Comparison between Modified Weighted Pareto Distribution and Many other Distributions

Mahmood A. Shamran*, Asmaa A. Mohammed, Iden H. Alkanani

Department of Mathematics, College of Science for Women, University of Baghdad, Baghdad, Iraq.
*Corresponding Author.

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Abstract

In 2020 one of the researchers in this paper, in his first research, tried to find out the Modified Weighted Pareto Distribution of Type I by using the Azzalini method for weighted distributions, which contain three parameters, two of them for scale while the third for shape. This research compared the distribution with two other distributions from the same family; the Standard Pareto Distribution of Type I and the Generalized Pareto Distribution by using the Maximum likelihood estimator which was derived by the researchers for Modified Weighted Pareto Distribution of Type I, then the Mont Carlo method was used—that is one of the simulation manners for generating random samples data in different sizes (n = 10, 30, 50), and in different initial values for each Pareto distribution family being used in the research. A comparison was done by using Akaike Information Criteria, Corrected Akaike Information Criterion, and Bayesian Information Criteria.

Keywords: Akaike Information Criterion, Bayesian Information Criterion, Corrected Akaike Information Criterion, Generalized Pareto Distribution, Maximum likelihood Estimator, Modified Weighted Pareto Distribution Type I, Standard Pareto Distribution Type I.

Introduction

The aim of this research boils down to three goals, the first goal is to find out and derive what was carried out by the researchers about the Maximum likelihood estimator (MLE) to estimate the three parameters for Modified Weighted Pareto Distribution of Type I (MWPDTI). The second goal is to compare the new distribution with two other distributions of the same family by using Akaike Information Criteria (AIC), (AICc), and (BIC). While the third goal is to generate data by using the Mont Carlo method in order to generate random samples of different sizes; with different initial values to compare the same Pareto family distribution.

Azzalini, found a method for computing the probability density function for weighted distributions according to Eq.1.

\[ f_w(x) = \frac{1}{p_{x < x_1} f(x_1) F(x_1)} \]

Where \( F(x_1) \) is the cumulative distribution function, this technique has opened up new challenges in the field of weighted probability distributions.

Para and Jan, they submitted a generalization of the Pareto distribution of the second type using the idea of weighted distributions, and the statistical properties of this generalization were derived.

Alkanani et al, used two methods to estimate the parameters of the Maxwell-Boltzmann distribution, one of which is the method of Maximum Likelihood estimator method used in this paper. Nagatsuka et al, 2021, they presented a new development of the inference for the generalized Pareto distribution operating on all values of the k-
shape parameter, in which they extracted new properties that lead to obtaining new confidence intervals. Omekam et al \(^5\), they used the method of introducing new parameters on the distribution of Pareto of the first type, resulting in a new distribution, and a more flexible distribution with new types of data. Pho KH et al \(^6\), in studying the problem of traffic accidents, using the comparison between Akaike’s criterion and Bayes’ criterion to get the best estimate of the parameters of the problem model. Porret S. \(^7\), used the Akaike information criterion to study some biological models to obtain the best model in terms of estimating highly accurate parameters.

Pham MH. et al \(^8\), estimated the parameters of the Generalized Pareto Distribution (GPD) and they tested the goodness of fit. Cavanaugh JE.et al \(^9\), they reviewed the Akaike criterion, its properties, its derivation, and its use in predicting the estimation of the parameters of the statistical model.

Sahmran MA. \(^10\), used the Azzalini method to find a modified and weighted distribution of a Pareto type I (MWPDTI) according to Eq. 2.

\[
f_w(x; \alpha, k, \theta) = \frac{2ak^a x^{a-1}}{\theta^a} \left(1 - \frac{k}{\theta x^a}\right) , \alpha > 0, x \geq \frac{k}{\theta} > 0, \theta, k > 0, k \neq 1
\]

Al Sarraf NM et al \(^11\), they studied some methods of parameter estimation for the standard Pareto distribution of type I.

**Preliminary:**

The probability density function (pdf) and the cumulative distribution function (cdf) for the MWPDTI are equations respectively:

\[
f_w(x; \alpha, k, \theta) = \frac{2ak^a x^{a-1}}{\theta^a} \left(1 - \frac{k}{\theta x^a}\right) , \alpha > 0, x \geq \frac{k}{\theta} > 0, \theta, k > 0
\]

\[
F_w(x) = \frac{(\theta x^a - k)^2}{\theta^2 a x^{2a}} x \geq \frac{k}{\theta} > 0, \theta > 1
\]

The mean and the variance of the MWPDTI are equations respectively:

\[
E_w(x) = \frac{2a^2 k}{\theta(1-2\alpha)(1-\alpha)} , \alpha > 1, k > 0
\]

\[
Var_w(x) = \frac{(5a-1)\alpha^2 k^2}{(\alpha-2)(\alpha-1)^2(2a-1)\theta^2} , \alpha > 2, \theta > 1, k > 0
\]

The survival function and the hazard functions are equations respectively

\[
S_w(x) = 1 - \frac{(\theta x^a - k)^2}{\theta^2 a x^{2a}} x \geq \frac{k}{\theta} > 0, \alpha, \theta > 1
\]

\[
h_w(x) = \frac{2ak^a}{\theta^2 x^{2a}} x \geq \frac{k}{\theta} > 0, \alpha > 0, \theta > 1, k > 0
\]

And the other properties of the MWPDTI are mentioned in \(^10\).

**Materials and Methods:**

**Maximum Likelihood Estimation Method (MLE):**

The maximum likelihood estimator is used to find the estimated values for the parameters of MWPDTI.

From Eq.1, obtaining to

\[
f_w(x; \alpha, \theta, k) = 2ak^a \theta^{-\alpha}(x^{\alpha-1} - k^a x^{-2\alpha -1} \theta^{-\alpha})
\]

Then \(L(x; \alpha, \theta, k) = 2^n a^n k^n \alpha^{-n\alpha} \prod_{i=1}^{n} (x^{\alpha-1} - k^a x^{-2\alpha -1} \theta^{-\alpha})
\]

Taken the log of Eq.10, getting

\[
\ln L(x; \alpha, \theta, k) = n \ln 2 + n \ln \alpha + n\alpha \ln k - n\alpha \ln \theta + \sum_{i=1}^{n} \ln (x^{\alpha-1} - k^a x^{-2\alpha -1} \theta^{-\alpha})
\]

Let \(f(\hat{\theta}) = \frac{\partial \ln L}{\partial \theta} = \frac{n}{\alpha} + n \ln k - n \ln \theta + \sum_{i=1}^{n} \frac{x^{\alpha-1} - k^a x^{-2\alpha -1} \theta^{-\alpha}}{(x^{\alpha-1} - k^a x^{-2\alpha -1} \theta^{-\alpha})}
\]

Let \(g(\hat{k}) = \frac{\partial \ln L}{\partial k} = \frac{n \alpha}{k} + \sum_{i=1}^{n} \frac{x^{\alpha-1} - k^a x^{-2\alpha -1} \theta^{-\alpha}}{(x^{\alpha-1} - k^a x^{-2\alpha -1} \theta^{-\alpha})}
\]

Let \(h(\hat{\theta}) = \frac{\partial \ln L}{\partial \theta} = \frac{-n \alpha}{\theta} + \sum_{i=1}^{n} \frac{x^{\alpha-1} - k^a x^{-2\alpha -1} \theta^{-\alpha}}{(x^{\alpha-1} - k^a x^{-2\alpha -1} \theta^{-\alpha})}
\]
When $\frac{\partial \ln L}{\partial \alpha} = \frac{\partial \ln L}{\partial k} = \frac{\partial \ln L}{\partial \theta} = 0$, in this case, there is no closed solution for Eqs. 12, 13, and 14, which must use numerical methods such as Newton–Raphson method.

\[
\begin{bmatrix}
\tilde{\alpha}_{i+1} \\
\tilde{k}_{i+1} \\
\tilde{\theta}_{i+1}
\end{bmatrix} =
\begin{bmatrix}
\tilde{\alpha}_i \\
\tilde{k}_i \\
\tilde{\theta}_i
\end{bmatrix} - J^{-1} 
\begin{bmatrix}
\frac{\partial f(\tilde{\alpha})}{\partial \alpha} \\
\frac{\partial f(\tilde{\alpha})}{\partial k} \\
\frac{\partial f(\tilde{\alpha})}{\partial \theta}
\end{bmatrix}
\]

\[
\frac{\partial f(\tilde{\alpha})}{\partial \alpha} = -\frac{n}{\alpha^2} + \sum_{i=1}^{n} \frac{(x^a - k^a x^{-2a - 1} \theta^{-a})(x^a - k^a x^{-2a - 1} \theta^{-a})(\ln k - \ln \theta - 2 \ln x)}{(x^a - k^a x^{-2a - 1} \theta^{-a})^2}
\]

\[
\frac{\partial f(\tilde{\alpha})}{\partial \theta} = -\frac{n}{\theta} + \sum_{i=1}^{n} \frac{(x^a - k^a x^{-2a - 1} \theta^{-a})(-k^a x^{-2a - 1} \theta^{-a})(-\alpha(\ln k - \ln \theta - 2 \ln x) - 1)}{(x^a - k^a x^{-2a - 1} \theta^{-a})^2}
\]

\[
\frac{\partial f(\tilde{\alpha})}{\partial k} = \frac{n}{k} + \sum_{i=1}^{n} \frac{(x^a - k^a x^{-2a - 1} \theta^{-a})(\alpha(\ln k - \ln \theta - 2 \ln x) + 1)}{(x^a - k^a x^{-2a - 1} \theta^{-a})^2}
\]

\[
\frac{\partial g(\tilde{k})}{\partial \alpha} = \frac{n}{k} + \sum_{i=1}^{n} \frac{(-x^a - k^a x^{-2a - 1} \theta^{-a})(k^a x^{-2a - 1} \theta^{-a})(\alpha(\ln k - \ln \theta - 2 \ln x) + 1)}{(x^a - k^a x^{-2a - 1} \theta^{-a})^2}
\]

\[
\frac{\partial g(\tilde{k})}{\partial \theta} = \sum_{i=1}^{n} \frac{a^2 x^{-3a - 2}_a k^{-2a - 1}_a \theta^{-a}}{(x^a - k^a x^{-2a - 1} \theta^{-a})^2}
\]

\[
\frac{\partial g(\tilde{k})}{\partial k} = \frac{-n a}{k^2} + \sum_{i=1}^{n} \frac{(-ax^a - k^a x^{-2a - 1} \theta^{-a})(\alpha(\ln k - \ln \theta - 2 \ln x) + 1)(-k^a x^{-2a - 1} \theta^{-a})}{(x^a - k^a x^{-2a - 1} \theta^{-a})^2}
\]

\[
\frac{\partial h(\tilde{\theta})}{\partial \alpha} = -\frac{n}{\theta} + \sum_{i=1}^{n} \frac{(x^a - k^a x^{-2a - 1} \theta^{-a})(-k^a x^{-2a - 1} \theta^{-a})(\alpha(\ln k - \ln \theta - 2 \ln x) - 1)}{(x^a - k^a x^{-2a - 1} \theta^{-a})^2}
\]

\[
\frac{\partial h(\tilde{\theta})}{\partial \theta} = \sum_{i=1}^{n} \frac{a^2 x^{-3a - 2}_a k^{-2a - 1}_a \theta^{-a}}{(x^a - k^a x^{-2a - 1} \theta^{-a})^2}
\]

\[
\frac{\partial h(\tilde{\theta})}{\partial k} = \sum_{i=1}^{n} \frac{a^2 x^{-3a - 2}_a k^{-2a - 1}_a \theta^{-a}}{(x^a - k^a x^{-2a - 1} \theta^{-a})^2}
\]
Because the matrix J is a non-singular symmetric matrix, then
\[
\frac{\partial f(\alpha)}{\partial \theta} = \frac{\partial h(\theta)}{\partial \alpha} \cdot \frac{\partial f(\alpha)}{\partial k} = \frac{\partial g(k)}{\partial \alpha} \quad \text{and} \quad \frac{\partial g(k)}{\partial \theta} = \frac{\partial h(\theta)}{\partial k}.
\]

The maximum likelihood function for SPDTI distribution as in form 11:
\[
L(\alpha|x_{n}) = \left(\alpha^{nk}k^{n}\right)\prod_{i=1}^{n}(x_{i})^{-(\alpha+1)}
\]
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And the GPD distribution Maximum likelihood function is in form 8:
\[
L(k, \alpha; X) = -n \ln \alpha + \frac{1}{\alpha} \sum_{i=1}^{n} x_{i} - n \ln \alpha + \left(\frac{1}{k} - 1\right) \sum_{i=1}^{n} \ln \left(1 - \frac{kx_{i}}{\alpha}\right) \quad k \neq 0
\]
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The Three Criteria (AIC, AICc and BIC):

The following statistical criteria are used (AIC, AICc and BIC) to compare the MWPDTI with two Pareto distributions (SPDTI, GPD).

AIC = 2m - \hat{L} \quad 28

AICc = AIC + \frac{2m(m+1)}{n-m-1} \quad 29

and BIC = m \ln (n) - 2 \ln (\hat{L}) \quad 30

Where m is the number of parameters in the distribution, \(\hat{L}\) is the maximum log-likelihood function value for the distribution and n represents the sample size.

The Simulation Technique:

The Monte Carlo method is the most popular simulation technique used to generate the observations (samples) for any distribution. The simulation method is flexible for the test of experiments by replications many times.

1. Applying the Monte Carlo method for Modified Weighted Pareto distribution Type I (MWPDTI), using the cumulative distribution function as follows:

\[
F(x) = \frac{\theta x^{\alpha k - k \alpha}}{\theta x^{2 \alpha} x^{2 \alpha}}, x \geq \frac{k}{\theta} > 0, \alpha > 0, \theta \geq 1
\]
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By substituting the \(F(x)\) as \(u\), with a random number \(u = F(x)\), that is \(x = F^{-1}(u)\)

\[
\frac{(\theta x^{\alpha k - k \alpha})^{2}}{\theta x^{2 \alpha} x^{2 \alpha}} = u
\]
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\[
\frac{(\theta x^{\alpha k - k \alpha})^{2}}{(\theta)^{2}(x)^{2}} = u
\]
33

\[
\frac{\theta x^{\alpha k - k \alpha}}{\theta x^{\alpha k}} = \sqrt{u}
\]
34

\[
1 - \frac{k \alpha}{\theta x^{\alpha k}} = \sqrt{u}
\]
35

\[
\left[1 - \sqrt{u}\right] = k \alpha
\]
36

\[
x^{\alpha} = \frac{k \alpha}{\theta x^{1 - \sqrt{u}}}
\]
37

\[
x = \left[\frac{k \alpha}{\theta x^{1 - \sqrt{u}}}ight]^{1/2}
\]
38

2. Generating random samples which are \(n = 10, 30, 50\).

3. The initial values of the (SPDTI),(GPD) and (MWPDTI) are:

<table>
<thead>
<tr>
<th>Table No.</th>
<th>(\alpha)</th>
<th>(k)</th>
<th>(\theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.002</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>0.008</td>
<td>2</td>
</tr>
</tbody>
</table>

4. The replicate is \(R = 1000\).

Then compare three distributions by (AICc, AICc, and BIC) criteria to find the best distribution than another’s.

The result Estimated Value of Parameters with corresponding Criteria AIC, AICc, and BIC as in Table 1-3.
Table 1. The result when $\alpha = 1, \theta = 1, k = 0.1$ with sample size 10, 30 and 50

<table>
<thead>
<tr>
<th>Initial values of parameters</th>
<th>distribution name</th>
<th>Sample Size</th>
<th>estimated parameters</th>
<th>AIC</th>
<th>AICc</th>
<th>BIC</th>
</tr>
</thead>
</table>
| $\alpha=1$  
$\theta=1$  
$k=0.1$   | SPDTI (2parameters) | 10          | $\hat{\alpha}=1.004799$  
$k=0.099993$ | 44.871 | 46.575 | 45.931 |
|                      |                   | 30          | $\hat{\alpha}=1.001398$  
$k=0.099982$ | 85.5466 | 82.8741 | 82.8453 |
|                      |                   | 50          | $\hat{\alpha}=1.000898$  
$k=0.0997536$ | 115.5643 | 115.8243 | 116.06 |
| $\alpha=1$  
$\theta=1$  
$k=0.1$   | GPD (2parameters)      | 10          | $\hat{\alpha}=0.94008147$ 
$k=0.99987687$ | 44.5949 | 48.5949 | 45.4182 |
|                      |                   | 30          | $\hat{\alpha}=1.000490$  
$k=0.9999265$ | 83.6325 | 84.5555 | 84.4183 |
|                      |                   | 50          | $\hat{\alpha}=0.9999400$ 
$k=0.9999792$ | 114.6952 | 115.2169 | 115.2830 |
| $\alpha=1$  
$\theta=1$  
$k=0.1$   | MWPDTI (3 parameters) | 10          | $\hat{\alpha}=1.000493$  
$k=0.994898$ | 42.8642 | 46.8642 | 44.772 |
|                      |                   | 30          | $\hat{\alpha}=1.0002314$ 
$k=0.1000213$ | 81.2756 | 84.5555 | 84.4183 |
|                      |                   | 50          | $\hat{\alpha}=1.00004996$ 
$k=0.10090915$ | 114.1271 | 114.6488 | 115.7854 |

Table 1 showing the value of the three criteria increases with the sample size increasing.

1. When $n = 10$ the MWPDTI distribution has minimum values of AIC, and BIC, but the SPDTI distribution has less value in AICc.

2. When $n = 30$ the MWPDTI distribution has less values AIC, AICc, and BIC.

3. When $n = 50$ the MWPDTI distribution has less values AIC, and AICc, but the GPD distribution has less BIC.

4. Although the results for AIC, AICc, and BIC are converged from each other.

Table 2. The result when $\alpha = 1, \theta = 0.1, k = 0.002$ with sample size 10, 30 and 50

<table>
<thead>
<tr>
<th>Initial values of parameters</th>
<th>distribution name</th>
<th>Sample Size</th>
<th>estimated parameters</th>
<th>AIC</th>
<th>AICc</th>
<th>BIC</th>
</tr>
</thead>
</table>
| $\alpha=1$  
$\theta=0.1$  
$k=0.002$ | SPDTI (2parameters) | 10          | $\hat{\alpha}=1.004799$  
$k=0.099993$ | 32.2321 | 33.9461 | 34.21735 |
|                      |                   | 30          | $\hat{\alpha}=1.001398$  
$k=0.099982$ | 53.4666 | 54.0801 | 54.8432 |
|                      |                   | 50          | $\hat{\alpha}=1.000898$  
$k=0.0997536$ | 87.3421 | 87.6021 | 88.1796 |
| $\alpha=1$  
$\theta=0.1$  
$k=0.002$ | GPD (2parameters)      | 10          | $\hat{\alpha}=0.94008147$ 
$k=0.99987687$ | 34.9765 | 38.9765 | 39.6276 |
|                      |                   | 30          | $\hat{\alpha}=1.000490$  
$k=0.9999265$ | 55.2381 | 56.1611 | 56.796 |
|                      |                   | 50          | $\hat{\alpha}=0.99400814$ 
$k=0.999701905$ | 86.1436 | 86.6653 | 87.0631 |
| $\alpha=1$  
$\theta=0.1$  
$k=0.002$ | MWPDTI (3 parameters) | 10          | $\hat{\alpha}=1.0000493$  
$k=0.994898$ | 30.3651 | 34.3651 | 34.7876 |
|                      |                   | 30          | $\hat{\alpha}=1.0000493$  
$k=0.994898$ | 30.3651 | 34.3651 | 34.7876 |
|                      |                   | 50          | $\hat{\alpha}=0.99400814$ 
$k=0.999701905$ | 86.1436 | 86.6653 | 87.0631 |
Showing in Table 2 that
1. When \( n = 10 \) the SPDTI distribution has less values in AICc, and BIC, but the MWPDTI distribution has less value in AIC.
2. When \( n = 30 \) the MWPDTI distribution has less values in AIC, AICc, and BIC.
3. When \( n = 50 \) the MWPDTI distribution has less values in AIC, AICc, and BIC.
4. Although the results for AIC, AICc, and BIC are converged from each other.

Table 3. The result when \( \alpha = 1.5, \theta = 2, k = 0.008 \) with sample sizes 10, 30 and 50

<table>
<thead>
<tr>
<th>Initial values of parameters</th>
<th>distribution name</th>
<th>Sample Size</th>
<th>estimated parameters</th>
<th>AIC</th>
<th>AICc</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha=1.5 )</td>
<td>SPDTI (2parameters)</td>
<td>10</td>
<td>( \hat{\alpha}=1.499950 ) ( \hat{k}=0.00099768 )</td>
<td>42.7364</td>
<td>44.4504</td>
<td>45.2376</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>( \hat{\alpha}=1.5007543 ) ( \hat{k}=0.00799983 )</td>
<td>73.546</td>
<td>74.0075</td>
<td>74.8326</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>( \hat{\alpha}=1.50000542 ) ( \hat{k}=0.00095360 )</td>
<td>118.396</td>
<td>118.656</td>
<td>119.879</td>
</tr>
<tr>
<td>( k=0.008 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha=1.5 )</td>
<td>GPD (2parameters)</td>
<td>10</td>
<td>( \hat{\alpha}=1.499905632 ) ( \hat{k}=0.00092743 )</td>
<td>43.247</td>
<td>47.247</td>
<td>48.6965</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>( \hat{\alpha}=1.500490 ) ( \hat{k}=0.007956 )</td>
<td>71.212</td>
<td>72.135</td>
<td>72.9342</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>( \hat{\alpha}=1.5000635 ) ( \hat{k}=0.00906854 )</td>
<td>118.164</td>
<td>118.6857</td>
<td>119.2365</td>
</tr>
<tr>
<td>( k=0.008 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta=2 )</td>
<td>MWPDTI (3parameters)</td>
<td>10</td>
<td>( \hat{\alpha}=1.4999873 ) ( \hat{k}=0.00800674 ) ( \hat{\theta}=1.99936724 )</td>
<td>41.641</td>
<td>43.641</td>
<td>44.3421</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>( \hat{\alpha}=1.5000039 ) ( \hat{k}=0.00814231 ) ( \hat{\theta}=2.00001999 )</td>
<td>70.584</td>
<td>71.507</td>
<td>72.1231</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>( \hat{\alpha}=1.50004998 ) ( \hat{k}=0.00800443 ) ( \hat{\theta}=1.99991681 )</td>
<td>117.465</td>
<td>117.9867</td>
<td>118.3843</td>
</tr>
</tbody>
</table>

Table 3 shows that
1. When \( n = 10 \) the MWPDTI distribution has less values in AIC, AICc, and BIC.
2. When \( n = 30 \) the MWPDTI distribution has less values in AIC, AICc, and BIC.
3. When \( n = 50 \) the MWPDTI distribution has less values in AIC, AICc, and BIC.
4. Although the results for AIC, AICc, and BIC are converged from each other.

Conclusion

The researchers found out and derived the (MLE) to estimate the three parameters for (MWPDTI), then they compared the three distributions used in this research by (AIC), (AICc) and (BIC) to represent the efficiency of the statistical distribution, the three criteria used in this research must be in minimum value i.e. (AIC, AICc, BIC). By using the simulation manner, it was found that the new (MWPDTI) contains a minimum value for all sizes of the generated random samples, except for a few samples. It was obvious that the new (MWPDTI) is more efficient and better 23 times, (GPD) is more efficient one time, and (SPDTI) is three times more effective.

In the future, it can be confirmed the efficiency of (MWPDTI) by using other estimation methods to estimate the distribution parameters.

Author’s Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore,
any Figures and images that are not ours have been included with the necessary permission for re-publication, which is attached to the manuscript.

Author’s Contribution Statement

M.A. and I. H. presented the idea and discussed it theoretically and checked the equations. Authors M. A. and A. A. made the derivations for the equations. Authors M. A. and I. H. discuss and validate the results. M. A. prepared the computer programs for the simulation method. I. H. Conceptualization (supporting the idea and goal, methodology, and reviewing).

References

مقارنة بين توزيع باريتو المعدل الموزون والعدد من التوزيعات الأخرى

محمود عريبي شمران، اسماء عبد الحسين محمد و ايدين حسن حسين
قسم الرياضيات، كلية العلوم للبنات، جامعة بغداد، بغداد، العراق.

الخلاصة
قام احد الباحثين في هذه الورقة في سنة 2020 في بحثه الأول إيجاد توزيع باريتو الموزون المعدل من النوع الأول من التوزيعات الموزونة، وبدعه قام باستنتاج كافة الخواص الرياضية والإحصائية للتوزيع الجديد. والذي يحتوي على ثلاثة معلمات اثنان منها معلمات القياس والثالثة معلمة الشكل، هذا البحث تمت مقارنة توزيع باريتو الموزون المعدل من النوع الأول مع توزيعين من نفس عائلة توزيع باريتو وهما توزيع باريتو القياس من النوع الأول وتوزيع باريتو المعمم من خلال استخدام طريقة الإمكان الأعظم التي قام الباحثين باستنتاجها، ومن ثم استخدمت طريقة مونت كارلو وهي أحد أساليب المحاكاة لتوليد بيانات عباث عشوائية بإحجام مختلفة n=10,30,50 ويقيم ابتدائية مختلفة لكل التوزيعات من عائلة توزيع باريتو المستخدمة في البحث وتمت المقارنة باستخدام معيار معلومات أكاكي، معيار معلومات البيزية، معيار معلومات أكاكي المصحح ومعيار المعلومات البيزية. الكلمات المفتاحية: معيار معلومات أكاكي، معيار معلومات البيزية، معيار معلومات أكاكي المصحح، دالة الإمكان، توزيع باريتو الموزون المعدل من النوع I، توزيع باريتو القياس من النوع I.