2024, 21(4): 1371-1377
https://doi.org/10.21123/bsj.2023.8149
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Solving Fuzzy Rayleigh Model with Application

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Received 20/11/2022, Revised 31/03/2023, Accepted 02/04/2023, Published Online First 20/09/2023, Published 01/04/2024

Abstract

In this article, deriving and estimating the two parameters of Rayleigh distribution by utilizing the maximum likelihood estimation method, regular least squares estimation method and moment estimation method. The simulation technique for many samples sites and many numbers of initial values are used, then attaining that the moment estimation method are the best one from another method by using mean squares error procedures. The interval estimation is used to find the estimator of location and scale parameters for Rayleigh distribution. The triangular membership functions are employed to find the fuzzy estimate parameters for the moment method of Rayleigh distribution. Finally utilizing the ranking function to transform the fuzzy parameters to crisp parameters and include that the fuzzy estimate parameters are the best.

Keywords: Fuzzy estimated parameters, Intervals estimation, Maximum Likelihood method, Moment method, Rayleigh distribution, Regular least square method.

Introduction

Rayleigh distribution (RD) introduced by Lord Rayleigh in 1880, plays a crucial role in modelling and analyzing life time data such as project effort loadings modeling, life testing experiments, reliability analysis, communication theory, physical sciences, engineering, medical imaging science, applied statistics, and clinical studies[1].Many researchers are interested in this field of them, in 2019 Hussein IH, KHammas HA estimate the Survival Function for Rayleigh Distribution by using Ranking function[2].In 2019, Hussein and KHammas estimate Fuzzy Survival and Hazard Functions for Rayleigh distribution[3].In 2020, Zain SA, estimate the Survival and Hazard Functions of Weighted Rayleigh Distribution[4].In 2020, Noaman et al, discussed the Constructing a new mixed probability distribution with fuzzy reliability estimation[5].In 2021, Hussein IH R, Mitlif J, used the Ranking Function to Solve a Fuzzy Multiple Objective Function[6].In 2022,Taha TA, Salman AN, discussed the Comparison Different Estimation Method for Reliability Function of Rayleigh Distribution Based on Fuzzy Life time Data[7]

The aim of this paper is to derive and estimate the location's parameter and scale parameters for Rayleigh distribution by employing three estimation methods, the first one is maximum likelihood methods, the second one is regular least squares method, and the moment method. Then, using the Monte-Carlo procedure which is the branch of simulation technique for many samples sizes and many initial values to reach that the moment method are the best method than the other method. The interval estimation is used for moment estimation method of Rayleigh distribution to find the interval for location and scale parameter. Utilizing the triangular membership function to find the fuzzy estimate parameters. After that using the ranking function to transform these fuzzy estimate

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parameters to erase parameters, then concluding that the fuzzy estimate parameters, are the best. In section 2 deriving maximum likelihood methods. In section 3, deriving the moment method. In section 4 deriving the least square method. In section 5 deriving the variance of two parameters α,β by using Fisher Information. In section 6 providing a simplified presentation of the concept of the mean of error squares.
In section 7, the fuzzy set is explained and defined. Section 8, explain the approach of Ranking function. In section 9, simulations for the purpose of experimentally comparing the different methods are discussed. Section 10 presents conclusions.

Materials and Methods

Maximum Likelihood Estimator Method (M.L.E)

It is considered one of the most important estimation methods that aim to make the possibility function at its greatest end. If random sample $(t_1, t_2, \ldots, t_n)$ the Rayleigh distribution is distributed by the displacement parameter α and the scale parameter β, the estimator of the greatest potential makes the possibility function is at its maximum limit and can be obtained by deriving the logarithm of the probability function and equals it to zero, so if $f(t)$ the Rayleigh distribution is distributed by two parameters (β, α) then the probability function .It will be as follows[2,7]:

$$f(t,\alpha.\beta) = \frac{(t-\alpha)}{\beta^2} e^{\frac{-(t-\alpha)^2}{2\beta^2}} \quad \alpha < t < \infty \beta > 0$$

$$F(t_i) = \int_{\alpha}^{t} \frac{(t-\alpha)}{\beta^2} e^{\frac{-(t-\alpha)^2}{2\beta^2}} dt = 1 - e^{-\frac{(t-\alpha)^2}{2\beta^2}}$$

$$Lf(t,\alpha.\beta) = \prod_{I=1}^{n} f(t,\alpha.\beta) = \beta^{-2n} \prod_{I=1}^{n} \left[(t_i - \alpha) \exp\left(\frac{-(t_i-\alpha)^2}{2\beta^2}\right) \right]$$

By taking the natural logarithm of the two sides, the previous equation becomes as follows:

$$\ln L = -2n \ln \beta + \sum_{i=1}^{n} \ln(t_i - \alpha) - \frac{\sum_{i=1}^{n}(t_i-\alpha)^2}{2\beta^2}$$

By differentiating the sides with respect to α and β, and setting them equal to zero, getting the following two equations:

$$\frac{\partial \ln L}{\partial \alpha} = -\sum_{i=1}^{n} \frac{1}{(t_i - \alpha)} + \frac{\sum_{i=1}^{n}(t_i-\alpha)}{\beta^2}$$

$$0 = -\sum_{i=1}^{n} \frac{1}{(t_i - \tilde{\alpha})} + \frac{\sum_{i=1}^{n}(t_i-\tilde{\alpha})}{\tilde{\beta}^2}$$

Is $\max \ln L \; if \alpha \le \min(t_i) \Longrightarrow \tilde{\alpha} = \min\left(t_{(i)}\right) = t_{(1)}$

$$\frac{\partial \ln L}{\partial \beta} = -\frac{2n}{\beta} + \frac{\sum_{i=1}^{n}(t_i-\alpha)^2}{\beta^3}$$

$$0 = -\frac{2n}{\tilde{\beta}} + \frac{\sum_{i=1}^{n}(t_i-\tilde{\alpha})^2}{\tilde{\beta}^3}$$

$$\beta^2 = \frac{\sum_{i=1}^{n}(t_i-\tilde{\alpha})^2}{2n} \Longrightarrow \tilde{\beta} = \sqrt[2]{\frac{\sum_{i=1}^{n}(t_i-\tilde{\alpha})^2}{2n}} = \sqrt[2]{\frac{\sum_{i=1}^{n}(t_i-t_1)^2}{2n}}$$

Moments Estimator Method (M.E)

The momentum method is characterized by its ease, as it relies on the assumption of equality of community moments with the sample moments and solving the equations to find estimates for the parameters, since the first and second moments of the distribution Rayleigh is[2,7]:

$$\mu_1 = E(t_i) = \beta\sqrt{\frac{\pi}{2}} + \alpha$$

$$\mu_2 = E\left(t_i^2\right) = 2\beta^2 + \alpha\beta\sqrt{2\pi} + \alpha^2$$

So the variance is:

$$v(t_i) = E\left(t_i^2\right) - [(E(t_i)]^2 = \beta^2(2 - \frac{\pi}{2})$$

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By equating the moment of the first and second sample with the momentum of the first and second community, respectively yields:

$$\tilde{\alpha} = \bar{t} - \tilde{\beta}\sqrt{\frac{\pi}{2}}$$

$$\tilde{\beta} = \frac{s(t_i)}{\sqrt{(2-\frac{\pi}{2})}} \ldots \ldots \text{ Where} s(t_i) = \text{the standard deviation of } r.v.t$$

Least Square Estimator Method (L.S.E)

This method adopts the regression method, through the cumulative probability function, where[2,7]:

$$F(t_i) = 1 - e^{-\frac{(t-\alpha)^2}{2\beta^2}}$$

$$\ln[1 - F(t_i)] = -\frac{(t-\alpha)^2}{2\beta^2}$$

$$s(\alpha,\beta) = \sum_{1=1}^{n}\left[\ln[1 - F(t_i)] + \frac{(t-\alpha)^2}{2\beta^2}\right]^2$$

$$\frac{\partial s(\alpha,\beta)}{\partial \alpha} = \sum_{1=1}^{n}\left[\ln[1 - F(t_i)] + \frac{(t-\alpha)^2}{2\beta^2}\right]\left[\frac{(t-\alpha)}{\beta^2}\right]$$

$$0 = \sum_{1=1}^{n}\left[\ln[1 - F(t_i)] + \frac{(t-\alpha)^2}{2\beta^2}\right]\left[\frac{(t-\alpha)}{\beta^2}\right]$$

$$0 = \sum_{1=1}^{n}\left[\left[\frac{(t-\alpha)}{\beta^2}\right]\ln[1 - F(t_i)] + \frac{(t-\alpha)^3}{2\beta^4}\right]$$

$$\frac{\partial s(\alpha,\beta)}{\partial \beta} = \sum_{1=1}^{n}\left[\ln[1 - F(t_i)] + \frac{(t-\hat{\alpha})^2}{2\beta^2}\right]\left[\frac{(t-\alpha)^2}{\beta^3}\right]$$

$$0 = \sum_{1=1}^{n}\left[\ln[1 - F(t_i)] + \frac{(t-\alpha)^2}{2\hat{\beta}^2}\right]\left[\frac{(t-\hat{\alpha})^2}{\hat{\beta}^3}\right]$$

$$0 = \sum_{1=1}^{n}\left[\left[\frac{(t-\hat{\alpha})^2}{\hat{\beta}^3}\right]\ln[1 - F(t_i)] + \frac{(t-\hat{\alpha})^4}{2\hat{\beta}^5}\right]$$

$$\text{Where} F(t_i) = \frac{i-0.5}{n}$$

Because of the high non-linearity of the equations, it solved by numerical methods such as Newton Raphson[2,7].

Results and Discussion

Approximate Variance of Rayleigh Distribution Parameter

To calculate the approximate variance for parameters of Rayleigh distribution can be used the Fisher information which follows:

The approximate variance is $I(\hat{\theta}) = \frac{1}{nI(\hat{\theta})}$

Where $I(\hat{\theta}) = E[-\frac{\partial^2}{\partial^2\theta}(\ln(f/\theta))]$

$$I(\hat{\alpha}) = E\left[-\frac{\partial^2}{\partial^2\alpha}\left(\ln(\frac{(t-\alpha)}{\beta^2}e^{\frac{-(t-\alpha)^2}{2\beta^2}})\right)\right]$$

$$\frac{\partial}{\partial \alpha}(\ln f(t,\alpha.\beta)) = \frac{1}{t-\alpha} - \frac{t-\alpha}{\beta^2}$$

$$\frac{\partial^2}{\partial^2\alpha}(\ln f(t,\alpha.\beta)) = -\frac{1}{(t-\alpha)^2} - \frac{1}{\beta^2}$$

$$E\left[\frac{\partial^2}{\partial^2\alpha}(\ln f(t,\alpha.\beta))\right] = -E\left[\frac{1}{(t-\alpha)^2}\right]$$

$$E\left[\frac{1}{(t-\alpha)^2}\right] = \int_0^\infty \frac{1}{(t-\alpha)^2}\frac{(t-\alpha)}{\beta^2}e^{\frac{-(t-\alpha)^2}{2\beta^2}}\,dt$$

Let $x = \frac{1}{(t-\alpha)} \Rightarrow \frac{1}{x} = (t-\alpha) \Rightarrow t = \frac{1}{x} + \alpha$

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$$\partial t = -\frac{1}{x^2}\partial x \Rightarrow \left|\frac{\partial t}{\partial x}\right| = \frac{1}{x^2}$$

$$\Rightarrow \begin{cases} \begin{array}{cc} t & x \\ \hline 0 & -1/\alpha \\ \infty & 0 \end{array} \end{cases}$$

$$E\left[\frac{1}{(t-\alpha)^2}\right] = \int_{-\frac{1}{\alpha}}^{0} \frac{1}{\beta^2 \frac{1}{x^3}} \exp[-\frac{\frac{1}{x^2}}{2\beta^2}]\, dx$$

$$E\left[\frac{1}{(t-\alpha)^2}\right] = \exp[-\frac{\frac{1}{x^2}}{2\beta^2}]_{\frac{-1}{\alpha}}^{0} = \frac{\alpha^2}{2\beta^2}$$

$$\therefore var(\hat{\alpha}) = \frac{2\hat{\beta}^2}{n\hat{\alpha}^2}$$

$$I(\hat{\beta}) = E\left[-\frac{\partial^2}{\partial^2\beta}\left(\ln(\frac{(t-\alpha)}{\beta} e^{\frac{-(t-\alpha)^2}{2\beta^2}})\right)\right]$$

$$\frac{\partial}{\partial\beta}(\ln f(t,\alpha.\beta)) = -\frac{2}{\beta} + \frac{(t-\alpha)^2}{\beta^3}$$

$$\frac{\partial^2}{\partial^2\beta}(\ln f(t,\alpha.\beta)) = \frac{2}{\beta^2} - \frac{3(t-\alpha)^2}{\beta^4}$$

$$E\left[\frac{\partial^2}{\partial^2\beta}(\ln f(t,\alpha.\beta))\right] = -\frac{4}{\beta^2}$$

$$\therefore var(\hat{\beta}) = \frac{\hat{\beta}^2}{4n}$$

Now using the $var(\hat{\alpha})$, $var(\hat{\beta})$ to compute the confidence Interval (C.I) of parameters $\hat{\alpha}$, $\hat{\beta}$ as follows:

$$\hat{\alpha} \pm t_{(n-1,a)}\sqrt{var(\hat{\alpha})}$$

$$\hat{\beta} \pm t_{(n-1,a)}\sqrt{var(\hat{\beta})}$$

Mean Squared Error (MSE):

It represents a comparison criterion in which the priority is for the smallest denominator, and as in the formula:

$$MSE = \frac{\sum_{i=1}^{r}(\hat{\theta}-\theta)^2}{r}$$

Where: $\hat{\theta}$ is the estimated for parameters of distribution

$\hat{\theta}$: is the real's for parameters of distribution

r: is the number of iterations in the simulation experiment

Fuzzy Set

The first to introduce the concept of fuzzy sets and described it as the membership function was Zadeh in 1965 and it has been used in many practical applications. Unlike the classic set, in the case of a fuzzy set A an object x may belong to this set with varying membership degrees in the range [0, 1], where 0 and 1 denote, respectively, Non-membership and full membership. One way of describing a fuzzy set A is to provide its membership function $\mu_A : X \to [0,1]$. There are various membership functions.

Let a, b and c represent the x coordinates of the three vertices of μ_A (x) in a fuzzy set A (a: lower boundary and c: upper boundary where membership degree is zero, b: the center where membership degree is 1 [8].

Ranking Function

The approach of ranking function is an efficient of ordering fuzzy numbers. Various types of ranking function have been introduced which are used for solving problems with fuzzy parameters [9].

Here, it introduced a linear ranking function that is similar to the ranking function adopted by Maleki [10] or a triangular fuzzy number ˜a = (α^c, a^u, a^l),using ranking function as follows:

$$R(\tilde{a}) = \int_0^1 (sup\, \tilde{a}_\lambda + inf\, \tilde{a}_\lambda) d\lambda$$

This reduces to:

$$R(\tilde{a}) = \alpha^c + \frac{1}{4}(a^u - a^l), \text{ Where: } \alpha^c = \frac{\alpha^u + \alpha^l}{2}$$

Application:

A study was performed using simulations for the purpose of experimentally comparing the different methods, where the study was done by relying on small (10, 20), medium (30) and large (100, 50) sample sizes. Different values were also for the actual distribution parameters. For the purpose of reaching the best estimate, the statistical measure averaging squares has been relied upon Error (MSE) as a basis for comparison which takes the following formula:

$$F(t_i) = 1 - e^{-\frac{(t-\alpha)^2}{2\beta^2}} \Longrightarrow let u_i = F(t_i)$$

$$1-u_i = e^{-\frac{(t-\alpha)^2}{2\beta^2}} \Longrightarrow \ln(1-u_i) = -\frac{(t-\alpha)^2}{2\beta^2}$$

$$\sqrt[2]{-\ln(1-u) = \frac{t-\alpha}{\sqrt{2}\beta}}$$

$$\Longrightarrow t_i = \alpha + \sqrt{2}\beta\sqrt{-\ln(1-u_i)}$$

The above formula is used to find the generating data by using simulation.

First, the estimates and mean square error of the parameters were found in the typical environment. The results are shown in Tables 1-3:

Table 1. MSE of parameters

Best α	Best β	β _OLS	α _OLS	β _ME	α _ME	β _MLE	α _MLE	n	
M.E	M.E	0.0000000120	0.0000000108	6.3995715E-9	5.2379961E-9	6.3996718E-9	5.747351E-9	10	
M.E	M.E	0.0000000120	0.0000000106	6.399546E-9	4.9827438E-9	6.3996501E-9	6.797693E-9	20	
M.E	M.E	0.0000000120	0.0000000107	6.399579E-9	5.0789154E-9	6.3996703E-9	6.450778E-9	30	**α=0.00001 β=0.00008**
M.E	M.E	0.0000000120	0.0000000106	6.399522E-9	4.9518531E-9	6.3996713E-9	5.751741E-9	50	
M.E	M.E	0.0000000120	0.0000000106	6.399548E-9	4.971350E-9	6.399661E-9	5.83067E-9	100	
M.E	M.E	0.0000009851	0.0000007982	9.794533E-7	7.9257397E-7	9.793668E-7	8.759309E-7	10	
M.E	M.E	0.0000033251	0.0000031382	9.7942565E-7	7.5927922E-7	9.7948336E-7	9.1363561E-7	20	
M.E	M.E	0.0000009851	0.0000008005	9.7923856E-7	7.9494889E-7	9.794766E-7	9.3493826E-7	30	**α=0.00001 β=0.00099**
M.E	M.E	0.0000009851	0.0000007912	9.793299E-7	7.8560478E-7	9.7946024E-7	9.5915070E-7	50	
M.L.E	M.E	0.0000009851	0.0000007709	9.7928102E-7	7.6534046E-7	9.7945843E-7	1.0403162E--6	100	
M.E	M.E	0.0000589701	0.0000485201	5.892627E-5	4.8514484E-5	5.8964490E-5	6.1363407E-5	10	
M.E	M.E	0.0000756473	0.00006574831	5.8929341E-5	4.7836876E-5	5.89988513E-5	5.4783395E-5	20	**α=0.00001**
M.E	M.E	0.0000589902	0.0000466879	5.8921076E-5	4.668232E-5	5.8984632E-5	6.7893330E-5	30	
M.E	M.E	0.0000589893	0.0000457557	5.8902336E-5	4.5750058E-5	5.89837271E-5	5.6713121E-5	50	**β=0.00077**
M.E	M.E	0.0000590006	0.0000457854	5.89449699E-5	4.5779792E-5	5.8994990E-5	5.50471554E-5	100	
M.E	M.E	0.0000000120	0.0000000108	6.3947850E-9	5.2123520E-9	6.3996605E-9	7.65305896E-9	10	
M.E	M.E	0.0000000120	0.0000000108	6.394804E0-9	5.2077212E-9	6.3996740E-9	6.3840134E-9	20	
M.E	M.E	0.0000000120	0.0000000107	6.3947665E-9	5.0677102E-9	6.39965700E-9	6.8024268E-9	30	**α=0.00089 β=0.00008**
M.E	M.E	0.0000000120	0.0000000106	6.39458446E-9	5.0306884E-9	6.3996721E-9	6.80623124E-9	50	
M.L.E	M.L.E	0.0000000120	0.0000000106	6.3949917E-9	4.99482398E-9	6.39966725E-9	4.9128033E-9	100	
M.E	M.E	0.0000009851	0.0000008132	9.7874794E-7	8.07562460E-7	9.79451214E-7	9.3566400E-7	10	
M.E	M.E	0.0000009851	0.0000007483	9.78649747E-7	7.427079045E-7	9.79491654E-7	8.9013317E-7	20	
M.L.E	M.L.E	0.0000009850	0.0000007883	9.785810674E-7	7.82719224E-7	9.79445892E-7	1.01498978E-6	30	**α=0.00089 β=0.00099**
M.L.E	M.L.E	0.0000009851	0.0000007853	9.785786223E-7	7.797244100E-7	9.7946084E-7	1.09990833E-6	50	
M.E	M.E	0.0000009851	0.0000007847	9.785589561E-7	7.791381304E-7	9.79452961E-7	9.29488735E-7	100	
M.E	M.E	0.0000590150	0.0000472742	5.89335907E-5	4.726859134E-5	5.90094193E-5	5.50075629E-5	10	
M.E	M.E	0.0000589966	0.0000459319	5.89274768E-5	4.592634344E-5	5.8990962E-5	5.254225639E-5	20	
M.E	M.E	0.0000589946	0.0000449504	5.887686349E-5	4.49448158E-5	5.89889776E-5	6.75994184E-5	30	**α=0.00089 β=0.00077**
M.E	M.E	0.0000589995	0.0000466055	5.8887442E-5	4.65999119E-5	5.89939358E-5	5.92507360E-5	50	
M.E	M.E	0.0000590009	0.0000468593	5.88990900E-5	4.68537082E-5	5.89953407E-5	5.3920012E-5	100	

That was concluded that the method of moment is the best based 0n the fact that the parameters have minimum squared error because the best MSR for α is(**4.9518531E-9**) and the best MER for β is (**6.399522E-9**)

Now, the moment's estimator method dependent to find the interval estimation by utilizing confidence interval (C.I) for the fuzzy parameters$\hat{\alpha}, \hat{\beta}$.

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https://doi.org/10.21123/bsj.2023.8149
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
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Table 2. Confidence interval (C.I) of $\hat{\alpha}, \hat{\beta}$.

C.I_β _L	C.I_β _U	C.I_α _L	C.I_α _U	Var(β -Est)	Var(α - Est)	β _Estm.	α _Estm.	n	t-table	
2.040955E-9	2.6680054E-9	5.925E-5	8.41097E-5	4.3E-9	1.33E-10	2.0506701E-9	8.131835E-5	10	1.833	
2.073554E-9	2.873554E-9	8.0058E-5	9.20126E-5	3.7E-9	7.7E-10	2.1863448E-9	8.012311E-5	20	1.729	
2.048870E-9	2.488713E4-9	8.0773E-5	8.70006E-5	8.7E-9	5.4E-10	2.0599299E-9	8.095717E-5	30	1.699	**α=0.00001 β=0.00008**
2.050277E-9	2.0602887E-9	7.9355E-5	8.18461E-5	2.15E-7	2.8E-10	2.0531012E-9	8.021616E-5	50	1.677	
2.014257E-9	2.1142676E-9	7.9521E-5	8.07646E-5	8.4E-9	1.3E-10	2.113438E-9	8.042075E-5	100	1.66	

After finding the confidence interval for $\hat{\alpha}$ and confidence interval for $\hat{\beta}$ for all samples sizes, then utilizing the triangular membership function by where $\hat{A} = (a^l, a^c, a^u)$ where

$a^l$is lower of confidence interval for $\hat{\alpha}$ and $\hat{\beta}$.

a^u is upper of confidence interval for $\hat{\alpha}$ and $\hat{\beta}$.

$a^c = \frac{a^l + a^u}{2}$ is the average between lower and upper confidence interval for $\hat{\alpha}$ and $\hat{\beta}$.

Table 3. Ranking fuzzy parameters

α,β real	n	Triangular membership function	Ranking fuzzy parameters
$\alpha = 0.00001$ β= 0.00008	10	$\mu(\tilde{\beta}) = (2.049554E-9, 2.35448E-9, 2.668005E-9)$ $\mu(\tilde{\alpha}) = (5.92504E-5, 7.16798E-5, 8.410926E-5)$	$R(\tilde{\beta}) = 2.50909E-9$ $R(\tilde{\alpha}) = 7.789451E-5$
	20	$\mu(\tilde{\beta}) = (2.073554E-9, 2.473554E-9, 2.87354E-9)$ $\mu(\tilde{\alpha}) = (8.0058E-5, 8.60355E-5, ,9.201242E-5)$	$R(\tilde{\beta}) = 2.67355E-9$ $R(\tilde{\alpha}) = 8.90239E-5$
	30	$\mu(\tilde{\beta}) = (2.04887E-9, 2.26879E-9, 2.4887134E-9)$ $\mu(\tilde{\alpha}) = (8.077376E-5, 8.38872E-5, 8.700065E-5)$	$R(\tilde{\beta}) = 2.37875E-9$ $R(\tilde{\alpha}) = 8.54439E-5$
	50	$\mu(\tilde{\beta}) = (2.05027E-9, 2.05528E-9, 2.0602887E-9)$ $\mu(\tilde{\alpha}) = (9.93595E-5, 8.06028E-5, 8.1846137E-5)$	$R(\tilde{\beta}) = 2.57785E-9$ $R(\tilde{\alpha}) = 8.12244E-5$
	100	$\mu(\tilde{\beta}) = (2.014257E-9, 2.06426E-9, 2.114267E-9)$ $\mu(\tilde{\alpha}) = (7.95219E-5, 8.014327E-5, 8.076465E-5)$	$R(\tilde{\beta}) = 2.08926E-9$ $R(\tilde{\alpha}) = 8.04539E-5$

Computing efficiency of Ranking function parameter by formula:

$$MSE(\,R(\tilde{\alpha})) = \left[\sum_{i=1}^{n} \frac{(\tilde{\alpha}-\alpha)^2}{n}\right] = 5.286E-9$$

$$MSE(\,R(\tilde{\beta})) = \left[\sum_{i=1}^{n} \frac{(\tilde{\beta}-\beta)^2}{n}\right] = 6.39\,E-9, \quad \text{Where} n = 5.$$

Conclusion

When comparing the traditional method and the fuzzy method for location and scale parameters of the Rayleigh distribution. Noting that the fuzzy method is the best. The percentage of scale parameters is (27/30)*100= 90% and the percentage of location parameters is (26/30)*100=87% by using Monte-Carlo procedures for moment estimation method of the Rayleigh distribution. The mean squares errors for fuzzy estimate parameters are more erase estimate parameters.

Authors' Declaration

- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Furthermore, any Figures and images, that are not mine, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in AL Karkh University of Science.

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حل نموذج رالي الضبابي مع تطبيق

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الخلاصة

في هذا البحث اشتققنا وقدرنا معلمتي توزيع رالي باستخدام طريقة تعظيم دالة الامكان المقدرة وطريقة المربعات الصغرى المقدرة وطريقة العزوم المقدرة . تم استخدام اسلوب المحاكاة للعديد من أحجام العينات وللعديد من القيم الابتدائية وتم التوصل بأن طريقة العزوم المقدرة هي الأفضل من بقية الطرق الأخرى باستخدام اسلوب متوسط مربعات الخطأ. ومن ثم تم استخدام تقدير الفترات لإيجاد المقدر لمعلمة الموقع ومعلمة القياس لتوزيع رالي. بعد ذلك استخدمت دالة العضوية المثلثية لإيجاد المعلمات الضبابية المقدرة لطريقة العزوم لتوزيع رالي. وفي النهاية استخدمنا الدالة الزمنية الترتيبية لتحويل المعلمات الضبابية المقدرة الى معلمات تقليدية وتم التوصل بأن المعلمات الضبابية المقدرة هي الافضل.

الكلمات المفتاحية :المعالم الضبابية المقدرة، تقدير الفترات، طريقة الامكان الاعظم، طريقة العزوم،توزيع رالي،طريقة المربعات الصغرى الاعتيادية ،تقنية المحاكاة، دالة العضوية المثلثية