Modifications for Quasi-Newton Method and Its Spectral Algorithm for Solving Unconstrained Optimization Problems

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Abstract

In this paper, two modifications for spectral quasi-Newton algorithm of type BFGS are imposed. In the first algorithm, named SQN^{EI}, a certain spectral parameter is used in such a step for BFGS algorithm differs from other presented algorithms. The second algorithm, SQN^{Ev-Iv}, has both new parameter position and value suggestion. In SQN^{EI} and SQN^{Ev-Iv} methods, the parameters are involved in a search direction after an approximated Hessian matrix is updated. It is provided that two methods are effective under some assumptions. Moreover, the sufficient descent property is proved as well as the global and superlinear convergence for SQN^{Ev-Iv} and SQN^{EI}. Both of them are superior the standard BFGS (QN^{BFGS}) and previous spectral quasi-Newton (SQN^{LC}). However, SQN^{Ev-Iv} is outstanding SQN^{EI} if it is convergent to the solution. This means that, two modified methods are in the race for the more efficiency method in terms less iteration numbers and consuming time in running CPU. Finally, numerical results are presented for the four algorithms by running list of test problems with inexact line search satisfying Armijo condition.

Keywords: BFGS algorithm, Inexact line search, Numerical optimization, Spectral quasi-Newton method, Unconstrained optimization.

Introduction

The optimization problem is a model among many mathematical approaches that deals with solving real life problems, solving exactly and numerically, for different branches; such as, statistics; physics and engineering. This leads to more attempted from the researchers to present more efficient methods continuously.

Now, consider a minimization of unconstrained problem:

$$\min f(x), \ x \in \mathcal{R}^n \qquad 1$$

where, $f: \mathcal{R}^n \to \mathcal{R}$ is bounded below and twice differentiable function. Many numerical methods are used for solving Eq.1. Quasi-Newton approaches are among the most recommended methods having the efficiency in solving these types of problems, due to their super-linear rate of convergence and global convergent property.

The minimization of problem Eq.1 would be by many iteration steps in order to gain an acceptable solution, that is:

$$x_{k+1} = x_k + \alpha_k d_k , \ k = 0, 1, \ldots \qquad 2$$

for, $\alpha_k \in \mathcal{R}$ is line search and $d_k \in \mathcal{R}^n$ is direction search toward the reduction of the function. The direction of quasi-Newton methods is given by:

$$d_k = -B_k^{-1} g_k \qquad 3$$

for which, g_k is gradient of f at the iteration step x_k and B_k^{-1} is an approximation of Hessian matrix $\nabla^2 f(x)$ at x_k, denoted by H_k. BFGS is one of effective procedure among quasi-Newton methods, in which its name takes from four developers named: Broyden, Fletcher, Goldfrab and Shanno;[1]. In this most popular algorithm, the inverse of Hessian matrix will be approximated by the formula:

$$H_{k+1} = H_k + \frac{(s_k + H_k y_k)' y_k s_k s_k'}{(s_k' y_k)^2} - \frac{H_k y_k s_k' + s_k y_k' H_k}{s_k' y_k} \qquad 4$$

as,

$$s_k = x_k - x_{k-1} \ , \ y_k = g_k - g_{k-1} \qquad 5$$

whereas, it begins with initial positive definite matrix H_0 up to the required steps. The positive definite property of Hessian matrix approximation or the preserve it after any modifications is the matter that many researchers deal with it. For instance, Mahmood [2] gave a modification BFGS update formula the inverse version, tried to show how it remains symmetric and positive definite, this is the reason to make the problem convergence to the solution in minimization problems. Additionally, this issue is serious in other quasi-Newton method types [3]. The self-correcting property is a technique to overcome the ill-conditioned problem, for this, Cheng and Li [4] scaled the quasi-Newton equation and suggested a new spectral scaling for BFGS method with this property. Their method has the property of self-correcting alike as conventional BFGS has with more efficiency in correcting a large eigenvalue of Hessian matrix approximation might suffer from. This leads to the improvement in BFGS method. Minutely, in their work, they use the exact line search to minimize strictly convex problem from the dimension n, in which it is terminated in n steps as steepest descent method. Also, in uniformly convex problems, the method with Wolfe condition is globally and R- linear convergent. Nakayama et al., [5] studied a symmetry rank 1 memoryless quasi-

Newton with a parameter of spectral scaling given in [4]. Later, the global convergence of the formula proposed by Nakayama and Narushima [6]. Finally, the hybridization of it with three-term conjugate gradient utilizing a spectral parameter was designed by Nakayama [7]. Additionally, the efficiency of memoryless BFGS method using spectral scaling of [4] in minimizing the eigenvalues was proved by Lv et al. [8].

However, conjugate gradient methods (CGM) have many studies in this area. Firstly, a gradient parameter is used in proposing a new spectral scaling parameter [9]. Another idea was nested between spectral parameter and CGM one and this was given by Wang et al., [10] with presenting the importance of it in solving a large-scale problems. Furthermore, a fast CGM is given with combining a new direction with spectral parameter and previous direction, this is proposed in [11]. Also, the convex combination idea take a place in this topic, the spectral scaling defined as a convex combination of two CGM coefficients [12]. In a constrained optimization problem with bounded condition, there is a spectral parameter with memoryless property for Broydon class presented by Nakayama et al. [13]. Eventually, for a real live example, scientist use the spectral algorithms to analyze the problems as a drug abuse problems; see [14].

After all the presented works, SQN^{EI} and SQN^{Ev-Iv} algorithms are proposed; that deal with new position for spectral scaling parameter and value. In more details, it is an idea to think how changing the involving parameters out of updating formula for Hessian matrix approximation in BFGS method is affecting in the optimization processing. This is aiming at finding more efficient algorithms in spectral scaling methods due to their importance in soling real-life problems. The article, presents two methods with their step algorithms in next section. The third section is proved the convergence of the two method and the relationship between them along with some mild assumptions. There is a proof for the sufficient decent, global and superlinear rate convergence. Finally, the numerical results are presented.

Materials and Methods

In this work, two spectral quasi-Newton methods have been suggested as follows:

The SQN$^{\text{Ev-Iv}}$ Algorithm

In the first part, the SQN$^{\text{Ev-Iv}}$ algorithm is suggested, in which the acceleration parameter is involved in search direction; after the Hessian matrix inverse approximation has done. In other words, the search direction contains all required update for BFGS Hessian matrix formula, then it multiples with the spectral scaling parameter. Now, our new suggestion algorithm SQN$^{\text{Ev-Iv}}$; which the spectral parameter sets in the direction is as following:

$$d_k = \begin{cases} -H_k g_k & k=0 \\ -\left(\dfrac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} H_k g_k & k\geq 1 \end{cases} \qquad 6$$

with, the condition:

$$y_k^T s_k > \|y_k\|^2 \qquad 7$$

Thus, the algorithm steps are given as:

Step 1: Initializations step: choose H_0 identity matrix or positive definite matrix $X_0 \in \mathbb{R}^n$, tolerance $\varepsilon = 1 \times 10^{-7}$

Step 2: Start with $d_0 = -H_0 g_0$, $k = 1$

Step 3: Termination criteria, if $\|g_k\| \leq \varepsilon$ or maximum number of iterations reached **Stop**.

Step 4: Find inexact step size α_k satisfies Wolfe conditions

Step 5: Compute $s_k = x_k - x_{k-1}$ and $y_k = g_k - g_{k-1}$

Step 6: Check weather $y_k^T s_k > \|y_k\|^2$, if not; **Stop**

Step 7: Update H_k in Eq.4.

Step 8: Evaluate search direction by Eq. 6

Step 9: set $k = k + 1$, go to step 3.

The SQN$^{\text{EI}}$ Algorithm

This subsection is about another new algorithm named SQN$^{\text{EI}}$. The spectral parameter of Cheng and Li (2010) [4] is used but in different step of algorithm. The direction is given as this formula:

$$d_k = \begin{cases} -H_k g_k & k=0 \\ -\dfrac{y_k^T s_k}{\|y_k\|^2} H_k g_k & k\geq 1 \end{cases} \qquad 8$$

So, the steps of the SQN$^{\text{EI}}$ algorithm are the same as previous subsection, but without step 6 and in step 8, d_k is defined as in Eq.8.

The Convergence Analysis of SQN$^{\text{Ev-Iv}}$ and SQN$^{\text{EI}}$

The convergence analysis of our algorithm is discussed in this part.

A list of Assumptions:

For conducting the analysis in section 3, some assumptions are needed as follows:

(i) Assume that f, an objective function, is twice continuously differentiable.

(ii) The Lipschitz continuous properties for the Hessian matrix at x^*, that is, it is satisfying the inequality:

$$\|\nabla^2 f(x) - \nabla^2 f(x^*)\| \leq c\|x - x^*\|$$

with the existing of a positive constant c and all x in neighborhood of x^*

(iii) If the objective function $f \in C^2$ and $D = \{x : f(x) \leq f(x_0)\}$ is a convex level set, then there exist two positives k_1 and k_2 satisfying $k_1\|u\|^2 \leq u'\nabla^2 f(x)u \leq k_2\|u\|^2$ $\forall u \in \Re^n$, $x \in D$ and $\nabla^2 f(x)$ is Hessian matrix of f.

The Relationship between SQN$^{\text{Ev-Iv}}$ and SQN$^{\text{EI}}$ Parameters

It is obvious that, SQN$^{\text{Ev-Iv}}$ parameter is a function of SQN$^{\text{EI}}$. However, there is a relation between them; as long as there is a condition for the one in SQN$^{\text{Ev-Iv}}$, this means that:

$$\frac{y_k^T s_k}{\|y_k\|^2} > 1 \qquad\qquad 9$$

Then, multiplying both sides of Eq.9 $\frac{y_k^T s_k}{\|y_k\|^2}$, and taking the square root to them to obtain the following:

$$\left(\frac{y_k^T s_k}{\|y_k\|^2}\right)^{1/2} < \frac{y_k^T s_k}{\|y_k\|^2}$$

$$\frac{1}{\sqrt{2}}\left(\frac{y_k^T s_k}{\|y_k\|^2}\right)^{1/2} < \frac{1}{\sqrt{2}}\frac{y_k^T s_k}{\|y_k\|^2}$$

$$\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} < \frac{y_k^T s_k}{\|y_k\|^2}$$

Which means the parameter of SQN^Ev-Iv is less than the parameter of SQN^EI.

Sufficient Descent Property of Two Algorithms

In this algorithm, assumed that the direction as given in Eq.6 and Eq.8, that is; it is used to prove the descent direction:

$$g_0^T d_0 = -g_0^T H_0 g_0$$

$$\le -c\|g_0\|^2$$

By Assumption 1 (iii)

This means that it has descent direction property for $k = 0$.

Now, it is wanted to prove for $k \ge 1$.

Since, in this part the direction search is given as

$$d_k = -\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} H_k g_k$$

and

$$g_k^T d_k = g_k^T\left(-\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} H_k g_k\right)$$

$$= -\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} g_k^T H_k g_k$$

$$g_k^T d_k \le -c\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} .$$

For positive constant c, since y_k^T and s_k are in the same direction, and by the assumption 1. (iii), one can get

$$g_k^T d_k \le -c\,\frac{y_k^T s_k}{\|y_k\|^2} .$$

Therefore, $g_k^T d_k$ has descent direction property in SQN^Ev-Iv. In the same way for SQN^EI the property holds for it is gotten.

The Global Convergence Analysis
In order to prove the global convergence of the proposed algorithms, Lemma 7 in [15] showed a line search with Armijo condition and the property of descent direction satisfies one or both following inequalities. That is,

if $h(x_{k+1}) = f(x_{k+1}) - f(x_k)$, then either:

$$h(x_{k+1}) \le -n_1 \frac{(g_k^T d_k)^2}{\|d_k\|^2} \qquad\qquad 10$$

or

$$h(x_{k+1}) \le -n_2\, g_k^T d_k \qquad\qquad 11$$

where n_1 and n_2 any positive constant.

Theorem 1: Let assumption 1 is hold; search direction d_k is given by Eq.8 satisfied descent condition and α_k has Armijo condition inequality. Then.

$$\lim_{k\to\infty} \inf \|g_k\| = 0$$

Proof: It will prove by contradiction. Let $\delta > 0$ and $\|g_k\| > \delta$ and

$$d_k = -\frac{y_k^T s_k}{\|y_k\|^2} H_k g_k$$

$$\|d_k\| = \left\|-\frac{y_k^T s_k}{\|y_k\|^2} H_k g_k\right\| \le \frac{\|y_k^T s_k\|}{\|y_k\|^2}\|H_k\|\|g_k\|$$

$$\frac{\|d_k\|}{\|g_k\|^2} \le \frac{\|y_k^T s_k\|\|H_k\|}{\|y_k\|^2\|g_k\|}$$

since H_k is bounded then $\|H_k\| < u$, therefore:

$$\frac{\|d_k\|}{\|g_k\|^2} \le \frac{\|y_k^T s_k\|\, u}{\|y_k\|^2\|g_k\|}$$

and by assumption 1 (iii), it is:

$$m\|s_k\|^2 \leq y_k^T s_k \leq M\|s_k\|^2$$

where, m and M are positive constant.

Since $y_k^T = s_k^T \nabla^2 f$, then:

$$\frac{y_k^T s_k}{\|y_k\|^2} = \frac{s_k^T \nabla^2 f \, s_k}{s_k^T (\nabla^2 f)^2 s_k} = \frac{r_k^T r_k}{r_k^T \nabla^2 f r_k} = \frac{1}{Ray(\nabla^2 f, r)} = \frac{1}{\lambda} > 0$$

with $M = \frac{1}{\lambda}$, therefore:

$$\frac{\|d_k\|}{\|g_k\|^2} \leq \frac{M\,u}{\|g_k\|}$$

Now, by our assumption, $\|g_k\| > \delta$; then it is obtained:

$$\frac{\|d_k\|}{\|g_k\|^2} \leq \frac{M\,u}{\delta}$$

So,

$$\frac{\|g_k\|^4}{\|d_k\|^2} \geq \frac{\delta^2}{M^2 u^2}$$

$$\sum_{k=0}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} \geq \sum_{k=0}^{\infty} \frac{\delta^2}{M^2 u^2} = \infty$$

Hence,

$$\sum_{k=0}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} \geq \infty$$

But there is, $g_k^T d_k = -\|g_k\|^2$, thus:

$$\sum_{k=0}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} = \sum_{k=0}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} \geq \infty.$$

This is contradiction to Eq.9 and Eq.10. Therefore:

$$\lim_{k \to \infty} \inf \|g_k\| = 0$$

Hence, it is global convergent.

Remark 1: Theorem 1 is hold, the global convergence, when the direction of SQN$^{\text{EI}}$ algorithm is used. Therefore, by subsection 2, the same result is obtained.

Superlinear Rate of Convergence
In this section, the superlinear convergence is presented and proved. In order to do that, some principles are need and recall them in [16] two lemmas

4.9 and 4.10. Then with holding the assumption 1, a sequence of numbers as $\{\epsilon_k\}$ such that

$$\frac{\|y_k - \nabla^2 f(x^*) s_k\|}{\|s_k\|} \leq \epsilon_k \quad \text{and} \quad \sum_{k=1}^{\infty} \epsilon_k < \infty$$

whenever, y_k and s_k are given in Eq.5. These all tend to the boundedness of the sequence of Hessian matrix approximation and its inverse, that is, $\{H_k\}$ and $\{H_k^{-1}\}$:

Therefore:

$$\lim_{k \to \infty} \frac{\|(H_k^{-1} - \nabla^2 f(x^*)) s_k\|}{\|s_k\|} = 0$$

Theorem 2: Suppose assumptions 1 (i) and (iii) are holds, $\{x_k\} \to x^*$ and the sequence $\{H_k\}$, $\{H_k^{-1}\}$ are bounded. If $x_{k+1} = x_k + d_k$ holds for all sufficiently large k with

$$\lim_{k \to \infty} \frac{\|(H_k^{-1} - \nabla^2 f(x^*)) d_k\|}{\|d_k\|} = 0, \quad \text{then} \quad \lim_{k \to \infty} \frac{\|x_{k+1} - x^*\|}{\|x_{k+1} - x_k\|} = 0.$$

Proof:

$$[H_k^{-1} - \nabla^2 f(x^*)] d_k = H_k^{-1} d_k - \nabla^2 f(x^*) d_k$$

$$= H_k^{-1} [-\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} H_k g_k] - \nabla^2 f(x^*) d_k$$

$$= -\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} H_k^{-1} H_k g_k -$$

$$\nabla^2 f(x^*) d_k = -\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} g_k - \nabla^2 f(x^*) d_k$$

$$= -\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} [g_k - \nabla^2 f(x^*) d_k] +$$

$$[-\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} - 1] \nabla^2 f(x^*) d_k$$

$$= -\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} g_{k+1} + \left[-\left(\frac{y_k^T s_k}{2\|y_k\|^2}\right)^{1/2} - \right.$$

$$\left. 1 \right] \nabla^2 f(x^*) d_k + o(\|d_k\|)$$

Therefore:

$$\|(H_k^{-1} - \nabla^2 f(x^*)) d_k\| = O(1)\|g_{k+1}\| + o(\|d_k\|)$$

as it has given that:

$$\lim_{k\to\infty} \frac{\left\|\left(H_k^{-1} - \nabla^2 f(x^*)\right)d_k\right\|}{\|d_k\|} = 0$$

so that:

$$\lim_{k\to\infty} \frac{\|g_{k+1}\|}{\|d_k\|} = \lim_{k\to\infty} \frac{\|g_{k+1}\|}{\|x_{k+1} - x_k\|} = 0$$

On the other hand:

$$g_{k+1} - \nabla^2 f(x^*) - \nabla^2 f(x^*)(x_{k+1} - x^*) = o(\|x_{k+1} - x^*\|), \text{ with } \nabla^2 f(x^*) = 0,$$

Hence, $\lim_{k\to\infty} \frac{\|x_{k+1} - x^*\|}{\|x_{k+1} - x_k\|} = 0$.

Remark 2: In the same manner of Theorem 1, easily it can prove that SQN^Ev-Iv has superlinear rate of convergence.

Results and Discussion

This section gives the results and presents all findings throughout the performance profile plots, the cumulative distribution function. In this way, the significant difference will only show in the interesting area. The time of CPU running and the number of iterations are two criterions in showing the effectiveness of the suggested algorithms in numerical optimization branch. This way is used in this paper. For the comparison, 46 function tests are used, that are in each of [17] and [18], as listed in Table 1. About the dimensions, the various dimensions for functions are taken when the function is multivariate. This means that, if n is the dimension number, then for the first three functions in Table 1, when ID = 1, 2 and 3, $n = 2$ is used, while for all other listed functions; $n = 2,4,6,8,15,30$ is used except Diagonal 2, ID = 14, the used dinamsions were $n = 2,3,4,10,15$. As a final result, the overall data becomes 260 for plotting the performance profile.

The Fig.1 presents plots for the four procedures. In details, Fig.1(a) shows how the SQN^Ev-Iv algorithm behaves in terms of iteration numbers for all test functions. The SQN^EI is preferable than QN^BFGS and SQN^LC methods for reducing iteration numbers. However, this result is inaccurate with SQN^Ev-Iv algorithm after satisfying the condition Eq.7. Whenever the problem is convergent, SQN^Ev-Iv is more recommended among the four algorithms. Meanwhile, Fig.1(b) reveals the time consuming of CPU for running all of contest algorithms. Again, the SQN^Ev-Iv procedure is better than others. For

conducting the analysis in Fig.1, the number of test functions was 43; that is, all functions identify in Table 1 is utlized excluding functions $ID = 24,37,39$. Furthermore, there were two functions, $ID = 5,11$, made a terrible for some dimension, for instance, function with $ID = 5$, $n = 6,20,30$ and $ID = 11$, $n = 20,30$ are excluded from the analysis. In other word, 43 test problems are filtered with Eq.7.

On the other hand, Fig.2 demonstrates how the cumulative distribution line changes when involves all 46 problems named in Table 1; but inequality Eq.7 is ignored in SQN^Ev-Iv algorithm. Along with this failure, SQN^Ev-Iv remains the selected algorithm comparing with QN^BFGS and SQN^LC except SQN^EI. Overall, SQN^EI is dominates the all engaged methods in this study without the inequality Eq.7 holds.

For all programs, the MATLAB 2018a codes are written with using inexact line search satisfying the strong Wolf condition and the error $\varepsilon = 1 \times 10^{-7}$ or iteration number reached at maximum number, in which it was $1000n$; where n is the dimension of objective function. Furthermore, the Table 1 contains the name of all functions used in the comparison with some suggested dimensions. For running programs, the suggested initial values for those functions given in [17]; is used however, there were some testing functions with no initial points; in this case, the border values of defined region is used as an initial point in [18].

Table 1. List of test functions with some dimension

ID	Function Name	ID	Function Name	ID	Function Name
1	Camel Six Hump	17	Diagonal 4	33	HIMMELH
2	Camel Three Hump	18	Diagonal 5	34	HIMMELBG
3	Brent	19	Diagonal 6	35	SINCOS
4	Raydan 1	20	Diagonal 7	36	BIGGSB1
5	Raydan 2	21	Diagonal 8	37	DIXON3DQ
6	Fletcher	22	Diagonal 9	38	LIARWHD
7	Ext. White & Holst	23	Hager	39	Ext. DENSCHNF
8	Gen. Quartic	24	Ext. Tridiagonal 1	40	NONSCOMP
9	Gen. Tridiagonal 1	25	Ext. TET	41	CUBE
10	Ext. Freudenstein Roth	26	ARWHEAD	42	Ext. Penalty
11	Ext. Beale	27	Ext. DENSCHNB	43	ENGVAL1
12	Cliff	28	COSINE	44	POWER
13	Diagonal 1	29	SINE	45	DQDRTIC
14	Diagonal 2	30	Full Hessian FH3	46	Gen. Quartic
15	Diagonal 3	31	Quartc		
16	Almost Perturbed Quadratic	32	Ext. BD1 Block Diagonal		

(a)　　　　　　　　　　　　　　　　　　　(b)
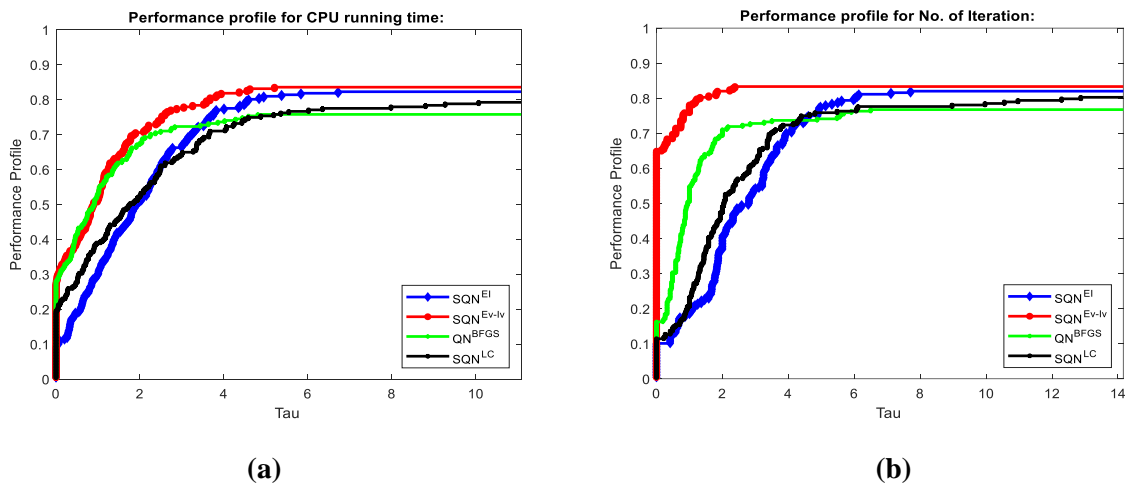
Figure 1. Performance profile for four algorithms, with filtered functions (a) Number of iterations and (b) CPU running time
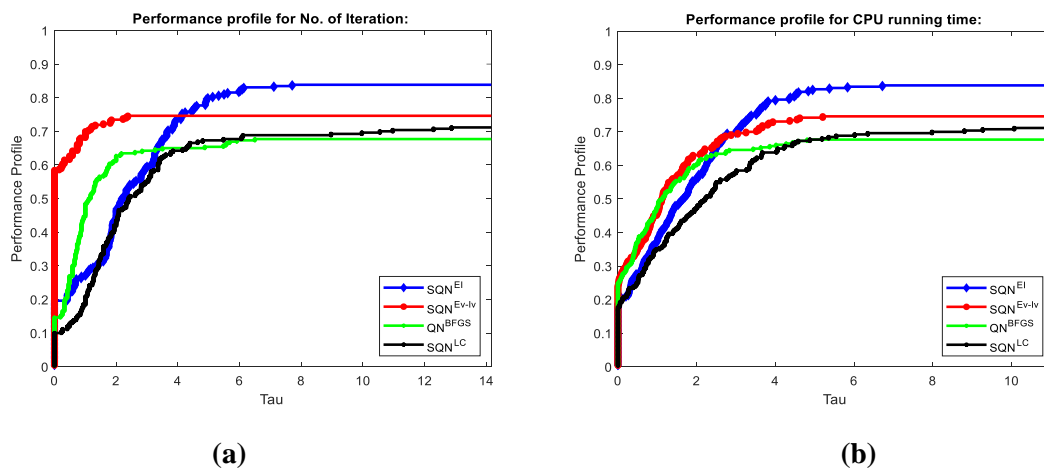

(a)　　　　　　　　　　　　　　　　　　　(b)

Figure 2. Performance profile for algorithms, with all functions (a) Number of iterations and (b) CPU running time

Conclusion

The paper suggested two new algorithms, SQN^{EI} and SQN^{Ev-Iv}. SQN^{EI} depended on the new position in the usage of defined spectral parameter for the past work while, SQN^{Ev-Iv} beside the new place of it; there is a new value to accelerate the process of solving problems. In the past, one or two spectral parameters were used in a Hessian approximation matrix updating formula. However, this new technique shows the effectiveness of approaches in optimizing problems. That is to say, SQN^{EI} and

SQN^{Ev-Iv} algorithms were preferable in comparison to each of QN^{BFGS} and SQN^{LC} according to running computer system processer and iteration numbers. In general, SQN^{EI} is better than others. However, there is a contest between the two proposed algorithms by a condition decision; it made SQN^{Ev-Iv} better than all competitor methods in convergence and filtering its condition. Finally, SQN^{Ev-Iv} is the best among the four algorithms if it success in finding the optimal solution.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are mine/ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary

permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Salahaddin.

Authors' Contribution Statement

E. L.S made, proved, figures, tested the function running through suggested algorithms, I. S.L. did

the supervision, investigation and reviewing the manuscript.

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تحديثات على طريقة شبه نيوتن وخوارزميتها الطيفية لحل مشاكل التحسين غير المقيدة

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الخلاصة

في هذا البحث تم فرض تعديلين لخوارزمية شبه نيوتن الطيفية من النوع BFGS. في إحدى الخوارزميات، المسماة SQN^{EI} يتم استخدام معلمة طيفية معينة في احدى خطوات الخوارزمية BFGS تختلف عن جميع الخوارزميات المعروضة. اما الخوارزمية الثانية SQN^{Ev-Iv} تقدم مكان جديد و قيمة جديدة لمعلمة طيفية. في كلا الطرقتين تستخدم المعلمات في اتجاه البحث لخوارزمية شبه نيوتن بعد تحديث مصفوفة Hessian التقريبية. وقد تم تبيان فعالية الطريقتين بوجود بعض الفرضيات. بالاضافة الى ذلك، تم اثبات خاصية الانحدار الكافي مع التقارب الشامل و التقارب الخطي الفائق للخوارزميات المقترحة و تفوقت الطرقتين على خوارزمية BFGS الاساسية والخوارزمية SQN^{LC} التي استخدمت مكان اخر للمعلمة ذاتها .

الكلمات المفتاحية: الخوارزمية BFGS، خط البحث غير تام، الامثلية العددية، طريقة كواسي- نيوتن الطيفي، الامثلية غير مقيدة.

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