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On Semigroup Ideals and Right n-Derivation in 3-Prime Near-Rings

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Abstract:

The current paper studied the concept of right n-derivation satisfying certified conditions on semigroup ideals of near-rings and some related properties. Interesting results have been reached, the most prominent of which are the following: Let $\mathcal{M}$ be a 3-prime left near-ring and $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n$ are nonzero semigroup ideals of $\mathcal{M}$, if d is a right n-derivation of $\mathcal{M}$ satisfies on of the following conditions,

(i) $d(\mathfrak{u}_1, \mathfrak{u}_2, \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, \mathfrak{u}_n) = 0 \ \forall \ \mathfrak{u}_1 \epsilon \mathcal{A}_1, \mathfrak{u}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n \epsilon \mathcal{A}_\mathfrak{u}$;

(ii) $d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) = 0$
$\forall \mathfrak{u}_1, \mathfrak{v}_1 \epsilon \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n, \mathfrak{v}_n \epsilon \mathcal{A}_\mathfrak{u}$;

(iii) $d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) = (\mathfrak{u}_j, \mathfrak{v}_j) \ \forall \ \mathfrak{u}_1, \mathfrak{v}_1 \epsilon \mathcal{A}_1$
$, \mathfrak{u}_2, \mathfrak{v}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n, \mathfrak{v}_n \epsilon \mathcal{A}_\mathfrak{u}$;

(iv) If $d + d$ is an n -additive mapping from $\mathcal{A}_1 \times \mathcal{A}_2 \times \dots \times \mathcal{A}_n$ to $\mathcal{M}$;

(v) $d(\mathfrak{u}_1, \mathfrak{u}_2, \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, \mathfrak{u}_n) \in \mathcal{Z}(\mathcal{M}) \ \forall \ \mathfrak{u}_1 \epsilon \mathcal{A}_1, \mathfrak{u}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n \epsilon \mathcal{A}_\mathfrak{u}$,;

(vi) $d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) \in \mathcal{Z}(\mathcal{M}) \ \forall \ \mathfrak{u}_1, \mathfrak{v}_1 \epsilon \mathcal{A}_1$
$, \mathfrak{u}_2, \mathfrak{v}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n, \mathfrak{v}_n \epsilon \mathcal{A}_\mathfrak{u}$;

Then $\mathcal{M}$ is a commutative ring.

Keywords: Generalized right derivations, Prime near-ring, Right derivations, Right $\mathfrak{n}$ –derivations, Semigroup ideals.

Introduction:

A left near-ring is a nonempty set $\mathcal{M}$ with two binary operations $(+)$ and $(\cdot)$ which satisfies (i) $(\mathcal{M}, +)$ is a group that is not necessarilv abelian, (ii) $(\mathcal{M}, \ \cdot)$ is a semi group, (iii) $a \cdot (b + c) = a \cdot b + a \cdot c$ for each $a, b, c \ \epsilon \mathcal{M}$ (recall that when $\mathcal{M}$ satisfies the right distributive law, $(a + b).c = a \cdot c + b \cdot c$ for each $a, b, c \ \epsilon \mathcal{M}$, then $\mathcal{M}$ will be called right near-ring), usually $\mathcal{M}$ will be 3-prime, if for $x.y \in \mathcal{M}$; $x\mathcal{M}y = \{0\}$ implies $x = 0$ or $y = 0$. A left near-ring $\mathcal{M}$ is called zero-symmetric if $0x = 0$, for all $x \in \mathcal{M}$ (left distributivity yields $x0 = 0$). $\mathcal{Z}(\mathcal{M})$ will refer to the multiplicative center of $\mathcal{M}$. Let $0 \neq \mathcal{A} \subseteq \mathcal{M}$, then $\mathcal{A}$ is said to be a semigroup ideal of $\mathcal{M}$ if $\mathcal{A}\mathcal{M} \subseteq \mathcal{M}$ and $\mathcal{M}\mathcal{A} \subseteq \mathcal{M}$. For each $\mathfrak{m}, \mathfrak{n} \in \mathcal{M}, then$ $(\mathfrak{m}, \mathfrak{n}) = \mathfrak{m} + \mathfrak{n} - \mathfrak{m} - \mathfrak{n}$, $[\mathfrak{m}, \mathfrak{n}] = \mathfrak{m}\mathfrak{n} - \mathfrak{n}\mathfrak{m}$ and $\mathfrak{m} \circ \mathfrak{n} = \mathfrak{m}\mathfrak{n} + \mathfrak{n}\mathfrak{m}$ will be denoted to the additive commutator, Lie product, and Jordan product, respectively. For more about near-ring, make reference to Pilz[1].

Certain mappings, involving some algebraic identities, defined on rings[2,3] or near- rings[4-6] and sometimes on an appropriate subset of them, and the effect of these mappings on the algebraic structure of the near-rings, how the near-rings can be converted into rings or commutative rings, was a study project that has attracted the interest of many researchers over the past three decades.

Different types of mappings, such as derivations, generalized derivations, left derivations, homoderivations and multipliers on near-rings or rings have been studied and some related properties have been discussed, see[7-10]. Also, the derivation concepts generalization has been studied by various means according to different authors such as n –derivations, $(\sigma, \ \tau)$-n-derivation, right n-derivation and generalized right n-derivation, on near-ring and obtained new interest results for researchers in this field[11-14]. Majeed and Farhan[15] are the first to define the concepts of right derivation and right n-derivation on the near-ring.

Let d be an additive mapping from $\mathcal{M}$ into itself, d is said to be a right derivation of $\mathcal{M}$ if $d(\mathfrak{m}\mathfrak{n}) = d(\mathfrak{m})\mathfrak{n} + d(\mathfrak{n})\mathfrak{m}$ for each $\mathfrak{m}, \mathfrak{n} \in \mathcal{M}$.

Let $d: \underbrace{\mathcal{M} \times \mathcal{M} \times \ldots \times \mathcal{M}}_{n-times} \longrightarrow \mathcal{M}$ be n-additive mapping (i.e. additive in each argument), d is said to be right n-derivation of $\mathcal{M}$ if the following equations hold for each $\mathfrak{m}_1, \mathfrak{m}_1', \mathfrak{m}_2, \mathfrak{m}_2', \ldots, \mathfrak{m}_n, \mathfrak{m}_n' \in \mathcal{M}$:

$$d(\mathfrak{m}_1\mathfrak{m}_1', \mathfrak{m}_2, \ldots, \mathfrak{m}_n) = d(\mathfrak{m}_1, \mathfrak{m}_2, \ldots, \mathfrak{m}_n)\mathfrak{m}_1' + d(\mathfrak{m}_1', \mathfrak{m}_2, \ldots, \mathfrak{m}_n)\mathfrak{m}_1$$

$$d(\mathfrak{m}_1, \mathfrak{m}_2\mathfrak{m}_2', \ldots, \mathfrak{m}_n) = d(\mathfrak{m}_1, \mathfrak{m}_2, \ldots, \mathfrak{m}_n)\mathfrak{m}_2' + d(\mathfrak{m}_1, \mathfrak{m}_2', \ldots, \mathfrak{m}_n)\mathfrak{m}_2$$

$$\vdots$$

$$d(\mathfrak{m}_1, \mathfrak{m}_2, \ldots, \mathfrak{m}_n\mathfrak{m}_n') = d(\mathfrak{m}_1, \mathfrak{m}_2, \ldots, \mathfrak{m}_n)\mathfrak{m}_n' + d(\mathfrak{m}_1, \mathfrak{m}_2, \ldots, \mathfrak{m}_n')\,\mathfrak{m}_n$$ [15]

In this line of inspection, this work will give new essential results in this field and generalize some known results presented.

Note that from now, $\mathcal{M}$ will be 3-prime left near-ring, the abbreviation $\mathcal{C.R}$ will refer to the commutative ring while $\mathcal{R.D}$ and $\mathcal{R}.\mathfrak{n}.\mathcal{D}$ are a brief to the right derivation and right n-derivation respectively.

Preliminaries

Lemma 1:[4] [Lemmas 1.2(ii) and 1.3(ii)]

(i) If $\mathfrak{z} \in \mathcal{Z}(\mathcal{M})/\{0\}$ for which $\mathfrak{z} + \mathfrak{z} \in \mathcal{Z}(\mathcal{M})$, then $(\mathcal{M}, +)$ is abelian.

(ii) If $\mathfrak{z} \in \mathcal{Z}(\mathcal{M})/\{0\}$ and $\mathfrak{z}\mathfrak{s} \in \mathcal{Z}(\mathcal{M})$ or $\mathfrak{s}\mathfrak{z} \in \mathcal{Z}(\mathcal{M})$, where $\mathfrak{s} \in \mathcal{M}$, then $\mathfrak{s} \in \mathcal{Z}(\mathcal{M})$.

Lemma 2: [4] [Lemma 1.3(i), 1.4(i) and 1.5] Let $\mathcal{A} \neq 0$ be a semigroup ideal of $\mathcal{M}$, $\mathfrak{s} \in \mathcal{M}$.

(i) If $\mathcal{A}\mathfrak{s} = \{0\}$ or $\mathfrak{s}\mathcal{A} = \{0\}$, then $\mathfrak{s} = 0$.

(ii) If $\mathfrak{t}\mathcal{A}\mathfrak{s} = \{0\}$, $\mathfrak{t}, \mathfrak{s} \in \mathcal{M}$, then either $\mathfrak{t} = 0$ or $\mathfrak{s} = 0$.

(iii) If $\mathcal{A} \subseteq \mathcal{Z}(\mathcal{M})$, then $\mathcal{M}$ is a $\mathcal{C.R}$.

(iv) If $[\mathfrak{s}, \mathfrak{v}] = 0$, for any $\mathfrak{v} \in \mathcal{A}$, then $\mathfrak{s} \in \mathcal{Z}(\mathcal{M})$.

Lemma 3: [14] [Lemma 2.6] $\mathcal{M}$ is zero symmetric if and only if $\mathcal{M}$ admitting $\mathcal{R}.\mathfrak{n}.\mathcal{D}$.

Lemma 4: Let $\mathcal{A} \neq 0$ be a semigroup ideal of $\mathcal{M}$. If $(\mathcal{A}, +)$ is abelian then $(\mathcal{M}, +)$ is abelian.

Proof: Since $\mathfrak{a} + \mathfrak{b} = \mathfrak{b} + \mathfrak{a}$ for any $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, substitute $\mathfrak{a}\mathfrak{s}$ for $\mathfrak{a}$ and $\mathfrak{a}\mathfrak{t}$ for $\mathfrak{b}$ to get $\mathfrak{a}\mathfrak{s} + \mathfrak{a}\mathfrak{t} = \mathfrak{a}\mathfrak{t} + \mathfrak{a}s$ for any $\mathfrak{a} \in \mathcal{A}, \mathfrak{s}, \mathfrak{t} \in \mathcal{M}$ it follows $\mathfrak{a}(\mathfrak{s} + \mathfrak{t} - \mathfrak{s} - \mathfrak{t}) = 0$ $\mathfrak{a} \in \mathcal{A}, \mathfrak{s}, \mathfrak{t} \in \mathcal{M}$, so $\mathcal{A}(\mathfrak{s} + \mathfrak{t} - \mathfrak{s} - \mathfrak{t}) = \{0\}$, thus $(\mathcal{M}, +)$ abelian by Lemma 2 (i).

Lemma 5: Let $\mathcal{A}$ and $\mathcal{B}$ be nonzero semigroup ideals of $\mathcal{M}$, if the additive commutator $(\mathfrak{a}, \mathfrak{b}) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{a} \in \mathcal{A}$ and $\mathfrak{b} \in \mathcal{B}$ then $\mathcal{M}$ is abelian.

Proof: From assumption $(\mathfrak{s}\mathfrak{a}, \mathfrak{s}\mathfrak{b}) = \mathfrak{s}(\mathfrak{a}, \mathfrak{b}) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{a} \in \mathcal{A}$, $\mathfrak{b} \in \mathcal{B}$, $\mathfrak{s} \in \mathcal{M}$, using Lemma 1(ii) leads to either $(\mathfrak{a}, \mathfrak{b}) = 0$ for any $\mathfrak{a} \in \mathcal{A}$, $\mathfrak{b} \in \mathcal{B}$ or $\mathfrak{s} \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{s} \in \mathcal{M}$, by Lemma 2(iii) the last result can be reduced to either $(\mathfrak{a}, \mathfrak{b}) = 0$ for any $\mathfrak{a} \in \mathcal{A}, \mathfrak{b} \in \mathcal{B}$ or $\mathcal{M}$ is a $\mathcal{C.R}$.

If $(\mathfrak{a}, \mathfrak{b}) = 0$ for any $\mathfrak{a} \in \mathcal{A}$, $\mathfrak{b} \in \mathcal{B}$, then substitute $\mathfrak{a}\mathfrak{s}$ for $\mathfrak{a}$ and $\mathfrak{a}b$ for $\mathfrak{b}$ to get $\mathfrak{a}\mathfrak{s} + \mathfrak{a}\mathfrak{b} = \mathfrak{a}\mathfrak{b} + \mathfrak{a}s$ for any $\mathfrak{a} \in \mathcal{A}, \mathfrak{b} \in \mathcal{B}, \mathfrak{s} \in \mathcal{M}$ it follows $\mathfrak{a}(\mathfrak{s} + \mathfrak{b} - \mathfrak{s} - \mathfrak{b}) = 0$ for any $\mathfrak{a} \in \mathcal{A}, \mathfrak{b} \in \mathcal{B}, \mathfrak{s} \in \mathcal{M}$ so $\mathcal{A}(\mathfrak{s} + \mathfrak{b} - \mathfrak{s} - \mathfrak{b}) = \{0\}$ for any $\mathfrak{b} \in \mathcal{B}, \mathfrak{s} \in \mathcal{M}$, thus $(\mathcal{M}, +)$ abelian by Lemma 2(i) and Lemma 4 and this complete the proof.

Corollary 1: Let $\mathcal{A}$ be a nonzero semigroup ideals of $\mathcal{M}$, if $(\mathfrak{a}, \mathfrak{b}) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{a}, \mathfrak{b} \in \mathcal{A}$, then $\mathcal{M}$ is abelian.

Corollary 2: Let $\mathcal{A}$ be a nonzero semigroup ideals of $\mathcal{M}$, if $(\mathfrak{a}, \mathfrak{s}) \in \mathcal{Z}(\mathcal{M})$ or $(\mathfrak{s}, \mathfrak{a}) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{a} \in \mathcal{A}$ and, $\mathfrak{s} \in \mathcal{M}$ then $\mathcal{M}$ is abelian.

Lemma 6: Let d be a $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are a nonzero semigroup ideals of $\mathcal{M}$, if $d(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n) = \{0\}$, then $d = 0$.

Proof: For any $\mathfrak{u}_1 \in \mathcal{A}_1$, $\mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_\mathfrak{j} \in \mathcal{A}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n} \in \mathcal{A}_\mathfrak{u}$,

$0 = d(m_1\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n}) = d(m_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n})\mathfrak{u}_1$, that is

$d(m_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n})\mathcal{A}_1 = \{0\}$ for any $m_1 \in \mathcal{M}$, $\mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_\mathfrak{j} \in \mathcal{A}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n} \in \mathcal{A}_\mathfrak{u}$, so

$d(m_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n}) = 0$

for any $m_1 \in \mathcal{M}$, $\mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_\mathfrak{j} \in \mathcal{A}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n} \in \mathcal{A}_\mathfrak{u}$ according to Lemma 2(i), replace $\mathfrak{u}_2$ by $m_2\mathfrak{u}_2$ in the last result, to obtain $d(m_1, m_2, \ldots, \mathfrak{u}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n}) = 0$ for any $m_1, m_2 \in \mathcal{M}$, $\ldots, \mathfrak{u}_\mathfrak{j} \in \mathcal{A}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n} \in \mathcal{A}_\mathfrak{u}$ and proceeding inductively to conclude $d = 0$.

Lemma 7: Let d be a $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are a nonzero semigroup ideals of $\mathcal{M}$, and $\mathfrak{s} \in \mathcal{M}$. $d(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n)\mathfrak{s} = \{0\}$, then $\mathfrak{s} = 0$.

Proof: For any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_\mathfrak{j} \in \mathcal{A}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n} \in \mathcal{A}_\mathfrak{u}$,

$d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n})\mathfrak{s} = 0$, that is

$$0 = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{s}\mathfrak{u}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n})\,\mathfrak{s} = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{s}, \ldots, \mathfrak{u}_\mathfrak{n})\mathfrak{u}_\mathfrak{j}\mathfrak{s},$$ it follows

$d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{s}, \ldots, \mathfrak{u}_\mathfrak{n})\mathcal{A}_\mathfrak{j}\mathfrak{s} = \{0\}$, using Lemma 2(ii) implies

Either $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{s}, \ldots, \mathfrak{u}_\mathfrak{n}) = 0$ or $\mathfrak{s} = 0$

if $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{s}, \ldots, \mathfrak{u}_\mathfrak{n}) = 0$, then for any $\mathfrak{u}_1 \in \mathcal{A}_1$, $\mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_\mathfrak{n} \in \mathcal{A}_\mathfrak{u}$, then

$$d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{p}_\mathfrak{j}\mathfrak{s})\mathfrak{q}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n}) = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{p}_\mathfrak{j}\mathfrak{s}, \ldots, \mathfrak{u}_\mathfrak{n})\mathfrak{q}_\mathfrak{j} + d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{q}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n})\mathfrak{p}_\mathfrak{j}\mathfrak{s} = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{q}_\mathfrak{j}, \ldots, \mathfrak{u}_\mathfrak{n})\mathfrak{p}_\mathfrak{j}\mathfrak{s}$$

As well,

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$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{p}_j(\mathfrak{s}\mathfrak{q}_j),\dots,\mathfrak{u}_n)=$
$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{p}_j,\dots,\mathfrak{u}_n)\mathfrak{s}\mathfrak{q}_j+$
$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{s}\mathfrak{q}_j,\dots,\mathfrak{u}_n)\mathfrak{p}_j=0$
Comparing the two last expressions to conclude
$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{q}_j,\dots,\mathfrak{u}_n)\mathfrak{p}_j\mathfrak{s}=0$
$d(\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_j,\dots,\mathcal{A}_n)\mathcal{A}_j\mathfrak{s}=\{0\}$, use Lemma 2(ii) and Lemma 6 to imply that $\mathfrak{s}=0$.

Corollary 3: Let d be $\mathcal{R}.n.\mathcal{D}$ (or, $\mathcal{R}.\mathcal{D}$) of $\mathcal{M}$ and $\mathcal{A}\neq 0$ be a semigroup ideal of $\mathcal{M}$, and $\mathfrak{s}\in\mathcal{M}$. If $d(\mathcal{A},\mathcal{A},\dots,\mathcal{A})\mathfrak{s}=\{0\}$(or, If $d(\mathcal{A})\mathfrak{s}=\{0\}$) then $\mathfrak{s}=0$.

Corollary 4: **[**Lemma 2.5[7]] Let $\mathcal{M}$ be a prime near-ring, $d\neq 0$ is a $\mathcal{R}.\mathfrak{n}.\mathcal{D}$ of $\mathcal{M}$ and $\mathfrak{s}\in\mathcal{M}$. If $d(\mathcal{M},\mathcal{M},\dots,\mathcal{M})\mathfrak{s}=\{0\}$, then $\mathfrak{s}=0$.

Corollary 5: **[**Lemma 2.5 [7]] Let $\mathcal{M}$ be a prime near-ring , $d\neq 0$ is a $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$ and $\mathfrak{s}\in\mathcal{M}$. If $d(\mathcal{M})\mathfrak{s}=\{0\}$, then $\mathfrak{s}=0$.

Lemma 8: **[**Lemma 2.5 [7]] "If $\mathcal{M}$ is referring to a $\mathcal{R}.\mathfrak{n}.\mathcal{D}$ $d\neq 0$ such that $d([x,y],x_2,\dots,x_n)=0$ for each $x,y,x_2,\dots,x_n\in\mathcal{M}$, then $\mathcal{M}$ is a C.R".

The following Lemma is a direct result of Lemma 8.

Lemma 9: If $\mathcal{M}$ is referring to a $\mathcal{R}.\mathfrak{n}.\mathcal{D}$ $d\neq 0$ and $(\mathcal{M},+)$ is abelian, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Proof: If $(\mathcal{M},+)$ is abelian, then $d([x,y],x_2,\dots,x_n)=0$ for each $x,y,x_2,\dots,x_n\in\mathcal{M}$ and thus $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$ according to Lemma 8.

Lemma 10: Let $\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_j,\dots,\mathcal{A}_n$ be nonzero simigroup ideals of $\mathcal{M}$ and d is a $\mathcal{R}.n,\mathcal{D}$ of $\mathcal{M}$. If there is $\mathfrak{z}\in\mathcal{A}_j$ such that $d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{z},\dots,\mathfrak{a}_n)=0$ for any $\mathfrak{a}_1\in\mathcal{A}_1,\mathfrak{a}_2\in\mathcal{A}_2,\dots\mathfrak{z}\in\mathcal{A}_j,\dots,\mathfrak{a}_n\in\mathcal{A}_n$, then either $d=0$ or $\mathfrak{z}\in\mathcal{Z}(\mathcal{M})$.

Proof: $\forall\mathfrak{a}_1\in\mathcal{A}_1,\mathfrak{a}_2\in\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j\in\mathcal{A}_j,\dots,\mathfrak{a}_n\in\mathcal{A}_n$,
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{u}_j(\mathfrak{z}\mathfrak{v}_j),\dots,\mathfrak{a}_n)=$
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{u}_j,\dots,\mathfrak{a}_n)\mathfrak{z}\mathfrak{v}_j+$
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{v}_j,\dots,u_n)\mathfrak{z}\mathfrak{u}_j$ 1
Also,
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,(\mathfrak{u}_j\mathfrak{z})\mathfrak{v}_j,\dots,\mathfrak{a}_n)=$
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{u}_j,\dots,\mathfrak{a}_n)\mathfrak{z}\mathfrak{v}_j+$
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{v}_j,\dots,u_n)\mathfrak{u}_j\mathfrak{z}$ 2
Combining Eq.1 and Eq.2 to get
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{v}_j,\dots,u_n)\mathfrak{u}_j\mathfrak{z}=$
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{v}_j,\dots,u_n)\mathfrak{z}\mathfrak{u}_j$. Put $\mathfrak{p}_j\mathfrak{u}_j$ instead of $\mathfrak{u}_j$ in last equation to and use it another time to get
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{v}_j,\dots,u_n)\mathfrak{p}_j\mathfrak{u}_j\mathfrak{z}=$
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{v}_j,\dots,u_n)\mathfrak{z}\mathfrak{p}_j\mathfrak{u}_j=$
$d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{v}_j,\dots,u_n)\mathfrak{p}_j\mathfrak{z}\mathfrak{u}_j$, which can be written as $d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{v}_j,\dots,u_n)\mathcal{A}_j[\mathfrak{z},,\mathfrak{u}_j]=0$, it follows either $d(\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_j,\dots,\mathcal{A}_n)=\{0\}$ or $[\mathfrak{z},,\mathfrak{u}_j]=0$ for any $\mathfrak{u}_j\in\mathcal{A}_j$ according to Lemma 2(ii) and use Lemma 6 to conclude either $d=0$ or $[\mathfrak{z},,\mathfrak{u}_j]=0$ for any $\mathfrak{u}_j\in\mathcal{A}_j$, i.e. $\mathfrak{u}_j\mathfrak{z}=\mathfrak{z}\mathfrak{u}_j$ for any $\mathfrak{u}_j\in\mathcal{A}_j$, put $\mathfrak{u}_j n$, where $n\in\mathcal{M}$, in last equation and use it to get $\mathfrak{u}_j n\mathfrak{z}=\mathfrak{z}\mathfrak{u}_j n=\mathfrak{u}_j\mathfrak{z}n$ and this result leads to $\mathcal{A}_j[\mathfrak{z},n]=\{0\}$ and Lemma 2(i) ensures that $\mathfrak{z}\in\mathcal{Z}(\mathcal{M})$.

Corollary 6: Let $\mathcal{A}$ be a nonzero simigroup ideal of $\mathcal{M}$and d is a $\mathcal{R}.n,\mathcal{D}$ of $\mathcal{M}$. If there is $\mathfrak{z}\in\mathcal{A}$ such that $d(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{z},\dots,\mathfrak{a}_n)=0$ for any $\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n\in\mathcal{A}$ then either $d=0$ or $\mathfrak{z}\in\mathcal{Z}(\mathcal{M})$.

Corollary 7: Let $\mathcal{A}$ be a nonzero simigroup ideal of $\mathcal{M}$and d is an $\mathcal{R},\mathcal{D}$ of $\mathcal{M}$. If there is $\mathfrak{z}\in\mathcal{A}$ such that $d(\mathfrak{z})=0$, then either $d=0$ or $\mathfrak{z}\in\mathcal{Z}(\mathcal{M})$.

Corollary 8: Let d be a $\mathcal{R}.n,\mathcal{D}$ of $\mathcal{M}$. If there is $\mathfrak{z}\in\mathcal{M}$ such that $d(m_1,m_2,\dots,\mathfrak{z},\dots,m_n)=0$ for any $m_1,m_2,\dots,m_n\in\mathcal{M}$ then either $d=0$ or $\mathfrak{z}\in\mathcal{Z}(\mathcal{M})$.

Corollary 9: Let d be an $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$. If there is $\mathfrak{z}\in\mathcal{M}$ such that $d(\mathfrak{z})=0$, then either $d=0$ or $\mathfrak{z}\in\mathcal{Z}(\mathcal{M})$.

Main Results:

Theorem 1: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_n$ are a nonzero semigroup ideals of $\mathcal{M}$, if $d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n)=0$ for any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j\in\mathcal{A}_j,\dots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{u}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Proof: By assumption
$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n)=0$ for any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j\in\mathcal{A}_j,\dots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{u}$, so,
$0=d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{s}\mathfrak{u}_j,\mathfrak{s}\mathfrak{v}_j),\dots,\mathfrak{u}_n)$
$=d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{s}(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n)$
$=d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{s},\dots,\mathfrak{u}_n)(\mathfrak{u}_j,\mathfrak{v}_j)$ for any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j\in\mathcal{A}_j,\dots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{u},\mathfrak{s}\in\mathcal{M}$.
Hence $(\mathfrak{u}_j,\mathfrak{v}_j)=0$ for any $\mathfrak{u}_j,\mathfrak{v}_j\in\mathcal{A}_j$ by Lemma 2.7, thus $(\mathcal{M},+)$ is abelian by Lemma 4. Therefore $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$ by Lemma 9.

Corollary 10: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}\neq 0$ be a semigroup ideal of $\mathcal{M}$, if $d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n)=0$ for any $\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j,\dots,\mathfrak{u}_n\in\mathcal{A}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 11: Let d be a nonzero $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}\neq 0$ be a semigroup ideal of $\mathcal{M}$, if $d((\mathfrak{a},\mathfrak{b}))=0$ for any $\mathfrak{a},\mathfrak{b}\in\mathcal{A}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 12: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, if $d(\mathfrak{s}_1,\mathfrak{s}_2,\dots,(\mathfrak{s},\mathfrak{m}),\dots,\mathfrak{s}_n)=0$ for any $\mathfrak{s}_1,\mathfrak{s}_2,\dots,\mathfrak{s},\mathfrak{m},\dots,\mathfrak{s}_n\in\mathcal{M}$, then $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$.

Corollary 13: Let d be a nonzero $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$, if $d((\mathfrak{s},\mathfrak{m}))=0$ for any $\mathfrak{s},\mathfrak{m}\in\mathcal{M}$, then $\mathcal{M}$ *is* $\mathcal{C}.\mathcal{R}$.

Theorem 2: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_n$ are a nonzero semigroup ideals of $\mathcal{M}$, if
$d\left((\mathfrak{u}_1,\mathfrak{v}_1),(\mathfrak{u}_2,\mathfrak{v}_2),\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,(\mathfrak{u}_n,\mathfrak{v}_n)\right)=0$
for any

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$\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_u$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Proof: By assumption

$$d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, (\mathfrak{u}_j, \mathfrak{v}_j), \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) = 0$$

for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_u$

Thus,

$$0 = d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, (\mathfrak{s}\mathfrak{u}_j, \mathfrak{s}\mathfrak{v}_j), \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right)$$
$$= d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, \mathfrak{s}(\mathfrak{u}_j, \mathfrak{v}_j), \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right)$$
$$= d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, \mathfrak{s}, \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right)(\mathfrak{u}_j, \mathfrak{v}_j)$$

for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$.

If $m \in \mathcal{M}$, then

$$0 = d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, \mathfrak{s}, \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right)(m\mathfrak{u}_j, m\mathfrak{v}_j) = d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, \mathfrak{s}, \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right)m(\mathfrak{u}_j, \mathfrak{v}_j),$$

It follows $d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, \mathfrak{s}, \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right)\mathcal{M}(\mathfrak{u}_j, \mathfrak{v}_j) = \{0\}$ for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$.

Three primeness of $\mathcal{M}$ implies either $d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, \mathfrak{s}, \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) = 0$ or $(\mathfrak{u}_j, \mathfrak{v}_j) = 0$ for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$, proceeding as above to arrive at $d = 0$ (a contradiction) or there is $i \in \{1,2,\ldots,n\}$ s.t $(\mathfrak{u}_j, \mathfrak{v}_j) = 0$ which implies $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$ because of Lemma 4 and Lemma 9.

Corollary 14: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, if $d\left((s_1, t_1), (s_2, t_2), \ldots, (s_n, t_n)\right) = 0$ for any $s_1, t_1, s_2, t_2, \ldots, s_n, t_n \in \mathcal{M}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Theorem 3: Let d be a $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are a nonzero semigroup ideals of $\mathcal{M}$, if $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_j, \mathfrak{v}_j), \ldots, \mathfrak{u}_n) = (\mathfrak{u}_j, \mathfrak{v}_j)$ for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_u$, then $\mathcal{M}$ *is* $\mathcal{C}.\mathcal{R}$.

Proof: For any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_u$

$$d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_j, \mathfrak{v}_j), \ldots, \mathfrak{u}_n) = (\mathfrak{u}_j, \mathfrak{v}_j) \qquad 3$$

Therefore

$$d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{s}\mathfrak{u}_j, \mathfrak{s}\mathfrak{v}_j), \ldots, \mathfrak{u}_n) = (\mathfrak{s}\mathfrak{u}_j, \mathfrak{s}\mathfrak{v}_j)$$

for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$.

It follows

$$\mathfrak{s}(\mathfrak{u}_j, \mathfrak{v}_j) = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{s}(\mathfrak{u}_j, \mathfrak{v}_j), \ldots, \mathfrak{u}_n) = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{s}, \ldots, \mathfrak{u}_n)(\mathfrak{u}_j, \mathfrak{v}_j) + (\mathfrak{u}_j, \mathfrak{v}_j)\mathfrak{s}$$

for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$.

Thus, for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$,

$$\mathfrak{s}(\mathfrak{u}_j, \mathfrak{v}_j) = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{s}, \ldots, \mathfrak{u}_n)(\mathfrak{u}_j, \mathfrak{v}_j) + (\mathfrak{u}_j, \mathfrak{v}_j)\mathfrak{s}$$

Put $(\mathfrak{p}_j, \mathfrak{q}_j)$ instead of s in last equation and use Eq.3 to get $(\mathfrak{u}_j, \mathfrak{v}_j)(\mathfrak{p}_j, \mathfrak{q}_j) = 0$ for any $\mathfrak{u}_j, \mathfrak{v}_j, \mathfrak{p}_j, \mathfrak{q}_j \in \mathcal{A}_j$. It follows $0 = (\mathfrak{u}_j, \mathfrak{v}_j)(m\mathfrak{p}_j, m\mathfrak{q}_j) = (\mathfrak{u}_j, \mathfrak{v}_j)m(\mathfrak{p}_j, \mathfrak{q}_j)$ for any $\mathfrak{u}_j, \mathfrak{v}_j, \mathfrak{p}_j, \mathfrak{q}_j \in \mathcal{A}_j, m \in \mathcal{M}$, three primeness of $\mathcal{M}$ implies $(\mathfrak{u}_j, \mathfrak{v}_j) = 0$ for any $\mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j$. Hence $(\mathcal{M}, +)$ abelian by Lemma 4, consequently $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$ by Lemma 9.

Corollary 15: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}$ is a nonzero semigroup ideal of $\mathcal{M}$, if $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_j, \mathfrak{v}_j), \ldots, \mathfrak{u}_n) = (\mathfrak{u}_j, \mathfrak{v}_j)$ for any for any $\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 16: Let d be a nonzero $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}$ is a nonzero semigroup ideal of $\mathcal{M}$, if $d\left((\mathfrak{u}_j, \mathfrak{v}_j)\right) = (\mathfrak{u}_j, \mathfrak{v}_j)$ for any for any $\mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 17: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, if $d(\mathfrak{s}_1, \mathfrak{s}_2, \ldots, (\mathfrak{s}, \mathfrak{m}), \ldots, \mathfrak{s}_n) = (\mathfrak{s}, \mathfrak{m})$ for any $\mathfrak{s}_1, \mathfrak{s}_2, \ldots, \mathfrak{s}, \mathfrak{m}, \ldots, \mathfrak{s}_n \in \mathcal{M}$, then $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$.

Corollary 18: Let d be a nonzero $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$, if $d((\mathfrak{s}, \mathfrak{m})) = (\mathfrak{s}, \mathfrak{m})$ for any $\mathfrak{s}, \mathfrak{m} \in \mathcal{M}$, then $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$.

Theorem 4: Let d be a $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are a nonzero semigroup ideals of $\mathcal{M}$, if $d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, (\mathfrak{u}_j, \mathfrak{v}_j), \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) = (\mathfrak{u}_j, \mathfrak{v}_j)$ for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_u$, then $\mathcal{M}$ *is* $\mathcal{C}.\mathcal{R}$.

Proof: From hypothesis,

$$d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, (s\mathfrak{u}_j, s\mathfrak{v}_j), \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) = (s\mathfrak{u}_j, s\mathfrak{v}_j)$$

for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$.

It follows

$$\mathfrak{s}(\mathfrak{u}_j, \mathfrak{v}_j) = d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, s(\mathfrak{u}_j, \mathfrak{v}_j), \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) = d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, s, \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right)(\mathfrak{u}_j, \mathfrak{v}_j) + (\mathfrak{u}_j, \mathfrak{v}_j)\mathfrak{s}$$

for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$

Therefore,

$$\mathfrak{s}(\mathfrak{u}_j, \mathfrak{v}_j) = d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \ldots, s, \ldots, (\mathfrak{u}_n, \mathfrak{v}_n)\right)(\mathfrak{u}_j, \mathfrak{v}_j) + (\mathfrak{u}_j, \mathfrak{v}_j)\mathfrak{s}$$

Put $(\mathfrak{p}_j, \mathfrak{q}_j)$ instead of s in last equation and use hypothesis to get $(\mathfrak{u}_j, \mathfrak{v}_j)(\mathfrak{p}_j, \mathfrak{q}_j) = 0$ for any

$\mathfrak{u}_j, \mathfrak{v}_j, \mathfrak{p}_j, \mathfrak{q}_j \in \mathcal{A}_j$. Using the same way as used in the end of proof of Theorem 3, implies the desired result.

Corollary 19: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, if $d\left((s_1,t_1),(s_2,t_2),\dots,(s_j,t_j),\dots,(s_n,t_n)\right) = (s_j,t_j)$ for any $s_1,t_1,\ s_2,t_2,\dots,s_j,t_j,\dots,s_n,t_n \in \mathcal{M}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Theorem 5: Let d be a $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_n$ are a nonzero semigroup ideals of $\mathcal{M}$, if $d+d$ is an n -additive mapping from $\mathcal{A}_1\times\mathcal{A}_2\times\dots\times\mathcal{A}_n$ to $\mathcal{M}$, then $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$.

Proof: From hypothesis: For any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j\in\mathcal{A}_j,\dots,\mathfrak{u}_n\in\mathcal{A}_u$

$(d+d)(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j+\mathfrak{v}_j,\dots,\mathfrak{u}_n) = (d+d)(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j,\dots,\mathfrak{u}_n) + (d+d)(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{v}_j,\dots,\mathfrak{u}_n)$

$= d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j,\dots,\mathfrak{u}_n) + d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j,\dots,\mathfrak{u}_n)$
$+ d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{v}_j,\dots,\mathfrak{u}_n) + d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{v}_j,\dots,\mathfrak{u}_n)$

As well,

$$(d+d)(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j+\mathfrak{v}_j,\dots,\mathfrak{u}_n) = d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j+\mathfrak{v}_j,\dots,\mathfrak{u}_n) + d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j+\mathfrak{v}_j,\dots,\mathfrak{u}_n)$$

$$= d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j,\dots,\mathfrak{u}_n) + d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{v}_j,\dots,\mathfrak{u}_n)$$
$$+d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{u}_j,\dots,\mathfrak{u}_n) + d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{v}_j,\dots,\mathfrak{u}_n)$$

Comparing the last two expressions to conclude $d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n) = 0$, the required result obtained by Theorem 1.

Corollary 20: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}$ is a nonzero semigroup ideals of $\mathcal{M}$,if $d+d$ is an n -additive mapping from $\mathcal{A}\times\mathcal{A}\times\dots\times\mathcal{A}$ to $\mathcal{M}$, then $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$.

Corollary 21: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, if $d+d$ is an n -additive mapping on $\mathcal{M}$, then $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$.

Corollary 22: Let d be a nonzero $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$ and $\mathcal{A}$ is a nonzero semigroup ideals of $\mathcal{M}$, if $d+d$ is an additive on $\mathcal{A}$ then $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$

Corollary 23: Let d be a nonzero $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$, if $d+d$ is an additive on $\mathcal{M}$ then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Theorem 6: Let d_1 and d_2 are two nonzero $\mathcal{R}.n.\mathcal{D}'s$ of $\mathcal{M}$ ($\mathcal{M}$ is two torsion free) and $\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_n,\mathcal{B}_1,\mathcal{B}_2,\dots,\mathcal{B}_n$ are nonzero semigroup ideals of $\mathcal{M}$, if $d_1(\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_n)d_2(\mathcal{B}_1,\mathcal{B}_2,\dots,\mathcal{B}_n)\subseteq\mathcal{Z}(\mathcal{M})$., then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Proof: By assumption: for any $\mathfrak{a}_1\in\mathcal{A}_1,\mathfrak{a}_2\in\mathcal{A}_2,\dots,\mathfrak{a}_n\in\mathcal{A}_n,\mathfrak{b}_1\in\mathcal{B}_1,\mathfrak{b}_2\in\mathcal{B}_2,\dots,\mathfrak{b}_j\in\mathcal{B}_j,\dots,\mathfrak{b}_n\in\mathcal{B}_n$.

$d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{b}_j,\dots,\mathfrak{b}_n)\in\mathcal{Z}(\mathcal{M})$..

Therefore

$d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{b}_j^2,\dots,\mathfrak{b}_n) = d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{b}_j,\dots,\mathfrak{b}_n)\mathfrak{b}_j$
$+d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{b}_j,\dots,\mathfrak{b}_n)\mathfrak{b}_j$
$= d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{b}_j,\dots,\mathfrak{b}_n)(\mathfrak{b}_j+\mathfrak{b}_j)\in\mathcal{Z}(\mathcal{M})$.

Use Lemma 1(ii) to get,
for any
$\mathfrak{a}_1\in\mathcal{A}_1,\mathfrak{a}_2\in\mathcal{A}_2,\dots,\mathfrak{a}_n\in\mathcal{A}_n,\mathfrak{b}_1\in\mathcal{B}_1,\mathfrak{b}_2\in\mathcal{B}_2,\dots,\mathfrak{b}_j\in\mathcal{B}_j,\dots,\mathfrak{b}_n\in\mathcal{B}_n$

$d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{b}_j,\dots,\mathfrak{b}_n) = 0$ or $\mathfrak{b}_j+\mathfrak{b}_j\in\mathcal{Z}(\mathcal{M})$. 4

From Lemma 7, Eq.4 can be written as
$d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{b}_j,\dots,\mathfrak{b}_n) = 0$ or $\mathfrak{b}_j+\mathfrak{b}_j\in\mathcal{Z}(\mathcal{M})$
for any $\mathfrak{b}_1\in\mathcal{B}_1,\mathfrak{b}_2\in\mathcal{B}_2,\dots,\mathfrak{b}_j\in\mathcal{B}_j,\dots,\mathfrak{b}_n\in\mathcal{B}_n$ 5

If there is $\mathfrak{b}\in\mathcal{B}_j$ and
$d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{b},\dots,\mathfrak{b}_n) = 0$
$\forall\ \mathfrak{b}_1\in\mathcal{B}_1,\mathfrak{b}_2\in\mathcal{B}_2,\dots,\mathfrak{b}_n\in\mathcal{B}_n$ 6

Hence $\mathfrak{b}\in\mathcal{Z}(\mathcal{M})$, because of Lemma 10.

As well, by assumption and Eq.6
$d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{s}\mathfrak{b}\dots,\mathfrak{b}_n) = d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{s},\dots,\mathfrak{b}_n)\mathfrak{b}\in\mathcal{Z}(\mathcal{M})$.

$\forall\mathfrak{a}_1\in\mathcal{A}_1,\mathfrak{a}_2\in\mathcal{A}_2,\dots,\mathfrak{a}_n\in\mathcal{A}_n,\mathfrak{b}_1\in\mathcal{B}_1,\mathfrak{b}_2\in\mathcal{B}_2,\dots,\mathfrak{b}_n\in\mathcal{B}_n,\mathfrak{s}\in\mathcal{M}$.

Again use Lemma 1(ii) to get: $\forall\mathfrak{a}_1\in\mathcal{A}_1,\mathfrak{a}_2\in\mathcal{A}_2,\dots,\mathfrak{a}_n\in\mathcal{A}_n,\mathfrak{b}_1\in\mathcal{B}_1,\mathfrak{b}_2\in\mathcal{B}_2,\dots,\mathfrak{b}_n\in\mathcal{B}_n,\mathfrak{s}\in\mathcal{M}$

$d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{s},\dots,\mathfrak{b}_n)\in\mathcal{Z}(\mathcal{M})$. or $\mathfrak{b}=0$

If
$d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{s},\dots,\mathfrak{b}_n)\in\mathcal{Z}\ \forall\mathfrak{a}_1\in\mathcal{A}_1,\mathfrak{a}_2\in\mathcal{A}_2,\dots,\mathfrak{a}_n\in\mathcal{A}_n,\mathfrak{b}_1\in\mathcal{B}_1,\mathfrak{b}_2\in\mathcal{B}_2,\dots,\mathfrak{b}_n\in\mathcal{B}_n,\mathfrak{s}\in\mathcal{M}$

Then
$d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{s}(\mathfrak{b}+\mathfrak{b}),\dots,\mathfrak{b}_n) = d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{s},\dots,\mathfrak{b}_n)(\mathfrak{b}+\mathfrak{b})\in\mathcal{Z}(\mathcal{M})$.

For the same reason as above, the following result can be satisfied
$d_1(\mathfrak{a}_1,\mathfrak{a}_2,\dots,\mathfrak{a}_n)d_2(\mathfrak{b}_1,\mathfrak{b}_2,\dots,\mathfrak{s},\dots,\mathfrak{b}_n) = 0$ or $(\mathfrak{b}+\mathfrak{b})\in\mathcal{Z}(\mathcal{M})$
$\forall\mathfrak{a}_1\in\mathcal{A}_1,\mathfrak{a}_2\in\mathcal{A}_2,\dots,\mathfrak{a}_n\in\mathcal{A}_n,\mathfrak{b}_1\in\mathcal{B}_1,\mathfrak{b}_2\in\mathcal{B}_2,\dots,\mathfrak{b}_n\in\mathcal{B}_n,\mathfrak{s}\in\mathcal{M}$

Because of Lemma 7, the last result can be reduce to $\mathfrak{b}+\mathfrak{b}\in\mathcal{Z}(\mathcal{M})$ and this last result, Eq.5 becomes $\mathfrak{b}_j+\mathfrak{b}_j\in\mathcal{Z}$ for every $\mathfrak{b}_j\in\mathcal{B}_j$, it follows $\mathfrak{s}\mathfrak{b}_j+\mathfrak{s}\mathfrak{b}_j = \mathfrak{s}(\mathfrak{b}_j+\mathfrak{b}_j)\in\mathcal{Z}(\mathcal{M})$ for every $\mathfrak{b}_j\in\mathcal{B}_j$, $\mathfrak{s}\in\mathcal{M}$. So $\mathfrak{b}_j+\mathfrak{b}_j=0$ or $\mathfrak{s}\in\mathcal{Z}(\mathcal{M})$ for any $\mathfrak{s}\in\mathcal{M}$, so two torsion freeness and Lemma 2(iii), leads to the required result.

Corollary 24: Let d_1 and d_2 are two nonzero $\mathcal{R}.n.\mathcal{D}'s$ of $\mathcal{M}$ ($\mathcal{M}$ is two torsion free) and $\mathcal{A},\mathcal{B}$ are nonzero semigroup ideals of $\mathcal{M}$, if $d_1(\mathcal{A},\mathcal{A},\dots,\mathcal{A})d_2(\mathcal{B},\mathcal{B},\dots,\mathcal{B})\subseteq\mathcal{Z}(\mathcal{M})$, then $\mathcal{M}$ *is* $\mathcal{C}.\mathcal{R}$.

Corollary 25: Let d_1 and d_2 are two nonzero $\mathcal{R}.n.\mathcal{D}'s$ of $\mathcal{M}$ ($\mathcal{M}$ is two torsion free) and $\mathcal{A}$ is a nonzero semigroup ideals of $\mathcal{M}$, if $d_1(\mathcal{A},\mathcal{A},\dots,\mathcal{A})d_2(\mathcal{A},\mathcal{A},\dots,\mathcal{A}) \subseteq \mathcal{Z}(\mathcal{M})$, then $\mathcal{M}$ *is* $\mathcal{C}.\mathcal{R}$.

Corollary 26: Let d_1 and d_2 are two nonzero $\mathcal{R}.\mathcal{D}'s$ of $\mathcal{M}$ ($\mathcal{M}$ is two torsion free), $\mathcal{A}$ and $\mathcal{B}$ are nonzero semigroup ideals of $\mathcal{M}$, if $d_1(\mathcal{A})d_2(\mathcal{B}) \subseteq \mathcal{Z}(\mathcal{M})$, then $\mathcal{M}$ *is* $\mathcal{C}.\mathcal{R}$.

Corollary 27: Let d_1 and d_2 are two nonzero $\mathcal{R}.\mathcal{D}'s$ of $\mathcal{M}$ ($\mathcal{M}$ is two torsion free), $\mathcal{A}$ is a nonzero semigroup ideals of $\mathcal{M}$, if $d_1(\mathcal{A})d_2(\mathcal{A}) \subseteq \mathcal{Z}(\mathcal{M})$, then $\mathcal{M}$ *is* $\mathcal{C}.\mathcal{R}$.

Corollary 28: Let d_1 and d_2 are two nonzero $\mathcal{R}.n.\mathcal{D}'s$ of $\mathcal{M}$ ($\mathcal{M}$ is two torsion free), if $d_1(\mathcal{M},\mathcal{M},\dots,\mathcal{M})d_2(\mathcal{M},\mathcal{M},\dots,\mathcal{M}) \subseteq \mathcal{Z}(\mathcal{M})$, then $\mathcal{M}$ *is* $\mathcal{C}.\mathcal{R}$.

Corollary 29: Let d_1 and d_2 are two $\mathcal{R}.\mathcal{D}'s$ of $\mathcal{M}$ ($\mathcal{M}$ is two torsion free), if $d_1(\mathcal{M})d_2(\mathcal{M}) \subseteq \mathcal{Z}(\mathcal{M})$, then $\mathcal{M}$ *is* $\mathcal{C}.\mathcal{R}$.

Theorem 7: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, where $\mathcal{M}$ is two torsion free, $\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_n$ are nonzero semigroup ideals of $\mathcal{M}$, if $d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{u}_1\epsilon\mathcal{A}_1$, $\mathfrak{u}_2\epsilon\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j,\dots,\mathfrak{u}_n\epsilon\mathcal{A}_u$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Proof: By assumption

$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{u}_1\epsilon\mathcal{A}_1,\mathfrak{u}_2\epsilon\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j,\dots,\mathfrak{u}_n\epsilon\mathcal{A}_u$,

Therefore,

$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{s}\mathfrak{u}_j,\mathfrak{s}\mathfrak{v}_j),\dots,\mathfrak{u}_n) = d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{s}(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n)$

$= d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\mathfrak{s},\dots,\mathfrak{u}_n)(\mathfrak{u}_j,\mathfrak{v}_j) + d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n)\mathfrak{s} \in \mathcal{Z}(\mathcal{M})$

for any $\mathfrak{u}_1\epsilon\mathcal{A}_1$, $\mathfrak{u}_2\epsilon\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j,\dots,\mathfrak{u}_n\epsilon\mathcal{A}_u,\mathfrak{s}\in\mathcal{M}$. Replace $\mathfrak{s}$ by $(\mathfrak{u}_j,\mathfrak{v}_j)$in last equation to get

$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n)(2(\mathfrak{u}_j,\mathfrak{v}_j)) \in \mathcal{Z}(\mathcal{M})$

for any $\mathfrak{u}_1\epsilon\mathcal{A}_1$, $\mathfrak{u}_2\epsilon\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j,\dots,\mathfrak{u}_n\epsilon\mathcal{A}_u$ 7

using Lemma 1(ii) implies

$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n) = 0$ or $\left(2(\mathfrak{u}_j,\mathfrak{v}_j)\right) \in \mathcal{Z}(\mathcal{M})$

for any $\mathfrak{u}_1\epsilon\mathcal{A}_1$, $\mathfrak{u}_2\epsilon\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j,\dots,\mathfrak{u}_n\epsilon\mathcal{A}_u$ 8

If there is $\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\epsilon\,\mathcal{A}_j$ such that $d\left(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right),\dots,\mathfrak{u}_n\right) = 0$ for any $\mathfrak{u}_1\epsilon\mathcal{A}_1$, $\mathfrak{u}_2\epsilon\mathcal{A}_2,\dots,\mathfrak{u}_n\epsilon\mathcal{A}_u$, so $\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right) \in \mathcal{Z}(\mathcal{M})$, according to Lemma 6.

Return to the hypothesis: for any $\mathfrak{u}_1 \in \mathcal{A}_1,\mathfrak{u}_2 \in \mathcal{A}_2,\dots,\mathfrak{u}_n \in \mathcal{A}_u,,\mathfrak{s}\in\mathcal{M}$.

$d\left(\mathfrak{u}_1,\mathfrak{u}_2,\dots,\left(s\mathfrak{u}_{j_0},s\mathfrak{v}_{j_0}\right),\dots,\mathfrak{u}_n\right) = d\left(\mathfrak{u}_1,\mathfrak{u}_2,\dots,s\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right),\dots,\mathfrak{u}_n\right)$

$= d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,s,\dots,\mathfrak{u}_n)\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right) \in \mathcal{Z}(\mathcal{M})$

Using Lemma 1(ii) in last result forces $d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,s,\dots,\mathfrak{u}_n) \in \mathcal{Z}(\mathcal{M})$ or $\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right) = 0$, if $d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,s,\dots,\mathfrak{u}_n) \in \mathcal{Z}(\mathcal{M})$, replace s by $2s\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right)$, last expression can be written as:

$d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,s,\dots,\mathfrak{u}_n)(2\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right)) \in \mathcal{Z}(\mathcal{M})$ or $\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right) = 0$, which conclude that $d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,s,\dots,\mathfrak{u}_n)\left(2\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right)\right) \in \mathcal{Z}(\mathcal{M})$, thus $2\left(\mathfrak{u}_{j_0},\mathfrak{v}_{j_0}\right) \in \mathcal{Z}(\mathcal{M})$ according to Lemma 1(ii) and Lemma 6.

Therefore, Eq.8 becomes $2(\mathfrak{u}_j,\mathfrak{v}_j) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j$, it follows $2(s\mathfrak{u}_j,s\mathfrak{v}_j) = \mathrm{s}(2(\mathfrak{u}_j,\mathfrak{v}_j)) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j,\mathfrak{s}\in\mathcal{M}$, Lemma 1(i) and two torsion freeness ensures that $(\mathfrak{u}_j,\mathfrak{v}_j) = 0$ for any $\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j$, or $s \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{s}\in\mathcal{M}$. Therefore $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$ by Lemma 4, Lemma 9 and Lemma 2(iii).

Corollary 30: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, where $\mathcal{M}$ is two torsion free, $\mathcal{A}$ is a nonzero semigroup ideals of $\mathcal{M}$, if $d(\mathfrak{u}_1,\mathfrak{u}_2,\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,\mathfrak{u}_n) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{u}_1$, $\mathfrak{u}_2,,\dots,\mathfrak{u}_j,\mathfrak{v}_j,\dots,\mathfrak{u}_n \in \mathcal{A}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 31: Let d be a nonzero $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$, where $\mathcal{M}$ is two torsion free, $\mathcal{A}$ is a nonzero semigroup ideals of $\mathcal{M}$, if $d((u,v)) \in \mathcal{Z}(\mathcal{M})$ for any u, v $\in \mathcal{A}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Corollary 32: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, where $\mathcal{M}$ is two torsion free, if $d(\mathfrak{s}_1,\mathfrak{s}_2,\dots,(\mathfrak{s},\mathfrak{m}),\dots,\mathfrak{s}_n) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{s}_1$, $\mathfrak{s}_2,\dots,\mathfrak{s},\mathfrak{m},\dots,\mathfrak{s}_n\in\mathcal{M}$, then $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$.

Corollary 33: Let d be a nonzero $\mathcal{R}.\mathcal{D}$ of $\mathcal{M}$, where $\mathcal{M}$ is two torsion free, if $d((s,t)) \in \mathcal{Z}(\mathcal{M})$ for any s, t $\in \mathcal{M}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Theorem 8: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, where $\mathcal{M}$ is two torsion free, $\mathcal{A}_1,\mathcal{A}_2,\dots,\mathcal{A}_n$ are nonzero semigroup ideals of $\mathcal{M}$, if $d((\mathfrak{u}_1,\mathfrak{v}_1),(\mathfrak{u}_2,\mathfrak{v}_2),\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,(\mathfrak{u}_n,\mathfrak{v}_n)) \in \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{u}_1,\mathfrak{v}_1\epsilon\mathcal{A}_1$, $\mathfrak{u}_2,\mathfrak{v}_2\epsilon\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j,\dots,\mathfrak{u}_n,\mathfrak{v}_n\epsilon\mathcal{A}_u$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Proof: By assumption: for any $\mathfrak{u}_1,\mathfrak{v}_1\epsilon\mathcal{A}_1$, $\mathfrak{u}_2,\mathfrak{v}_2\epsilon\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j,\dots,\mathfrak{u}_n,\mathfrak{v}_n\epsilon\mathcal{A}_u$,

$d\left((\mathfrak{u}_1,\mathfrak{v}_1),(\mathfrak{u}_2,\mathfrak{v}_2),\dots,(\mathfrak{s}\mathfrak{u}_j,\mathfrak{s}\mathfrak{v}_j),\dots,(\mathfrak{u}_n,\mathfrak{v}_n)\right) = d\left((\mathfrak{u}_1,\mathfrak{v}_1),(\mathfrak{u}_2,\mathfrak{v}_2),\dots,s(\mathfrak{u}_j,\mathfrak{v}_j),\dots,(\mathfrak{u}_n,\mathfrak{v}_n)\right)$

$= d((\mathfrak{u}_1,\mathfrak{v}_1),(\mathfrak{u}_2,\mathfrak{v}_2),\dots,s,\dots,(\mathfrak{u}_n,\mathfrak{v}_n))(\mathfrak{u}_j,\mathfrak{v}_j)$

$+d\left((\mathfrak{u}_1,\mathfrak{v}_1),(\mathfrak{u}_2,\mathfrak{v}_2),\dots,(\mathfrak{u}_j,\mathfrak{v}_j),\dots,(\mathfrak{u}_n,\mathfrak{v}_n)\right)\mathfrak{s} \in \mathcal{Z}(\mathcal{M})$

for any $\mathfrak{u}_1,\mathfrak{v}_1\epsilon\mathcal{A}_1$, $\mathfrak{u}_2,\mathfrak{v}_2\epsilon\mathcal{A}_2,\dots,\mathfrak{u}_j,\mathfrak{v}_j \in \mathcal{A}_j$,

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$\dots, u_n, v_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$.
Replace $\mathfrak{s}$ by (u_j, v_j) in last equation to get:
$d\left((u_1,v_1),(u_2,v_2),\dots,(u_j,v_j),\dots,(u_n,v_n)\right)(2(u_j,v_j)) \in \mathcal{Z}(\mathcal{M})$
For any
$u_1,v_1 \in \mathcal{A}_1, u_2,v_2 \in \mathcal{A}_2,\dots,u_j,v_j \in \mathcal{A}_j,\dots,u_n,v_n \in \mathcal{A}_u$ 9
Using Lemma 1(ii) implies
$d\left((u_1,v_1),(u_2,v_2),\dots,(u_j,v_j),\dots,(u_n,v_n)\right) = 0$ or $\left(2(u_j,v_j)\right) \in \mathcal{Z}(\mathcal{M})$
for any
$u_1,v_1 \in \mathcal{A}_1, u_2,v_2 \in \mathcal{A}_2,\dots,u_j,v_j \in \mathcal{A}_j,\dots,u_n,v_n \in \mathcal{A}_u$
10
If there is $u_{j_0}, v_{j_0} \in \mathcal{A}_j$ such that
$d\left((u_1,v_1),(u_2,v_2),\dots,\left(u_{j_0},v_{j_0}\right),\dots,(u_n,v_n)\right) = 0$
for any $u_1,v_1 \in \mathcal{A}_1, u_2,v_2 \in \mathcal{A}_2,\dots,u_n,v_n \in \mathcal{A}_u$.
Return to the hypothesis: for any $u_1,v_1 \in \mathcal{A}_1, u_2,v_2 \in \mathcal{A}_2,\dots,u_n,v_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$.

$d\left((u_1,v_1),(u_2,v_2),\dots,\left(su_{j_0},sv_{j_0}\right),\dots,(u_n,v_n)\right) = d\left((u_1,v_1),(u_2,v_2),\dots,s\left(u_{j_0},v_{j_0}\right),\dots,(u_n,v_n)\right)$
$= d\left((u_1,v_1),(u_2,v_2),\dots,s,\dots,(u_n,v_n)\right)\left(u_{j_0},v_{j_0}\right) \in \mathcal{Z}(\mathcal{M})$ 11
Replace s by (u_j,v_j) where $u_j,v_j \in \mathcal{A}_j$ and use Lemma 1(ii) in Eq.11 to get:
Either
$d\left((u_1,v_1),(u_2,v_2),\dots,(u_j,v_j),\dots,(u_n,v_n)\right) = 0$ or $\left(u_{j_0},v_{j_0}\right) \in \mathcal{Z}(\mathcal{M})$.
If $d\left((u_1,v_1),(u_2,v_2),\dots,(u_j,v_j),\dots,(u_n,v_n)\right) = 0$ for any
$u_1,v_1 \in \mathcal{A}_1, u_2,v_2 \in \mathcal{A}_2,\dots,u_j,v_j \in \mathcal{A}_j,\dots,u_n,v_n \in \mathcal{A}_u$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$ because of Theorem 2.
If $\left(u_{j_0},v_{j_0}\right) \in \mathcal{Z}(\mathcal{M})$, then using Eq.11 leads to
$d\left((u_1,v_1),(u_2,v_2),\dots,s,\dots,(u_n,v_n)\right) \in \mathcal{Z}(\mathcal{M})$
for any $u_1,v_1 \in \mathcal{A}_1, u_2,v_2 \in \mathcal{A}_2,\dots,u_n,v_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$ or $\left(u_{j_0},v_{j_0}\right) = 0$.
If $d\left((u_1,v_1),(u_2,v_2),\dots,s,\dots,(u_n,v_n)\right) \in \mathcal{Z}(\mathcal{M})$
Replace s by $2s\left(u_{j_0},v_{j_0}\right)$, the last expression can be written as:
$d\left((u_1,v_1),(u_2,v_2),\dots,s,\dots,(u_n,v_n)\right)(2\left(u_{j_0},v_{j_0}\right)) \in \mathcal{Z}(\mathcal{M})$
Use Lemma 1(ii) to get
$d\left((u_1,v_1),(u_2,v_2),\dots,s,\dots,(u_n,v_n)\right) = 0$
for any $u_1,v_1 \in \mathcal{A}_1, u_2,v_2 \in \mathcal{A}_2,\dots,u_n,v_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$ or $2\left(u_{j_0},v_{j_0}\right) \in \mathcal{Z}(\mathcal{M})$.
Therefore, Eq.9 becomes $d\left((u_1,v_1),(u_2,v_2),\dots,s,\dots,(u_n,v_n)\right) = 0$ for any $u_1,v_1 \in \mathcal{A}_1, u_2,v_2 \in \mathcal{A}_2,\dots,u_n,v_n \in \mathcal{A}_u, \mathfrak{s} \in \mathcal{M}$ or $2(u_j,v_j) \in \mathcal{Z}(\mathcal{M})$ for any $u_j,v_j \in \mathcal{A}_j$, proceeding inductively as above indicates $d = 0$ (a contradiction) or there is $i \in \{1,2,\dots,n\}$ s.t $2(u_j,v_j) \in \mathcal{Z}(\mathcal{M})$ for any $u_j,v_j \in \mathcal{A}_j$, hence $2(su_j,sv_j) = s(2(u_j,v_j)) \in \mathcal{Z}(\mathcal{M})$ for any $u_j,v_j \in \mathcal{A}_j, \mathfrak{s} \in \mathcal{M}$, Lemma 1(i) and two torsion freeness ensures that $(u_j,v_j) = 0$ for any $u_j,v_j \in \mathcal{A}_j$, or $s \in \mathcal{Z}(\mathcal{M})$for any $\mathfrak{s} \in \mathcal{M}$. Therefore $\mathcal{M}$ is $\mathcal{C}.\mathcal{R}$ by Lemma 4, Lemma 9 and Lemma 2(iii).

Corollary 34: Let d be a nonzero $\mathcal{R}.n.\mathcal{D}$ of $\mathcal{M}$, where $\mathcal{M}$ is two torsion free, if $d((s_1,t_1),(s_2,t_2),\dots,(s_n,t_n)) \in \mathcal{Z}(\mathcal{M})$ for any $s_1,t_1, \ ,s_2,t_2,\dots,s_n,t_n \in \mathcal{M}$, then $\mathcal{M}$ is a $\mathcal{C}.\mathcal{R}$.

Conclusion:

By using the semigroup ideals with right n-derivations involving some algebraic identities, this work gives very attractive results about the commutativity of the near ring.

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- Conflicts of Interest: None.
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References:

1. Pilz G. Near-Rings: The Theory and Its Applications. 2nd ed. Amsterdam: North Holland; 1983. 541 p. https://worldcat.org/en/title/849794415
2. Faraj A, Hashim R. On Skew Left n-Derivations with Lie Ideal Structure. Baghdad Sci J. 2019; 16(2): 389-394. http://dx.doi.org/10.21123/bsj.2019.16.2.0389.
3. Said A. Jordan left (θ-θ) -derivations of $\boldsymbol{\sigma}$ -prime rings. Baghdad Sci J. 2011; 8(3): 826-831. https://doi.org/10.21123/bsj.2011.8.3.826-831
4. Bell H. On Derivations in Near-Rings II, Near-rings, Near-fields and k-loops. Kluwer Academic Publishers. Dordrecht. 1997; 426: 191–197. https://link.springer.com/book/10.1007/978-94-009-1481-0
5. Boua A, Ali A, UL Huque I. Several Algebraic Identities in 3-Prime Near-Rings. Kragujev J Math. 2018; 42(2): 249-258.

http://elib.mi.sanu.ac.rs/files/journals/kjm/52/kjmn52p249-258.pdf

6. Boua A. Commutativity of Jordan Ideal in 3-Prime Near-Rings with Derivations. Commun Korean Math Soc. 2018, 33(1): 37–44. https://www.koreascience.or.kr/article/JAKO201809951099852.page
7. Enguady A, Boua A. On Lie ideals with left derivations in 3-prime near-rings. An S tiint Univ Al. I. Cuza Ia¸ si. Mat. (N.S.). 2022; LXVIII, f. 1: 123-132
8. Boua A. Homoderivations and Jordan right ideals in 3-prime near-rings. AIP Conf Proc. 2019; 020010. https://aip.scitation.org/doi/10.1063/1.5090627
9. Mouhssine S, Boua A. Homoderivations and Semigroup Ideals in 3-Prime Near-Rings. Algebr Struct their Appl. 2021; 8(2): 177-194. http://as.yazd.ac.ir/article_2110_f9680f36382e46fed705244ba6b17c04.pdf
10. Boua A, Ashraf M. identities in 3-prime near-rings with left multiplier. J Algebra Relat Top. 2018; 6(1): 67-77. https://jart.guilan.ac.ir/article_3080.html
11. Ashraf. M. Siddeeque Mohammad A, , Parveen Nazia, et al. On semigroup ideals and n-derivations in near-rings. J Taibah Univ Sci. 2015; 9(1): 126–132. https://www.sciencedirect.com/science/article/pii/S165836551400082X
12. Aroonruviwat P, Leerawat U. On outer (σ, τ)-n-derivations and commutativity in prime near-rings. Int J Math Comput Sci. 2021; 16(2): 563–575. http://ijmcs.future-in-tech.net/16.2/R-Leerawat-Aroonruviwat.pdf.
13. Ali A. Boua A, Ul Huque I. STRUCTURE OF 3-PRIME NEAR RINGS WITH GENERALIZED (σ, τ)-n-DERIVATIONS. Kragujev J Math. 2023; 47(6): 891–909. https://imi.pmf.kg.ac.rs/kjm/pdf/accepted-finished/3d3e097d33ce6381fdde16a84764b69d_2536_02142021_080256/kjm_47_6-6.pdf
14. Farhan E. Near – Rings with Generalized Right n-Derivations. Iraqi J Sci. 2021; 62(7): 2334-2342. https://ijs.uobaghdad.edu.iq/index.php/eijs/article/view/2810
15. Majeed. A, Farhan.E. Right n-derivations in prime near–rings. Al-Qadisiyah J Pure Sci. 2019; 21(3): 31-41 https://qu.edu.iq/journalsc/index.php/JOPS/article/view/32.

على مثاليات شبه الزمرة والاشتقاقات n- اليمينية على الحلقات المقتربة

انعام فرحان

قسم الاشراف الاختصاص، مديرية تربية القادسية، القادسية، العراق.

الخلاصة:

في الورقة الحالية تم دراسة مفهوم الاشتقاقات اليمينية -n والتي تحقق شروط معينة على مثاليات شبه الزمرة على الحلقات المقتربة وتم مناقشة بعض الخصائص المرتبطة بها. وقد تم التوصل إلى نتائج مثيرة للاهتمام ، من أبرزها ما يلي : لتكن $\mathcal{M}$ حلقة مقتربة يسارية ثلاثية اولية وكل من $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n$ شبه زمرة مثالية من $\mathcal{M}$, اذا كان d اشتقاق-n يميني على $\mathcal{M}$ يحقق احد الشروط التالية :

(i) $d(\mathfrak{u}_1, \mathfrak{u}_2, \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, \mathfrak{u}_n) = 0 \ \forall \ \mathfrak{u}_1 \epsilon \mathcal{A}_1, \mathfrak{u}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n \epsilon \mathcal{A}_\mathfrak{u};$

(ii) $d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) = 0$
$\forall \ \mathfrak{u}_1, \mathfrak{v}_1 \epsilon \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n, \mathfrak{v}_n \epsilon \mathcal{A}_\mathfrak{u};$

(iii) $d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) = (\mathfrak{u}_j, \mathfrak{v}_j) \ \forall \ \mathfrak{u}_1, \mathfrak{v}_1 \epsilon \mathcal{A}_1$
$, \mathfrak{u}_2, \mathfrak{v}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n, \mathfrak{v}_n \epsilon \mathcal{A}_\mathfrak{u};$

(iv) $d + d$ is an n -additive mapping from $\mathcal{A}_1 \times \mathcal{A}_2 \times \dots \times \mathcal{A}_n$ to $\mathcal{M}$;

(v) $d(\mathfrak{u}_1, \mathfrak{u}_2, \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, \mathfrak{u}_n) \in \mathcal{Z}(\mathcal{M}) \ \forall \ \mathfrak{u}_1 \epsilon \mathcal{A}_1, \mathfrak{u}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n \epsilon \mathcal{A}_\mathfrak{u}$

(vi) $d\left((\mathfrak{u}_1, \mathfrak{v}_1), (\mathfrak{u}_2, \mathfrak{v}_2), \dots, (\mathfrak{u}_j, \mathfrak{v}_j), \dots, (\mathfrak{u}_n, \mathfrak{v}_n)\right) \in \mathcal{Z}(\mathcal{M}) \ \forall \ \mathfrak{u}_1, \mathfrak{v}_1 \epsilon \mathcal{A}_1$
$, \mathfrak{u}_2, \mathfrak{v}_2 \epsilon \mathcal{A}_2, \dots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \dots, \mathfrak{u}_n, \mathfrak{v}_n \epsilon \mathcal{A}_\mathfrak{u}$

فان $\mathcal{M}$ حلقة ابدالية.

الكلمات المفتاحية: تعميم الاشتقاقات اليمينية، الحلقات المقتربة الاولية، الاشتقاقات اليمينية، الاشتقاقات-n اليمينية، مثاليات شبه الزمرة.