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Numerical Solutions of Linear Abel Integral Equations Via Boubaker Polynomials Method

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Abstract

In this article, a numerical method based on Boubaker polynomials (BPs) was presented to solve the Linear Abel integral (LAI) Eqs of first and second types. The matrices were used to form the (LAI) Eq into a system of linear Eqs. To get Boubaker parameters, solve this system of Eqs using the Guess elimination method. To explain the results of this method, four examples have been provided and compared with the results of many methods mentioned in previous research. MATLAB R2018$_b$ program was used to perform all calculations and graphs.

Keywords: Abel integral Eqs, Boubaker polynomial, Numerical solutions, Numerical method, Singular Voltarra Eq.

Introduction

The Abel problem is summarized in the following sentences. In 1823, Abel studied how a particle moves when it is pulled downward by gravity along a smooth, unknowing curve in a vertical plane[1, 2, 3]. Numerous domains of science use the Abel integral Eq, such as Seismology, radio astronomy, radar ranging, plasma diagnostics, X-ray radiography, optical, electron emission, atomic scattering[1], Seismology, scattering theory, metallurgy, fluid flow, chemical processes, electrochemistry, semi-conductors, mathematical physics, chemistry, electrochemistry, population dynamics, and chemistry[3]. The following are the typical forms of the (LAI) Eqs of the 2nd and 1st types[1, 4] respectively:

$$T(\mu) = y(\mu) + \int_{s}^{\mu} \frac{1}{\sqrt{\mu-\theta}} T(\theta)d\theta, \qquad \mu \in [a,b] \qquad 1$$

$$y(\mu) = \int_{s}^{\mu} \frac{1}{\sqrt{\mu-\theta}} T(\theta)d\theta, \qquad 2$$

where s is the value of a real number, $y(\mu)$ is a well-known function and $T(\mu)$ is an unidentified function to be determined. In recent years, the (LAI) Eq numerical solution has been the focus of intense research by academics. Among their works are some of the following methods: Taylor-collocation approach[5], Lagrangian matrix technique[6], Chebyshev polynomials[7], Touchard Polynomials[8], Orthogonal polynomials method[9], the product integration and Haar Wavelet techniques[10], Hermite wavelet approach[11], Babenko and fractional integrals methods[12], Babenko and fractional calculus techniques[13], Mechanical quadrature method[14]. The following is how the rest of the article is structured: the proposed method, approximation function, solution accuracy, solutions Abel integral Eq, and numerical examples. Finally, conclusions and references.

Materials and Methods

Method of Boubaker Polynomial

Boubaker polynomials belonging to the Banach space and was first developed by Boubaker et al.[15-18] as a reference for solving the heat Eq within a physical model. The following Eq introduces the first monomial definition of the Boubaker polynomial:

$$w_r(\mu) = \sum_{h=0}^{\gamma(r)} \frac{(r-4h)}{(r-h)} C_{r-h}^{h} (-1)^h \mu^{2r-h}, \quad r = 0,1,2, \qquad 3$$

where $C_{r-h}^{h} = \frac{(h)!}{(r-h)!(2h-r)!}$, $\gamma(r) = \left\lfloor \frac{r}{2} \right\rfloor = \frac{2r+((-1)^r-1)}{4}$, $\gamma(r) = \lfloor . \rfloor$ represents the floor function, and r is the degree of the polynomials. The following first four polynomials of this polynomial are defined as follows:

$$w_0(\mu) = 1$$
$$w_1(\mu) = \mu$$
$$w_2(\mu) = \mu^2 + 2$$
$$w_3(\mu) = \mu^3 + \mu,$$

while the following recurrence relation holds for $r > 2$:

$$w_r(\mu) = \mu w_{r-1}(\mu) - w_{r-2}(\mu)$$

Approximation Function

Suppose that the approximate numerical solution to Eq be a linear combination of the (BPs) as follows:

$$T_r(\mu) = \sum_{c=0}^{r} x_c w_c(\mu) \ , \quad r = 1,2,3,\ldots, \quad 0 \le \mu \le 1, \qquad 4$$

the function $\{w_c(\mu)\}_{c=0}^{r}$ specify the basis of the (BPs) of r-th degree, as determined by Eq3, the unknown parameters x_c's will be determined later. Eq 4 can also be written as follows:

$$T_r(\mu) = [w_0(\mu) \ \ w_1(\mu) \ldots w_r(\mu)] \cdot \begin{bmatrix} x_0 \\ x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_r \end{bmatrix}, \qquad 5$$

So, Eq 5 can be written as follows:

$$T_r(\mu) = [1 \ \ \mu \ \ \mu^2 \ldots \mu^r] \cdot \begin{bmatrix} z_{00} & z_{01} & z_{02} & \cdots & z_{0r} \\ 0 & z_{11} & z_{12} & \cdots & z_{1r} \\ 0 & 0 & z_{22} & \cdots & z_{2r} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & z_{rr} \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_r \end{bmatrix}, \qquad 6$$

where the matrix in Eq 6 is invertible and $\{z_{ss}\}_{s=0}^{r}$ are used to specify the (BPs) parameters, which are power base amounts.

Solution Accuracy

A precision of the solutions can be verified using the proposed method through the following procedures:

The truncated Boubaker series[18, 19] in Eq 4 have to be approximately satisfying Eq 1. For every $\mu = \mu_\beta \in [a,b], \beta = 0,1,2,\ldots,r.$ therefore, the error functions:

$$Ef\,(\mu_\beta) = \left| \sum_{c=0}^{r} x_c w_c(\mu_\beta) - y(\mu_\beta) - \int_{s}^{\mu_\beta} \frac{1}{\sqrt{\mu_\beta - \theta}} \sum_{c=0}^{r} x_c w_c(\theta)\, d\theta \right| \cong 0, \text{then}$$

$Ef\,(\mu_\beta) \le \epsilon$, for all μ_β in the interval [a, b] and $\epsilon > 0$, therefore, the truncation limit r is increased until the error function $Ef\,(\mu_\beta)$ at each μ_β can be smaller than any positive number ϵ, so, the following Eq represents the error function:

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$$\mathrm{Ef}\,(\mu)=\sum_{c=0}^{r}x_c w_c(\mu)-y(\mu)-\int_s^{\mu}\frac{1}{\sqrt{\mu-\theta}}\sum_{c=0}^{r}x_c w_c(\theta)\,d\theta\,,$$

$\mathrm{Ef}_r\,(\mu)\to 0$ when the r value is large enough, the error decreases.
Hence, $\mathrm{Ef}\,(\mu)\leq\epsilon$.

Remark 1: The same previous procedure can be achieved on Eq 2.

Solutions Abel Integral Equation

With aid of the (BPs) to give approximate solution to Eq 1, assuming that Eq 4 will be used:

$$T(\mu)\cong T_r(\mu)=\sum_{c=0}^{r}x_c w_c\,(\mu), \qquad 7$$

when the Eq 7 is substituted into Eq 1, the result is:

$$\sum_{c=0}^{r}x_c w_c(\mu)=y(\mu)+\int_s^{\mu}\frac{1}{\sqrt{\mu-\theta)}}\sum_{c=0}^{r}x_c w_c(\theta)\,d\theta\,, \qquad 8$$

after Eq 5 is applied, Eq 8 becomes:

$$[w_0(\mu)\ \ w_1(\mu)\dots w_r(\mu)]\,.\begin{bmatrix}x_0\\x_1\\.\\.\\.\\x_r\end{bmatrix}=y(\mu)+\int_s^{\mu}\frac{1}{\sqrt{\mu-\theta}}\,[w_0(\theta)\ \ w_1(\theta)\dots w_r(\theta)]\,.\begin{bmatrix}x_0\\x_1\\.\\.\\.\\x_r\end{bmatrix}d\theta\,, \qquad 9$$

when using Eq 6, then Eq 9 transforms to the following:

$$[1\ \ \mu\ \ \mu^2\dots\mu^r].\begin{bmatrix}z_{00}&z_{01}&z_{02}&\dots&z_{0r}\\0&z_{11}&z_{12}&\cdots&z_{1r}\\0&0&z_{22}&\cdots&z_{2r}\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&z_{rr}\end{bmatrix}.\begin{bmatrix}x_0\\x_1\\.\\.\\.\\x_r\end{bmatrix}=y(\mu)+\int_s^{\mu}\frac{1}{\sqrt{\mu-\theta}}\,[1\ \ \theta\ \ \theta^2\dots\theta^r].\begin{bmatrix}z_{00}&z_{01}&z_{02}&\dots&z_{0r}\\0&z_{11}&z_{12}&\cdots&z_{1r}\\0&0&z_{22}&\cdots&z_{2r}\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&z_{rr}\end{bmatrix}.\begin{bmatrix}x_0\\x_1\\.\\.\\.\\x_r\end{bmatrix}d\theta\,, \qquad 10$$

After simplifying Eq 10, the unknown parameters $\{x_c\}_{c=0}^{r}$ can be calculated by selecting a number of values from the given interval [a, b]. In this case, Eq 10 is transformed into a system of linear Eqs made up of (r+1) from Eqs with unknown parameters. The Gauss elimination technique can be used to solve this system to determine the parameters that are unique. Finally, to obtain the approximate solution of Eq 1, these values are replaced into Eq 4.

Remark 2: The same previous procedure can be achieved on Eq 2.

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Results and Discussion

Numerical Examples

Four numerical examples are given in this section to demonstrate the capability and effectiveness of the method utilized to locate solutions using the MatlabR2018b program. The graphs solutions have all been converged.

Example 1: Solving the 1st type of the (LAI) Eq [8, 9, 19, 20]

$$\frac{2\sqrt{\mu}}{105}(105-56\mu^2-48\mu^3) = \int_0^{\mu}\frac{1}{\sqrt{\mu-\theta}}T(\theta)d\theta, \quad 0\le\mu\le 1$$

$T(\mu)=\mu^3-\mu^2+1$ is the precise solution. By applying the proposed method for this example when the degree of polynomials r=3 as mentioned in the Eq 10, getting:

$$\frac{2\sqrt{\mu}}{105}(105-56\mu^2-48\mu^3) = \int_0^{\mu}\frac{1}{\sqrt{\mu-\theta}}.\begin{bmatrix}1&0&2&0\\0&1&0&1\\0&0&1&0\\0&0&0&1\end{bmatrix}.\begin{bmatrix}\theta_0\\\theta_1\\\theta_2\\\theta_3\end{bmatrix}d\theta\ ,$$

Rewriting this would be:

$$\frac{2\sqrt{\mu}}{105}(105-56\mu^2-48\mu^3 = \int_0^{\mu}\frac{1}{\sqrt{\mu-\theta}}(\theta_0+\theta_1\mu+\theta_2(2+\mu^2))d\theta\ ,$$

Consequently, after performing the integrations, choosing $\delta_{\sigma}\,(\sigma=0,1,2,3)$ in the [0, 1], getting a system of four Eqs. This system can be solved in MATLABR2018b to obtain the (BPs) parameters. To obtain the approximate solution, these parameters are replaced into Eq 4 as follows:

$$T_3(\mu)=(3)\,w_0(\mu)+(-1)\,w_1(\mu)+(-1)w_2(\mu)+(1)w_3(\mu)=\mu^3-\mu^2+1\,.$$

The comparison appeared that the suggested technique gives the same precise solution as this example. Also[8, 9, 19, 20], the precise solution he had when r =3. As a result, these five techniques are identical in terms of accuracy of the results. Fig. 1 provided a comparison with the precise solution for r =3.

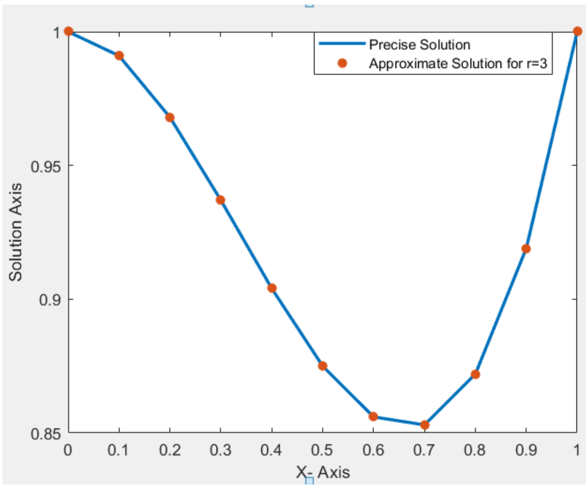

Figure 1. Compare with the precise solution, r=3.

Example 2: Solving the 1st type of the (LAI) Eq[19, 21]

$$\frac{4}{3}\mu^{3/2}-\frac{32}{35}\mu^{7/2} = \int_0^{\mu}\frac{1}{\sqrt{\mu-\theta}}T(\theta)d\theta, \quad 0\le\mu\le 1$$

$T(\mu)=\mu-\mu^3$ is the precise solution. By employing the suggested technique with r =3, and choosing the points $\mu_0=0.1$, $\mu_1=0.2$ and $\mu_3=0.3$ in the interval [0, 1]. The system in Eq 10 can be solved to obtain the (BPs) parameters. To obtain the approximate solution, these parameters are replaced into Eq 4 as follows:

$$T_3(\mu)=(0)\,w_0(\mu)+(2)\,w_1(\mu)+(0)\,w_2(\mu)+(-1)\,w_3(\mu)=\mu-\mu^3\,.$$

The comparison showed that the suggested technique yield the same precise solution as this example. The references[19, 21] obtained the precise solution when r ≥3. As a result, these three techniques are identical in terms of accuracy of the results. Fig. 2 compares the precise solution for r = 3 and shows the comparison.

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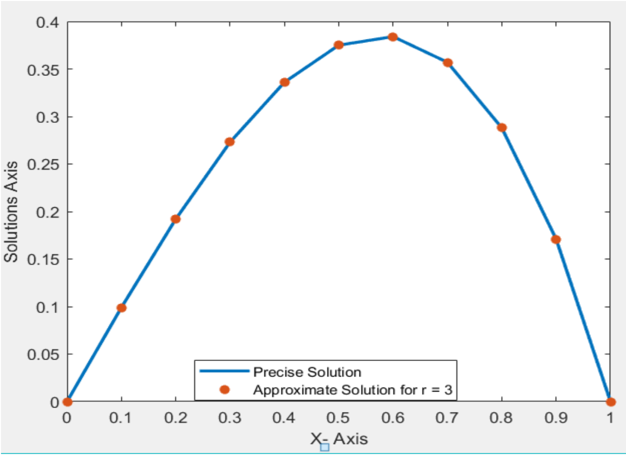

Figure 2. Compare with the precise solution, r=3.

Example 3: Solving the 2nd type of the (LAI) Eq[12, 19, 21, 22, 23]

$$T(\mu) = \mu + \frac{4}{3}\mu^{3/2} - \int_0^{\mu} \frac{1}{\sqrt{\mu-\theta}} T(\mu)d\mu\,, \quad 0 \le \mu \le 1$$

$T(\mu) = \mu$ is the precise solution. By employing the suggested technique for r =1 and choosing the points the values $\mu_0 = 0.1$ and $\mu_1 = 0.2$. Solving the algebraic system in Eq 10 using the Gauss elimination technique in the range [0, 1], in MATLAB R2018b to obtain the (BPs) parameters. To obtain the approximate solution, these parameters are replaced into Eq 4 as follows:

$$T_1(\mu) = (0)\, w_0(\mu) + \ (1)\, w_1(\mu) = \mu\,.$$

The comparison showed that the suggested technique yield the same precise solution as this example. The precise solution was obtained by the references[12, 19, 21] for r =1. Additionally, the reference[22] found the greatest absolute error and a relative error of approximately $5.1*10^{-3}$ and $5.1*10^{-1}$ respectively for r =18. Moreover[23], found an approximate solution of nearly $1 * 10^{-10}$ absolute of error for r =14. As a result, the suggested technique is superior to the approaches in the references[22, 23] and comparable with the approaches in the references[12, 19, 21] that have the same precision. The comparison with the precise solution for r = 1 is shown in Fig. 3.

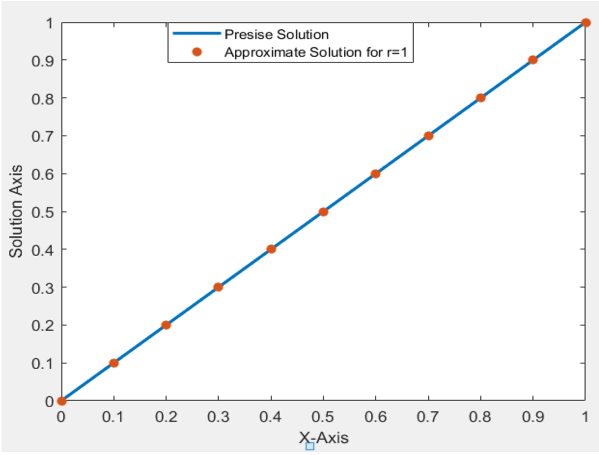

Figure 3. Compare with the precise solution, r=1.

Example 4: Solving the 2nd type of the (LAI) Eq[2, 3, 5, 19, 21, 23]

$$T(\mu) = \mu^2 + \frac{16}{15}\mu^{5/2} - \int_0^{\mu} \frac{1}{\sqrt{\mu-\theta}} T(\theta)d\theta\,, \quad 0 \le \theta \le 1$$

$T(\mu) = \mu^2$ is the precise solution. Using the suggested technique in Eq 10 for r = 2 and choosing the values $\mu_0 = 0.1, \mu_1 = 0.2$ and $\mu_2 = 0.3$. Solving the algebraic system in Eq 10 using the Gauss elimination technique in the range [0, 1] in MATLAB R2018b to obtain the (BPs) parameters. To obtain the approximate solution, these parameters are replaced into Eq 4 as follows:

$$T_2(\mu) = (-2)\, w_0(\mu) + \ (0)\, w_1(\mu) + (1) w_2(\mu) = \mu^2$$

The comparison shows that the proposed methods yield the same precise solution as this example. For r = 6 the largest absolute error of order 10^{-8}, 10^{-13} and 10^{-14} were obtained using the reference[2] for k= 8, 16 and k=32 respectively. Furthermore[3, 19], found the precise solution for r =2. Also[5], found the precise solution with r = 5. Besides[21], obtained the precise solution for r ≥2. Moreover[23], obtained the maximum absolute error of order 10^{-11} for r =14. Therefore, the suggested strategy is better than the techniques in the references[2, 23], and is comparable to others with the same level of precision. Fig. 4 displays the comparison with the precise solution for r = 2.

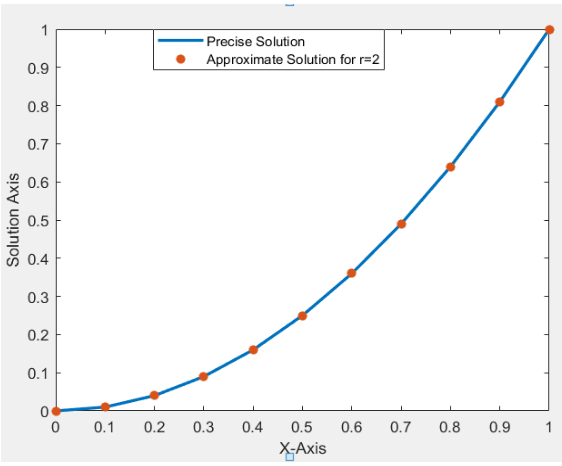

Figure 4. Compare with the precise solution, r=2.

Conclusion

In this paper, Boubaker polynomials were used to solve (LAI) Eqs of the 1st and 2nd types. The MATLAB program was used to calculate the integrals as well as to solve the systems of linear Eqs for the given numerical examples, where the precise solutions were obtained as a result of these examples. All the results obtained from solving these four examples were compared with the results of seven numerical methods in the literature and it was found that the proposed method is either identical in accuracy to some methods or better. In each given example, its results were compared with the precise solution as shown in the graphs.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Wasit University.

Authors' Contribution Statement

J. T. A., H. S. A. and W. S. A. participated in the planning and carrying out of the study, the analysis of the findings, and the preparation of the manuscript.

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الحلول العددية لمعادلات أبيل التكاملية الخطية بطريقة متعددات حدود بوبكر

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الخلاصة

في هذه المقالة، تم تقديم طريقة عددية تستند الى عديدات حدود بوبكر لحل معادلات آبيل التكاملية الخطية من النوع الاول والثاني. تم تحويل معادلات آبيل التكاملية الى نظام جبري من المعادلات الخطية باستخدام المصفوفات. للحصول على معالم بوبكر، تم حل هذا النظام باستخدام طريقة كاوس للحذف. ولتوضيح نتائج هذه الطريقة تم تقديم اربعة امثلة ومقارنتها مع نتائج العديد من الطرق المذكرة في البحوث السابقة. تم استخدام برنامج الماتلاب R2018b لأجراء جميع الحسابات والرسوم البيانية.

الكلمات المفتاحية: معادلة أبيل التكاملية، متعددات حدود بوبكر، الحل العددي، الطريقة العددية، معادلة فولتيرا الشاذة..