On The Dynamical Behavior of a Prey-Predator Model With The Effect of Periodic Forcing

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Abstract:

The dynamical behavior of a two-dimensional continuous time dynamical system describing by a prey predator model is investigated. By means of constructing suitable Lyapunov functional, sufficient condition is derived for the global asymptotic stability of the positive equilibrium of the system. The Hopf bifurcation analysis is carried out. The numerical simulations are used to study the effect of periodic forcing in two different parameters. The results of simulations show that the model under the effects of periodic forcing in two different parameters, with or without phase difference, could exhibit chaotic dynamics for realistic and biologically feasible parametric values.

1. Introduction:

The nonlinear mechanical and electronic systems described by Duffing and Van de Pol equations have a very simple dynamic behavior in the constant parameter case, but become very complex (i.e. have a multiplicity of attractors, catastrophes and chaos) when they are periodically perturbed [5]. Another important example in a different field is the classical SEIR epidemic model which has a globally stable equilibrium in the constant parameter case and a great number of modes of behavior in the periodically varying case [8, 12].

In the last three decades or so, a number of studies have been performed on the interactions between periodic forcing (seasonality) on the parameters and internal biological species of ecosystems, for recent review see [7]. Without seasonal variations the two-species non-linear autonomous dynamical system can either have limit cycle or stability. However, due to seasonal variation the behavior of the system becomes complex and can depict chaos [3, 4, 10, 11]. In this paper, we show that the prey predator ecological system, which is based on a modified version of the Leslie-Gower scheme, is also very sensitive to seasonality. In the constant parameter case the model has a simple Hopf bifurcation and therefore has only one mode of behavior for each set of parameters: a globally stable equilibrium or a globally stable limit cycle. The seasonality is assumed in two parameters: the growth rate of the prey and the predator. The effects of periodic variations on the parameters of real ecological models are often in different phases. Its reaches their maximum influence at different times. Therefore, seasonality in two different parameters will be considered simultaneously with different phase angles between them.

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2. The Mathematical Model

Consider a prey-predator system for which it is assumed that the prey X_1 grows logistically in the absence of predation. The predator X_2 consumes the prey according to Holling type-II functional response. The interaction between species X_2 and its prey X_1 has been modeled by modified Leslie-Gower scheme in which the loss in a predator population is proportional to the reciprocal of per-capita availability of its most favorite food. The state equations, which cover this model, can be written as follows:

$$\frac{dX_1}{dT} = r_1 X_1\left(1-\frac{X_1}{K}\right) - \frac{A X_1 X_2}{1+B X_1}, \qquad (1a)$$

$$\frac{dX_2}{dT} = r_2 X_2^2 - \frac{C X_2^2}{D X_1 + E}, \qquad (1b)$$

with $X_1(0) \geq 0$ and $X_2(0) \geq 0$. Where X_1 and X_2 represent the population densities at time T; r_1, r_2, K, A, B, C, D and E are the model parameters assuming only positive values and its defined as follows: r_1 represent the growth rate of prey, K is the carrying capacity in the absence of predation, A is the search rate, $1/B$ is the half saturation constant, r_2 describes the growth rate of predator X_2, which is assumed to be a sexually reproducing species. The square term signifies the fact that mating frequency is directly proportional to the number of males as well as to that of females. The last term in the right-hand side measures the loss of predator population due to rarity of its favorite food X_1, the constant E normalizes the residual reduction in the predator population X_2 due to severe scarcity of its favorite food X_1 [1]. In order to reduce the number of parameters in system (1) from 8 to 4, we assume the following non-dimensional variable and parameters, $t = r_1 T$, $x = X/K$, $y = A X_2/r_1$, $w_1 = BK$, $w_2 = r_2/A$, $w_3 = C/EA$, and $w_4 = DK/E$. Then system (1) takes the non-dimensional form

$$\frac{dx}{dt} = x\, f(x,y) = x\left[1 - x - \frac{y}{1+w_1 x}\right] \qquad (2a)$$

$$\frac{dy}{dt} = y\, g(x,y) = y\left[w_2 y - \frac{w_3 y}{1+w_4 x}\right] \qquad (2b)$$

In order to be a biologically meaningful system, system (2) should be qualify as a Kolmogorov system [2]. Applying the conditions of the Kolmogorov theorem, we obtain that the system (2) is a Kolmogorov system under the following conditions:

$$\frac{w_3}{(1+w_4 x)^2} < w_2 < \frac{w_3}{1+w_4 x} \qquad (3a)$$

$$0 < \frac{w_3 - w_2}{w_2 w_4} < 1 \qquad (3b)$$

Further more, the solution of system (2) with nonnegative initial values is uniformly bounded as shown in the following theorem.

Theorem 1. The Komogorov system (2) has uniformly bounded solution in the positive quadrant $R_+^2 = \{(x,y):\ x \geq 0,\ y \geq 0\}$.

Proof: From the Eq. (2a), we have

$$\frac{dx}{dt} \leq x(1-x)$$

So, according to the differential inequality theorem [6], for $x(0) = x_0 > 0$ we get

$$x(t) \leq \frac{1}{1+c\,e^{-t}} = \bar{x},\ t \geq 0 \text{ and } c = \frac{1-x_0}{x_0}.$$

Thus, for sufficiently large $t > 0$, we have $Sup\, x(t) \leq 1$.

Now, let $M(t) = x(t) + \alpha\, y(t)$, with $\alpha = \frac{w_3}{1+w_4 \bar{x}} - w_2 > 0$. Then straight forward computations yields:

$$\frac{dM}{dt} = \frac{dx}{dt} + \alpha \frac{dy}{dt} \le x(1-x) + x - M + \alpha\, y(1-\alpha\, y),$$

or $\dfrac{dM}{dt} + M \le x(1-x) + x + \alpha\, y(1-\alpha\, y)$.

Since the logistic terms have a maximum value at 1/4 and $Sup\, x(t) \le 1$. Thus we get

$$\frac{dM}{dt} + M \le \frac{3}{2}$$

Again, using the differential inequality theorem yields

$$M \le \frac{3}{2} - \left(\frac{3}{2} - M(0)\right) e^{-t}, \text{ for } t \ge 0.$$

Hence, for sufficiently large value of t and for any initial value $M(0)$ we have

$$Sup\, M(t) \le \frac{3}{2}$$

This shows the bounded ness of $M(t) = x(t) + \alpha\, y(t)$, which implying to the bounded ness of $y(t)$ in R_+^2. Hence the Kolmogorov system has uniformly bounded solution in R_+^2. ■

Note that due to the above theorem, the Kolmogorov system (2) is dissipative in R_+^2. Further more, the interaction functions of Kolmogorov system (2) are C^2 on the domain R_+^2 and hence they are Lipschizion on R_+^2. Accordingly the solution of the Kolmogorov system (2) with nonnegative initial condition exists and is unique.

3. The Analysis of model and Hopf Bifurcation

The Kolmogorov system (2) has three non-negative equilibrium points namely $E_0 = (0,0)$ and $E_1 = (1,0)$ and $E_2 = (x^*, y^*)$, where $x^* = \dfrac{w_3 - w_2}{w_2\, w_4}$ and $y^* = (1 - x^*)(1 + w_1\, x^*)$.

The equilibrium points $E_0 = (0,0)$ and $E_1 = (1,0)$ are always exists. However the coexistence state point $E_2 = (x^*, y^*)$ exists under Komogorov condition (3b).

Now, in order to investigate the local dynamical behavior of model system (2) around each of the equilibrium points, the Variational matrix J at the point (x, y) is computed as:

$$J = \begin{pmatrix} x\dfrac{\partial f}{\partial x} + f & x\dfrac{\partial f}{\partial y} \\ y\dfrac{\partial g}{\partial x} & y\dfrac{\partial g}{\partial y} + g \end{pmatrix}$$

Therefore, evaluating J at the equilibrium points $E_i\,;\, i = 0,1,2$ yield the following results respectively.

$$J_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$J_1 = \begin{pmatrix} -1 & -\dfrac{1}{1+w_1} \\ 0 & 0 \end{pmatrix}$$

$$J_2 = \begin{pmatrix} x^*\left(-1 + \dfrac{w_1 y^*}{(1+w_1 x^*)^2}\right) & -\dfrac{x^*}{1+w_1 x^*} \\ \dfrac{w_3 w_4 {y^*}^2}{(1+w_4 x^*)^2} & 0 \end{pmatrix}$$

Clearly, the Variational matrix at the equilibrium points $E_0 = (0,0)$ and $E_1 = (1,0)$ has zero eigenvalue, and hence they are non-hyperbolic points. Thus the dynamical behavior near them can be stable or periodic. However, the eigenvalues of the $J_2 = \lfloor a_{ij} \rfloor_{2\times 2}$ are the roots of :

$$\lambda^2 + a\lambda + b = 0$$

With

$$a = -(a_{11} + a_{22}) \tag{4a}$$

$$b = a_{11}a_{22} - a_{12}a_{21} \tag{4b}$$

The Kolmogorov system (2) is locally stable, if the eigenvalues are negative or have negative real parts. Therefore, a necessary and sufficient condition for locally stable at E_2 is $a > 0$ and $b > 0$, where a and b are defined in Eqs (4a) and (4b), respectively.

$$a > 0 \Rightarrow -x^*\left(-1 + \frac{w_1 y^*}{(1+w_1 x^*)^2}\right) > 0$$

Substituting the values of x^* and y^* and then simplifying the terms give:

$$x^* = \frac{w_3 - w_2}{w_2 w_4} > \frac{w_1 - 1}{2w_1} \qquad (5)$$

Also, it is clear that

$$b = \left(\frac{x^*}{1+w_1 x^*}\right)\left(\frac{w_3 w_4 y^{*2}}{(1+w_1 x^*)^2}\right) > 0$$

Thus, for suitable choices of the parameters satisfying Eq. (5) will lead to a solution with stable equilibrium point E_2 and choices violating this condition will lead to limit cycles.

Now, in order to investigate the Hopf bifurcation of Kolmogorov system (2), we will follow the Liu approach [9]. According to Liu approach, the simple Hopf bifurcation at $\theta = \theta_*$ can occurs for a 2-dimensional system provided that:

- $a(\theta_*) = 0;\ b(\theta_*) > 0$ and
- $\left.\frac{d\,a(\theta)}{d\theta}\right|_{\theta=\theta_*} \neq 0$

Where a and b are given by Eqs. (4a) and (4b) respectively.

Now, let w_1, is the bifurcation parameter. Therefore, if the following condition holds

$\frac{w_1^* y^*}{(1+w_1^* x^*)^2} = 1$, this yields

$$w_1^* = \frac{1}{1-2x^*} \text{ with } 0 < x^* < \frac{1}{2} \qquad (6)$$

Then, obviously $a(w_1^*) = 0$ and $b(w_1^*) > 0$. Further it is easy to verify that

$$\left.\frac{d\,a(w_1)}{dw_1}\right|_{w_1=w_1^*} = -\frac{x^*(1-2x^*)^2}{1-x^*} \neq 0$$

Accordingly, the following theorem establishes the Hopf bifurcation conditions.

Theorem 2. The Kolmogorov system (2) admits a simple Hopf bifurcation of the positive equilibrium point E_2 at the critical value of the parameter w_1 given by Eq. (6).

Proof: Follows directly from above analysis.

Finally, we give a sufficient condition for the global stability of the positive equilibrium point E_2.

Theorem 3. If

$$x^* > \frac{w_1 - 1}{w_1} \qquad (7)$$

Then the positive equilibrium point E_2 is globally stable.

Proof. Consider the following positive definite function

$$V(x,y) = \xi\left(\alpha - x^* \log(1 + \frac{\alpha}{x^*})\right) + \eta\left(\beta - y^* \log 91 + \frac{\beta}{y^*})\right)$$

Where ξ and η are arbitrarily chosen positive constants to be determined, however $\alpha = x - x^*$ and $\beta = y - y^*$ are the perturbation about the equilibrium point E_2. Along any trajectory of Kolmogorov system (2), we have

$$\frac{dV}{dt} = \frac{\partial V}{\partial \alpha}.\frac{d\alpha}{dt} + \frac{\partial V}{\partial \beta}.\frac{d\beta}{dt}$$

$$= \xi\alpha\left[1 - x - \frac{y}{1+w_1 x}\right] + \eta\beta\left[w_2 y - \frac{w_3 y}{1+w_4 x}\right]$$

$$= \xi(x-x^*)\left[1 - x - \frac{y}{1+w_1 x} - (1 - x^* - \frac{y^*}{1+w_1 x^*})\right]$$

$$+ \eta(y-y^*)\left[w_2 y - \frac{w_3 y}{1+w_4 x} - (w_2 y^* - \frac{w_3 y^*}{1+w_4 x^*})\right]$$

$$= -\frac{\xi}{1+w_1 x}(1 + w_1 x^* - w_1 + w_1 x)(x-x^*)^2$$

$$-\eta\left[-w_2 + \frac{w_3}{1+w_4 x}\right](y-y^*)^2$$

$$+\left[-\frac{\xi(1+w_1 x^*)}{M_1} + \frac{\eta\, w_3 w_4 y^*}{M_2}\right](x-x^*)(y-y^*)$$

Choose the arbitrarily positive constants ξ and η so that

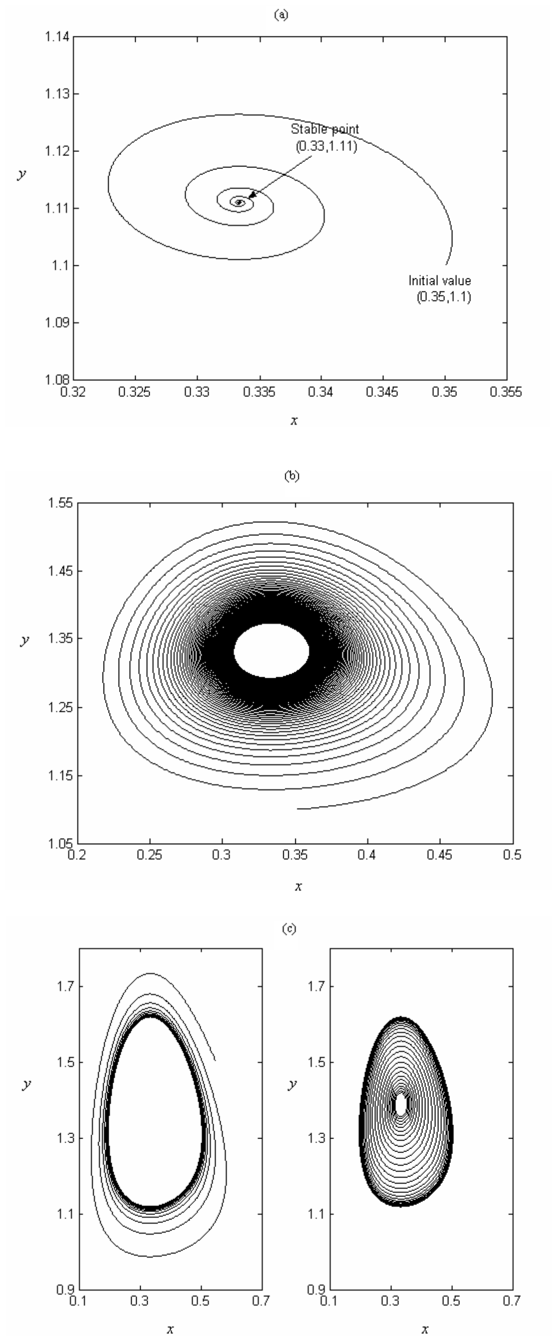
$$\xi = \frac{w_3 w_4 y^* M_1}{(1 + w_1 x^*) M_2} \text{ and } \eta = 1$$

Where: $M_1 = (1 + w_1 x)(1 + w_1 x^*)$ and $M_2 = (1 + w_4 x)(1 + w_4 x^*)$. Hence

$$\frac{dV}{dt} = -\frac{w_3 w_4 y^*}{M_2}(1 + w_1 x^* - w_1 + w_1 x)(x - x^*)^2 - \left[-w_2 + \frac{w_3}{1 + w_4 x} \right](y - y^*)^2$$

Note that, according to Kolmogorov condition (3a), the coefficient for $(y - y^*)^2$ is always negative. Further, if condition (7) is satisfied, then dV/dt is negative definite and hence the function V is Lyapunov function. Thus the positive equilibrium point E_2 of system (2) is globally stable. ■

For the following set of parameter values $w_2 = 0.3$, $w_3 = 0.4$, $w_4 = 1.0$, with $w_1 < 2.99$ the local stability condition (5) for $E_2 = (x^*, y^*)$ is satisfied. However, according to theorem 2, system (2) admits a simple Hopf bifurcation at $w_1 \cong 2.99$. Further, condition (5) is not satisfied and hence there is a stable limit cycle solution for the above set of data with $w_1 \geq 3.0$. Finally, theorem 3 shows that the system (2) has a globally stable positive equilibrium point $E_2 = (x^*, y^*)$ at $w_1 < 1.49$ keeping other parameter values as given above. The numerical simulations in Fig. 1a show the asymptotically stable solution of E_2 at $w_1 = 2$. Fig. 1b shows the transferring from stability to periodic at the bifurcation point $w_1 = 2.99$. However, the presence of a stable limit cycle at $w_1 = 3.25$ is shown in Fig. 1c. Finally, the globally asymptotically stable solution is clearly shown in Fig. 1d for different initial conditions at $w_1 = 1.25$.



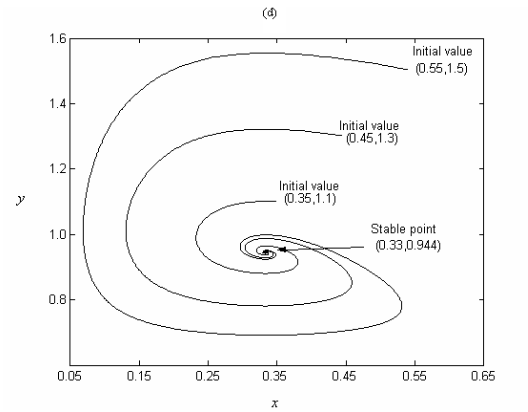


Fig. 1. Dynamical behavior of system (2). (a) Asymptotically stable point at $w_1 = 2$; (b) Hopf bifurcation at $w_1 = 2.99$; (c) Asymptotically stable limit cycle at $w_1 = 3.25$ starting from outside as well as from inside; (d) Global stable point for different initial values at $w_1 = 1.25$.

4. The periodic forced system

We consider the intrinsic growth rates $r_i\ (i=1,2)$ in system (1) as periodically varying functions of time due to seasonal variations. For these parameters sinusoidal perturbations are used with the same periodicity T. Therefore, the periodic forcing (or seasonality) is superimposed as follows:

$$r_1' = r_1\left(1+\varepsilon_1 \sin(\Omega T)\right); \qquad (8a)$$

$$r_2' = r_2\left(1+\varepsilon_2 \sin(\Omega T+\phi)\right). \qquad (8b)$$

Here $r_i\ (i=1,2)$ are the average values of the forced intrinsic growth rates $r_i'\ (i=1,2)$ respectively. The parameters $\varepsilon_i\ (i=1,2)$ represent the degree of seasonality; $r_i\,\varepsilon_i$ are the magnitude of the perturbation in r_i' respectively and Ω is the angular frequency of the fluctuations caused by seasonality. Since $r_i\ (i=1,2)$ are assumed to be positive, therefore $0 \le \varepsilon_i \le 1$. Finally the parameter ϕ, where $0 \le \phi \le 2\pi$, can be interpreted as a difference in phase angle between the seasonality in the intrinsic growth rate of prey and predator. Therefore, according to (8), the original system (1) can be written in non-dimensional non-autonomous form as:

$$\frac{dx}{dt} = x(1-x) - \left(\frac{xy}{1+w_1 x}\right), \qquad (9a)$$
$$+\varepsilon_1 x \sin(\theta t)$$

$$\frac{dy}{dt} = w_2 y^2 - \left(\frac{w_3 y^2}{1+w_4 x}\right). \qquad (9b)$$
$$+\lambda y^2 \sin(\theta t+\phi)$$

Where $w_i\ (i=1,2,3,4)$ as before, $\theta = \Omega/r_1$ and $\lambda = r_2\,\varepsilon_2/A = w_2\,\varepsilon_2$. Further, using $z=\theta t$, the non-autonomous system (9) transfer to the three-dimensional autonomous system.

$$\frac{dx}{dt} = x(1-x) - \left(\frac{xy}{1+w_1 x}\right), \qquad (10a)$$
$$+\varepsilon_1 x \sin(z)$$

$$\frac{dy}{dt} = w_2 y^2 - \left(\frac{w_3 y^2}{1+w_4 x}\right), \qquad (10b)$$
$$+\lambda y^2 \sin(z+\phi)$$

$$\frac{dz}{dt} = \theta. \qquad (10c)$$

With $z(0)=0$.

5. The numerical simulation results for the periodic forced system

The global dynamics of the prey-predator system (10) with seasonality is studied. The solution of the system with initial conditions in the first octant is obtained numerically for biologically feasible range of parametric values. The range of parametric values is selected so that the unforced system is Kolmogorov. The system being nonlinear and three dimensional, variety of behavior in the solution are expected in contrast to the corresponding two-dimensional system without seasonality. The bifurcation diagram provides a summary of essential dynamical behavior of the system.

Indeed the points that are plotted will represent either fixed or periodic sinks or other attracting sets including chaos. It shows the birth, evolution, and death of the attracting sets. The term "bifurcation" refers to significant changes in the set of fixed or periodic points or other sets of interest.

Number of bifurcation diagrams is obtained in three different cases $\phi = 0,\ \pi/2$ and π respectively. The case $\phi = 0$ corresponds to the synchronous periodic forcing or seasonal variations, however $\phi = \pi$ corresponds to the anti-synchronous case for periodic forcing. In each of these cases, for a fixed value of the key parameter, the maximum value of prey species x is plotted after removing the transient effect. Then increment the key parameter and begin the procedure again. We first assume the critical parameter as w_1. Fig. 2 shows the bifurcation diagram for $\varphi = 0$ as a function of w_1 in the range $1 \le w_1 \le 4$ keeping other parameters fixed as

$$\begin{array}{l} w_2 = 0.3, w_3 = 0.4, w_4 = 1.0, \\ \varepsilon_1 = 0.5, \lambda = 0.2, \theta = 0.5 \end{array} . \qquad (11)$$

As shown from Fig. 2, the evidence for cascade of period doubling leading to chaos can be seen clearly for $1.9 < w_1 < 2.8$, the solution becomes chaotic for $w_1 > 2.8$, in between there is periodic windows as for example $3.025 < w_1 < 3.03$, $3.155 < w_1 < 3.165$, and $3.27 < w_1 < 3.282$. Finally, the prey predator system faces extinction in prey species for $3.625 < w_1 < 4.0$ due to the effect of periodic forcing.

Bifurcation diagrams, for the above set of data, are also drawn for the cases $\phi = \pi/2$ and $\phi = \pi$ in Fig. 3 and Fig. 4 respectively. It is observed that, for $\phi = \pi/2$ the solution admits period doubling leading to chaos for the range $2 < w_1 \le 2.9$ then the solution becomes chaotic for the range $2.9 < w_1 \le 3.175$. For the range $3.175 < w_1 \le 3.3$ the solution exhibits period three dynamics, then again period doubling leading to chaos take please for the range $3.3 < w_1 \le 3.475$. However, for the $\phi = \pi$ cascade of period doubling take please, for the range $1.85 < w_1 < 2.7$, leading to chaos. The periodic windows are visible through the chaotic region with narrow intervals see for example $2.915 < w_1 < 2.925$ and $3.04 < w_1 < 3.05$. For the range $3.78 < w_1 < 3.875$ the solution becomes periodic, then again after a cascade of period doubling, in the range $3.875 < w_1 < 3.925$, the solution becomes chaotic in the interval (3.925, 4.0).

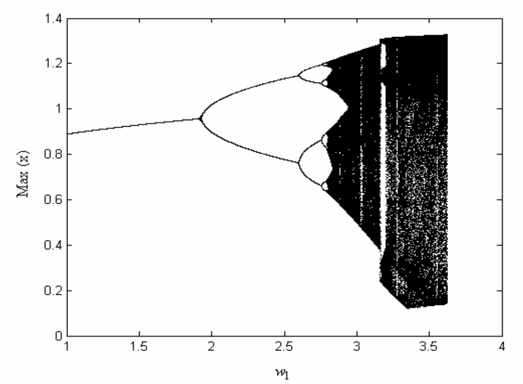


Fig. 2. Bifurcation diagram as a function of w_1 for $\varphi = 0$.

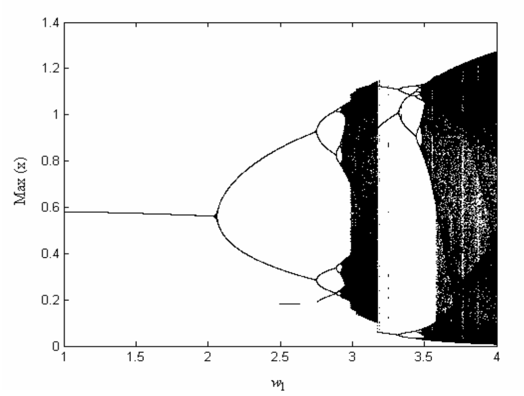


Fig. 3. Bifurcation diagram as a function of w_1 for $\varphi = \pi/2$.

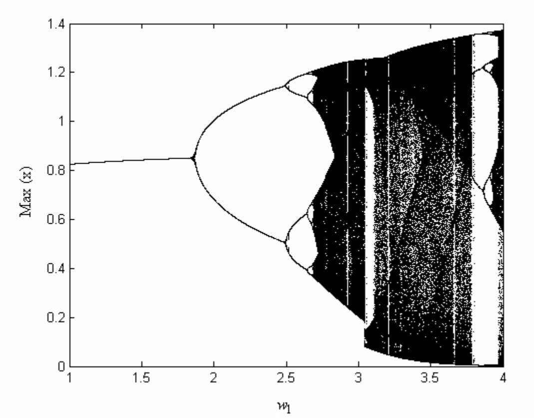


Fig. 4. Bifurcation diagram as a function of w_1 for $\varphi = \pi$.

According to the above bifurcation diagrams, it is observed that the appearance of first period doubling and hence the regions of chaotic are get delayed from the case $\varphi = 0$ to the case $\varphi = \pi/2$. Indeed, the periodic window becomes wider as phase angle changes from 0 to $\pi/2$. Moreover the prey species, which is facing extinction in case $\varphi = 0$ for $3.625 < w_1 < 4.0$, still survive in case $\varphi = \pi/2$. However, for the anti-synchronous case $\varphi = \pi$, the chaotic region becomes denser.

The effect of periodic forcing on the dynamical behavior of unforced system (2) is further investigated using the attracting set of the solution of the system (10). In case $\varphi = 0$ the projection of the attracting set for the solution of system (10) on the $x - y$ plane is plotted, after removing the transient effect, in Fig. 5(a-d) for the parameter values given in Eq. (11) with $w_1 = 2$, $w_1 = 2.7$, $w_1 = 2.775$, and $w_1 = 3$ respectively. The figures show the evidence of the cascade of period doubling leading to chaos.

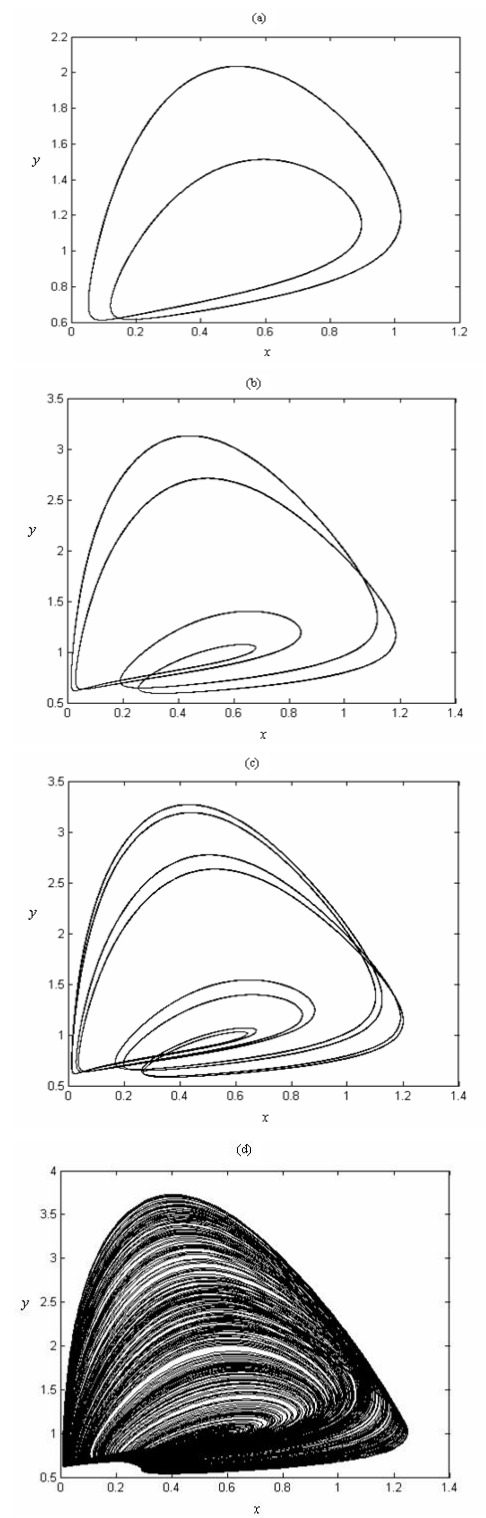
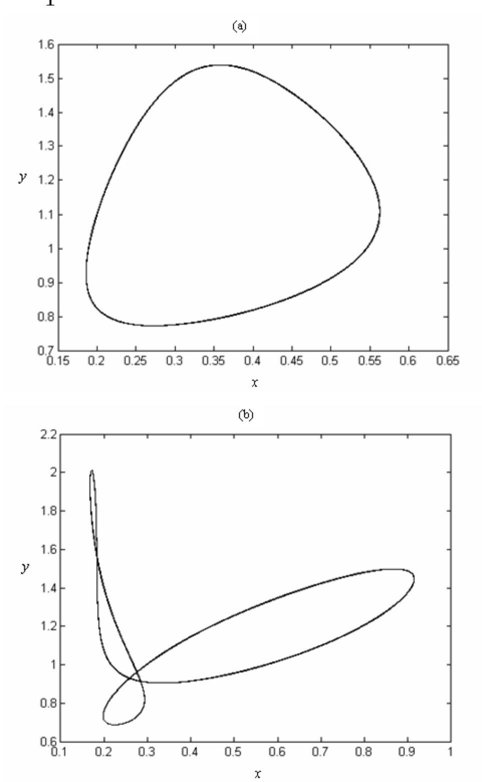


Fig. 5. the attracting set of the solution of system (10) for the parameter values given in Eq. (11). (a) Period 2 at $w_1 = 2$. (b) Period 4 at $w_1 = 2.7$. (c) Period 8 at $w_1 = 2.775$. (d) chaotic attractor at $w_1 = 3$.

In case $\varphi = \pi/2$, Fig. 6(a-f) presence the projection of attracting set of the solution of system (10) on the $x-y$ plane for the parameter values given in Eq. (11) with $w_1 = 2$, $w_1 = 2.7$, $w_1 = 2.775$, $w_1 = 3$, $w_1 = 3.4$, and $w_1 = 3.6$ respectively. In additional to the transition of the solution of system (10) to chaos through cascade of period doubling, the figures show clearly the delay in appearing of period doubling in comparison with the case $\varphi = 0$.

Further, the survival of the prey species is also shown due to the transition of the solution between the periodic and chaotic dynamics along the range $1 \le w_1 \le 4$.



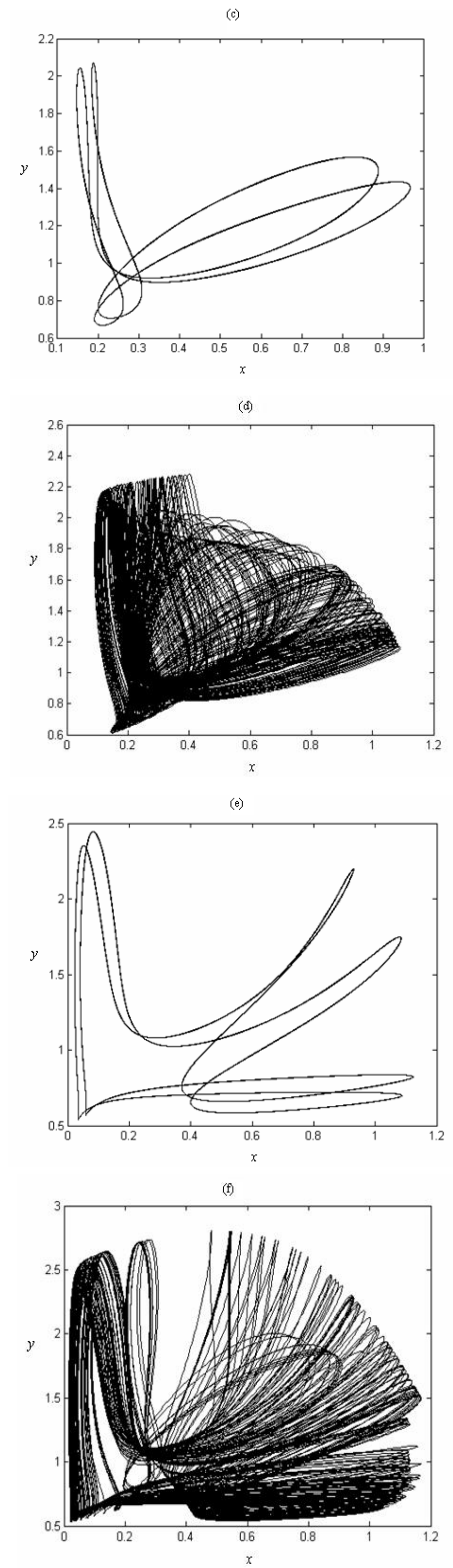
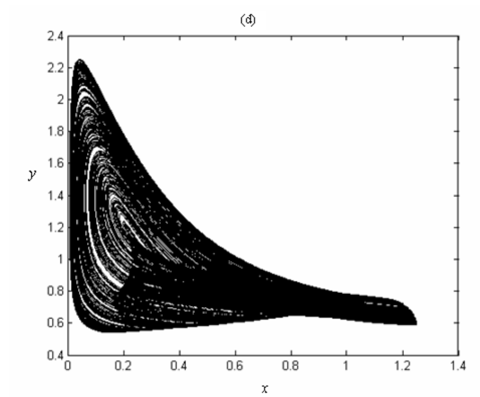


Fig. 6. the attracting set of the solution of system (10) for the parameter values given in Eq. (11). (a) Limit cycle at $w_1 = 2$. (b) Period 2 at $w_1 = 2.7$. (c) Period 4 at $w_1 = 2.775$. (d) Chaotic attractor at $w_1 = 3$. (e) Period 6 at $w_1 = 3.4$. (f) Chaotic attractor at $w_1 = 3.6$.

The attracting set of the solution of system (10), in case $\varphi = \pi$, is shown in Fig. 7(a-f). Again the route to chaos through cascade of period doubling is clearly shown in these figures; also the chaotic region becomes denser. Further, on contrast to the case $\varphi = 0$, both the species of unforced system (2) still survive for all the range $1.0 < w_1 < 4.0$.



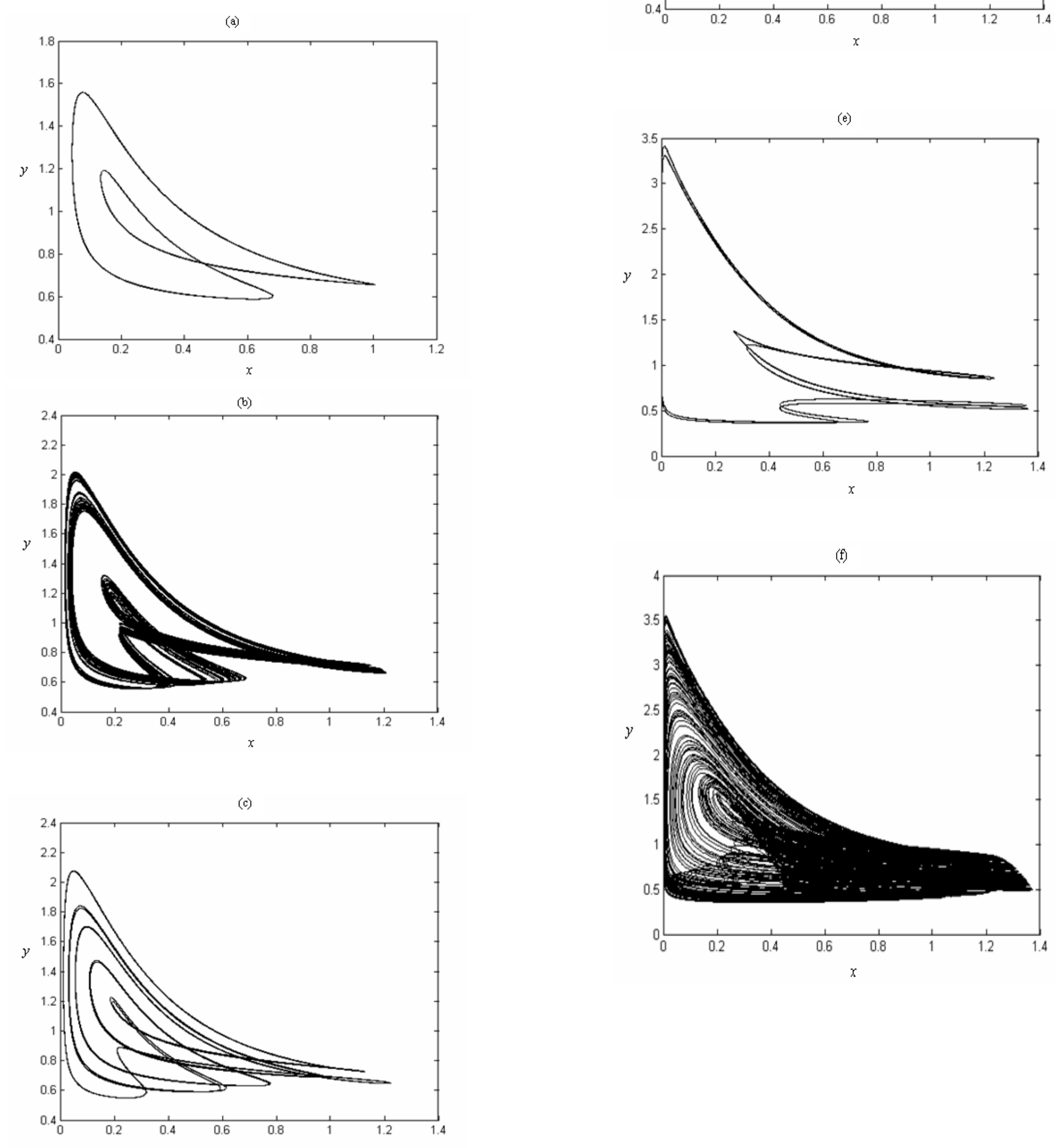


Fig. 7. the attracting set of the solution of system (10) for the parameter values given in Eq. (11). (a) Period 2 at $w_1 = 2$. (b) Chaotic attractor at $w_1 = 2.7$. (c) Long periodic at $w_1 = 2.775$. (d) Chaotic attractor at $w_1 = 3$. (e) Period 6 at $w_1 = 3.9$. (f) Chaotic attractor at $w_1 = 4.0$.

Finally, for further investigation to the effects of periodic forcing on the unforced system (2), different key parameters are also considered separately keeping the other parameters fixed at specific values. The results obtained are almost inline with that of the bifurcation diagrams and attracting sets given above. This shows that the ecological model under consideration, which has a globally stable equilibrium in the constant parameter case (unforced case), has a great number of modes of behavior in the periodically varying case including long periodic and chaos.

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حول السلوك الديناميكي لنظام الفريسة والمفترس مع تأثير القوة الدورية

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الخلاصـة:

تناول البحث دراسة السلوك الديناميكي للنظام الثنائي المستمر المتمثل بنظام الفريسة والمفترس. تم ايجاد الشرط الكافي للاستقرارية الشاملـة للنظام باستخـدام دالة ليابانوف. كما تناول البحث دراسة التفرع من نوع هوبف للنظام. استخدمت المحاكاة لدراسة تأثيرالقوة الدورية على معلمتين مختلفتين عند وجود او عدم وجود اختلاف في زاوية التاثير. لقد اظهرت نتائج المحاكاة ان النظام، تحت تاثير القوة الدورية، يمتلك ديناميكية الفوضى.