratio of ion to electron  $\sigma_i = 0.07$  (solid line),  $\sigma_i = 0.05$  (dashed line),  $\sigma_i = 0.03$  (dot line), Other parameters being  $\gamma_1 = 0.01$ ,  $\mu_i = \mu_e = 2$ .**

Fig 5 demonstrated that wave amplitude decreases with time for different parameters. Furthermore, for a given value of time, the amplitude of the DA solitary wave increases with the increase in the value of the nonextensive parameter  $q$ .

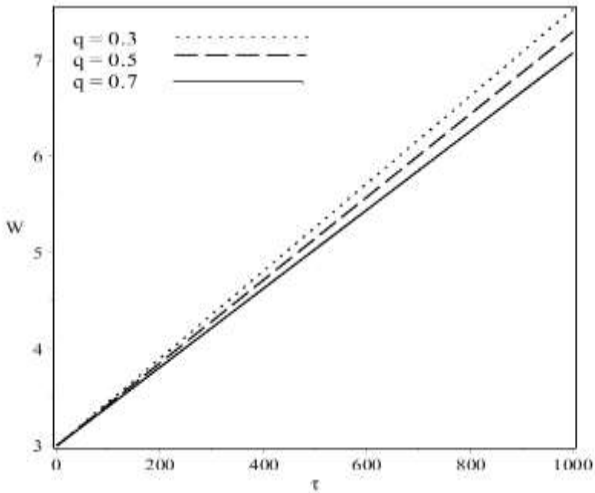
Relations  $W(\tau) = \sqrt{\frac{12B}{\phi(\tau)A}}$  and  $\phi(\tau) = \phi_0 \left(1 + \frac{\tau}{\tau_0}\right)^{-2}$  showed, that with time evolution the width  $W(\tau)$  of DA soliton wave increases when its amplitude  $\phi(\tau)$  decreases, as illustrated in Figs 5 and 6.



**Figure 5. Variations of solitary wave amplitude with time  $\tau$  for different values of nonextensive parameter  $q$   $q = 0.7$  (solid line),  $q = 0.5$  (dashed line),  $q = 0.3$  (dot line), Other parameters being  $\gamma_1 = 0.01$ ,  $\mu_i = \mu_e = 2$ ,  $\sigma_i = 0.03$ .**

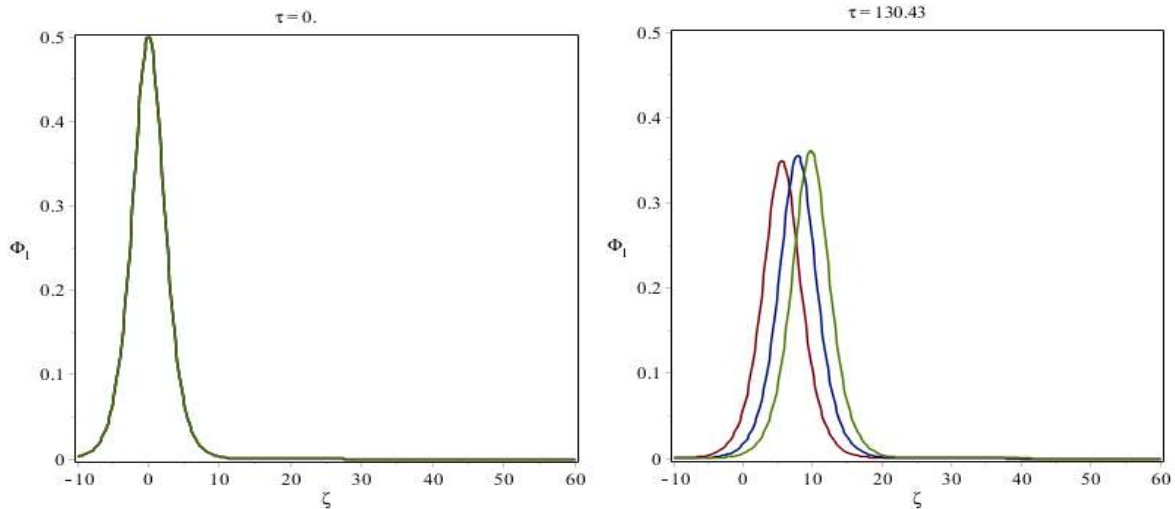
Fig 5 demonstrates the wave amplitude decreases with time for different parameters. For a given value of time, the amplitude of the DA solitary wave increases with the increase in the value of the nonextensive parameter  $q$ . As shown in Fig 5, the parameter  $q$  does not play any role in changing the amplitude of the DA solitary wave near the moment of the start of the wave  $t = 0$ , while its effect increases with the passage of time represented in the shift of the three curves toward each other as we approach  $q = 0.3$ . The amplitude of the wave approaches zero with the time evolution of  $\phi(\tau)$  near value  $q = 0.3$ .

Fig 6 shows linear change and increase of wave width with time defined by a linear equation  $W(\tau) = m\tau + \alpha$ , where constants  $m$  and  $\alpha$  are determined initially. Figure shows the slope of the graph increases as the value of the parameter  $q$  decreases and curve shift for different values of parameter  $q$ .



**Figure 6. Variations of solitary wave width with time  $\tau$  for different values of nonextensive parameter  $q$   $q = 0.7$  (solid line),  $q = 0.5$  (dashed line),  $q = 0.3$  (dot line), Other parameters being  $\gamma_1 = 0.01$ ,  $\mu_i = \mu_e = 2$ ,  $\sigma_i = 0.03$ .**

Physically, the state corresponding to  $q < 1$  is called superextensivity. This condition mainly

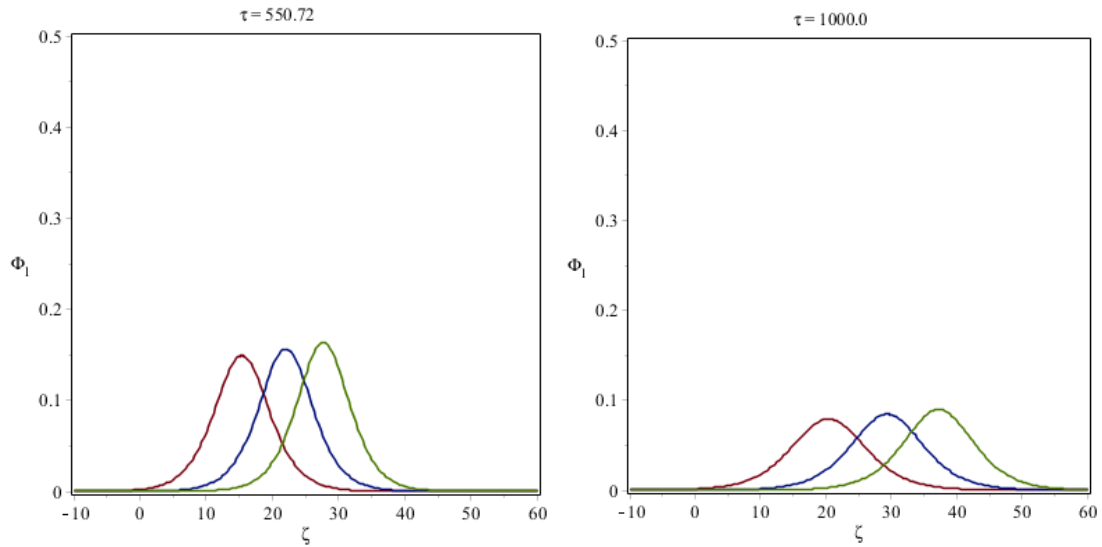


describes astrophysical plasmas in such fields as: (interstellar media, magneto sheaths, ionosphere, solar wind, interstellar media, planetary magnetospheres, cometary tails) and laboratory plasmas (The area between the plasma and the tokamak walls)<sup>14-16</sup>.

An increase in the nonextensive parameter  $q$  of the ions is accompanied by a decrease in the number of active ions in the tail of the energy spectrum, which leads to a decrease in the number of ions that extracts energy from the wave, and decreases of the Landau damping.

Fig 7 investigated the time evolution of the solitary wave under the influence of the Landau damping for three different parameters of  $q$  values. It shows the solitary wave shifts to the right with the passage of time. It also shows a decrease in the wave amplitude and increase in its width as the wave moves forward. In its turn, phase shift of the wave increases with the value of the parameter  $q$





**Figure 7. Time evolution of the dust acoustic solitary wave under the effect of Landau damping due to  $q$ -distribution ions. (Red line)  $q = 0.3$ , (Blue line)  $q = 0.5$ , (Green line)  $q = 0.7$ , Other parameters being  $\gamma_1 = 0.01$ ,  $\sigma_i = 0.03$   $\mu_i = \mu_e = 2$ .**

In the moment  $\tau = 0$ , there is not effect of the  $q$  parameter in the solitary wave profile because it is not found in the expression of the initial amplitude  $\Phi_0$ . Over time, the effect of the  $q$  parameter has appeared through increase of Landau damping for lower values of the  $q$  parameter. Table 1 allows us to conclude that DA solitary wave amplitude increases with the increases of the

nonextensive parameter  $q$  (decreases nonextensivity of ions). The physical explanation for this is that the presence of active ions enhances the energy exchange between the wave and the particles and increases the extraction of the wave energy, which results in a faster damping velocity and a greater decrease in the DA solitary wave<sup>23</sup>.

**Table 1. Variations of solitary wave amplitude with time  $\tau$  for different values of nonextensive parameter.**

Amplitude of DA solitary waves				
damping time → nonextensivity ↓	$\tau = 0$	$\tau = 130.43$	$\tau = 550.72$	$\tau = 1000$
$q = 0.3$	0.5	0.348879	0.148907	0.079256
$q = 0.5$	0.5	0.355019	0.156342	0.084557
$q = 0.7$	0.5	0.360622	0.163519	0.089823

Table 2 tabulated DA solitary wave width that calculated at different times for different values of the nonextensive parameter  $q$ . It is noticed that the width of the DA solitary wave increases with

time. For a constant value of time, the width decreases with the increase of the nonextensive parameter  $q$  (decreasing nonextensivity of ions).

**Table 2. Variations of solitary wave width with time  $\tau$  for different values of nonextensive parameter.**

Width of DA solitary waves				
damping time → nonextensivity ↓	$\tau = 0$	$\tau = 130.43$	$\tau = 550.72$	$\tau = 1000$
$q = 0.3$	3	3.591413	5.497148	7.534334
$q = 0.5$	3	3.560032	5.364649	7.293741
$q = 0.7$	3	3.531779	5.245352	7.077122

Table 3 the phase of the DA solitary waves was calculated at different times, for different values of the nonextensive parameter  $q$ . It is noticed that there is a phase shift that increases with the passage of time and with the increase in the value of the parameter  $q$ . Physically, a lower phase means

less time and higher damping speed. In other words, the presence of active ions accelerates the Landau damping, in other words, the presence of energetic ions speeds up Landau's damping. This speeds up the extraction of DA solitary wave energy.

**Table 3. Variations of solitary wave Phase with time  $\tau$  for different values of nonextensive parameter.**

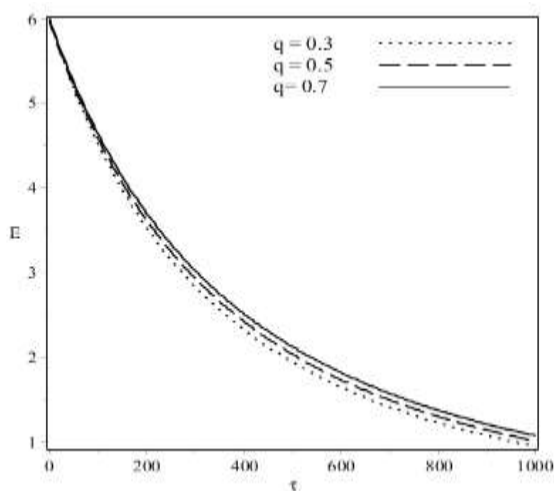
		Phase of DA solitary waves			
damping time $\rightarrow$	nonextensivity $\downarrow$	$\tau = 0$	$\tau = 130.43$	$\tau = 550.72$	$\tau = 1000$
	$q = 0.3$	0	5.574348	15.377132	20.372160
	$q = 0.5$	0	7.846697	21.986337	29.363851
	$q = 0.7$	0	9.723668	27.644090	37.203984

The previous discussion showed that the nonextensive parameter  $q$  has a clear effect on the energy of the DA solitary wave and to prove this we calculated the energy of the DA solitary wave at

different times for different values of the parameter  $q$  Table 4 and plotted the energy changes with time for different values of the parameter  $q$  (Fig 8).

**Table 4. Variations of solitary wave energy with time  $\tau$  for different values of nonextensive parameter.**

		Energy of DA solitary waves			
damping time $\rightarrow$	nonextensivity $\downarrow$	$\tau = 0$	$\tau = 130.43$	$\tau = 550.72$	$\tau = 1000$
	$q = 0.3$	6	4.186614	1.786976	0.951270
	$q = 0.5$	6	4.260747	1.876338	1.015063
	$q = 0.7$	6	4.329190	1.962656	1.078152



**Figure 8. Variations of solitary wave energy with time for different values of nonextensive parameter  $q$   $q = 0.7$  (solid line),  $q = 0.5$  (dashed line),  $q = 0.3$  (dot line), Other parameters being  $\gamma_1 = 0.01$ ,  $\mu_i = \mu_e = 2$ ,  $\sigma_i = 0.03$ .**

We notice from the Table 4 and Fig 8 that the energy of the DA solitary wave decreases with time. On the other hand, for a specific time, the energy increases with the increase of  $q$  parameter (decreasing nonextensivity of ions). This means that the presence of active ions enhances the extraction of DA solitary wave energy and the occurrence of damping.

The results obtained in this research indicate the important role played by the ratio  $\mu_i$  in the emergence of the solitary dust acoustic wave within the domain  $0 < q < 1$ . Ratio  $\mu_i$  influences the thermal turbulence of dusty plasma and electrostatic potential generated on the separation of positive ions from negative dust grains. The existence of thermal turbulence is reflected in the elasticity and the particle response of the plasma medium to periodic compression and rarefaction. Thermal turbulence causes the positive ions to move periodically around the negative and semi-static dust grain due to its greater inertia compared to the

ions. When the density of the dust grains approaches the density of positive ions, the separation distance between the dust grains

decreases, becoming almost pockets, the inability of the ions passes the dust grains and forms a negatively charged plasma (sheath)<sup>21</sup>.

## Conclusion

In this work, the propagation of dust acoustic solitary waves in dusty plasma in the presence of Landau Damping has been investigated. A reductive perturbation technique has been applied to dust fluid equations to derive the KdV equation with the addition of Landau damping limit. This study shows Landau damping decreases with increasing the  $q$  parameter. Research results can be summarized as follows:

- Landau damping decreases with increasing of the  $q$  parameter, and for  $\mu_i \geq 6$  the phase velocity get to zero, when  $q \rightarrow 0.9$ . These result is very useful in determining dust density in plasma, in which DA soliton Waves disappeared.
- The phase velocity of the DA solitary waves decreases by increasing the nonextensive parameter  $q$  of two ratios  $\sigma_i$  and  $\mu_i$ .
- The amplitude and energy of the DA solitary wave decreases with time  $\tau$  for

different values of nonextensive parameter  $q$

- The width of the DA solitary wave increases with time. the increase is enhanced for lower values of the parameter  $q$ .

Consequently, one can conclude that, the presence of energetic particles in the plasma enhances the mechanism of energy exchange between the waves and these particles, which causes an increase in the wave energy extraction and the occurrence of Landau damping. Similar results have been found in the references<sup>23-24</sup>. Since the space plasma described by the nonextensive distribution in Tsallis statistics is not in the state of thermodynamic equilibrium, the results can be helpful for understanding the nonextensive effects on Landau damping of the DA solitary waves in dusty nonextensive plasma regions of planetary rings, comets, and other space plasma environments.

## Data Availability

No data were used to support this study.

## Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine ours. Besides, the Figures and images, which are not mine ours, have been given the permission for re-publication attached with the manuscript.

- Authors sign on ethical consideration's approval
- Ethical Clearance: The project was approved by the local ethical committee in University of Tishreen.

## Authors' Contribution Statement

A.A. Present mathematical calculations and discuss results. N.K. and M.A Supervising the audit of mathematical calculations, discussing the results of

the work, and improving the quality of the manuscript in terms of readability and linguistic audit.



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## تخامد لاندوا للأموح السوليتونية الصوتية الغبارية في بلازما غبارية غير شاملة

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### الخلاصة

تم دراسة الأموح السوليتونية الصوتية الغبارية تحت تأثير تخامد لاندوا للبلازما الغبارية الفضائية تحوي أيونات خاضعة لتوزع سرعة غير شاملة. قمنا بالتحقيق في تأثير تخامد لاندوا والبارمتر غير الشامل  $q$  على البنى السوليتونية الصوتية الغبارية من خلال الحل العددي لمعادلة كورديفيك- دي فريس المعدلة بحد تخامد لاندوا. باستخدام طريقة الاضطراب الاختزالية قمنا باستنتاج معادلة كورتييفيك - دي فريس مع إضافة حد تخامد لاندوا. أظهرت هذه الدراسة أن كثافة حبيبات الغبار تلعب دوراً أساسياً في ظهور أو عدم ظهور الأموح السوليتونية DA في البلازما الغبارية، والتأثير الواضح للخاصية غير الشاملة للأيونات على ظاهرة تخامد لاندوا للبنى اللاخطية المتشكلة في البلازما الغبارية. تفيد نتائج هذه الدراسة في محاولة فهم الآلية الفيزيائية للانتشار الأموح السوليتونية تحت تأثير تخامد لاندوا في بيئات البلازما المغبرة غير الشاملة كالمناطق الكوكبية، والأغلفة المغناطيسية، وبيئات البلازما الفضائية.

**الكلمات المفتاحية:** بلازما غبارية، تخامد لاندوا، توزع غير شاملة، تقنية الاضطراب المختزلة، أموح سوليتونية