

Employing Novel Ranking Function for Solving Fully Fuzzy Fractional Linear Programming Problems

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Abstract

Fuzzy programming is especially useful in cases where the coefficients are ambiguous. Because of this feature, in recent years, numerous techniques have emerged for addressing uncertainty. This paper proposes a novel ranking function technique with variables of type decagonal fuzzy numbers for solving fully fuzzy fractional linear programming (FFFLP) problems. This technique is dependent on introducing a new membership function for a decagonal fuzzy number and using a fully fuzzy simplex method. After converting the FFFLP problem to the fully fuzzy linear programming (FFLP) problem by a complementary method, then solved with the fully fuzzy simplex tables, in which all the values are fuzzy decagonal numbers. With the aid of the arithmetic operations of decagonal numbers, the new iteration of the simplex table is reached. Steps are repeated until the optimal fuzzy solution is reached. To demonstrate the proposed method a numerical example is provided to illustrate the steps of finding an optimal fuzzy solution to the problem.

Keywords: Arithmetic Decagonal Operations, Decagonal membership function, Fully Fuzzy Fractional Linear Programming, Fully Fuzzy Simplex Method, and Ranking function.

Introduction

The significance of fractional programming problems (FPP) is derived from the fact that many important problems are based on the trading of economic or material values, such as cost/volume, profit/cost, and cost/time in both production and financial planning. The FPP is defined as a fractional linear programming problem when both the objective function and the constraints are linear. It is crucial for decision-makers to be able to include uncertainty and imprecision in their optimization models, and this is where the fuzzy fractional

programming problem comes in. This is especially helpful when the available data is unclear or lacking in detail, or when different objectives must be weighed. Solutions that are both stable and adaptable over time can be found with the help of fuzzy logic and fractional programming. This can aid businesses in making more informed decisions, decreasing vulnerability, and increasing efficiency. In addition, fuzzy fractional programming can be used in many contexts, such as economics, technology, logistics, and ecology. Using membership degrees for each

variable based on their relative importance, the ranking function fuzzy enables the representation of uncertain or imprecise data. This allows decision-makers to account for the inherent uncertainty and imprecision in the data, resulting in better conclusions. In addition, the fuzzy ranking function can help find optimal solutions that weren't obvious when using strict linear programming techniques. Decision-makers are able to find solutions that are strong and resilient to changes in the underlying data when they analyze multiple scenarios and assign a probability to each one. Bellman RE, Zadeh LA¹ initially put the notion of fuzzy decision-making, and due to its significant use in real-world situations, there has been a quick growth in theoretical approaches to examine the theory and draw beneficial results. Kumar-Das S² developed a new ranking function to solve a full FLPP, the ranking function was derived by replacing the non-parallel sides of the trapezoidal fuzzy number with a nonlinear function. In order to solve fully fuzzy linear fractional programming problems, Loganathan T, Ganesan K³ proposed a method in which the original fractional programming problem was being transformed into a single objective linear programming problem in parametric form, with all parameters and variables expressed in triangular fuzzy numbers.

Linear fractional programming problems with fully fuzzy normalized heptagonal fuzzy numbers were solved by Alharbi MG, Khalif HA⁴ using the closed interval approximation of normalized heptagonal fuzzy numbers. Mitlif RJ⁵ recently presented a new approach to dealing with triangular fuzzy integers. The fuzzy fractional programming problem was initially reduced to a fractional programming problem as the technique's application. Gupta JD, et al.⁶ proposed a unique trapezoidal fuzzy number ranking function to solve Fully Fuzzy Linear Fractional Programming Problems utilizing trapezoidal fuzzy numbers as the objective function and constraints. We propose the following strategy using neat linear fractional programming and the simplex method. Mustafa R, Sulaiman N⁷ developed two innovative ranking function strategies for problems in fully fuzzy linear fractional programming (FFLFP), where the coefficients of the objective function and constraints are viewed as

triangular fuzzy numbers. Fuzzy values are converted to discrete ones with the help of the suggested ranking algorithm. Zhang C, et al.⁸ presented a fuzzy credibility-based multi-objective linear fractional mathematical programming for establishing the link between the agricultural water-food-environment nexus and crop area planning. This technique was created by incorporating fuzzy credibility-constrained programming into based multi - objective linear fractional programming within the optimization model planning.

The aim of this paper is to show that, the ratio optimization problems can be solved in an efficient and straightforward manner, reducing computational difficulties. Here, we proposed a novel ranking function technique depending on the decagonal membership function for addressing the FFLP problem with the aid of the development of a Simplex method for solving the FFLP problem, which enables the optimal fuzzy solution to be obtained when all variables are decagonal fuzzy numbers. This paper is divided into nine sections. In section 2, the simple preface of fuzzy set theory. Section 3 proposes a decagonal fuzzy function and its (σ -cut) function. The ranking function is derived in section 4. The fuzzy mathematical operations of decagonal fuzzy numbers are shown in section 5. Section 6 shows the mathematical model of fully fuzzy fractional linear programming problems. The fully fuzzy simplex method is presented in section 7; a numerical example is given in section 8. Finally, section 9 presents conclusions.

Preface of Fuzzy Set

This section includes some basic definitions.

Definition 1:⁹ Let $\mathcal{X} = \{x\}$ be a set of objects. The fuzzy set $\tilde{\mathcal{A}}$ in \mathcal{X} is defined by the membership function $\mathcal{M}_{\tilde{\mathcal{A}}}(x)$, where $\mathcal{M}_{\tilde{\mathcal{A}}}(x): \mathcal{X} \rightarrow [0, 1]$, is the degree of membership of $x \in \mathcal{X}$ in the set $\tilde{\mathcal{A}}$ and is denoted by $\tilde{\mathcal{A}}(x) = \{(x, \mathcal{M}_{\tilde{\mathcal{A}}}(x)) \mid x \in \mathcal{X}\}$.

Definition 2:¹⁰ A fuzzy set $\tilde{\mathcal{A}}$ is a fuzzy number if satisfies the following conditions:

- $\tilde{\mathcal{A}}$ is a normal fuzzy set if there exists at least one x_0 in \mathcal{R} with $\mathcal{M}_{\tilde{\mathcal{A}}}(x) = 1$
- $\mathcal{M}_{\tilde{\mathcal{A}}}(x)$ is piecewise continuous.

- $\tilde{\mathcal{A}}(x)$ is convex if $\mathcal{M}_{\tilde{\mathcal{A}}}(x) \cdot [\sigma x_1 + (1 - \sigma)x_2] \geq \text{Min}(\mathcal{M}_{\tilde{\mathcal{A}}}(x_1), \mathcal{M}_{\tilde{\mathcal{A}}}(x_2))$ $x_1, x_2 \in X, \sigma \in [0, 1]$.

Definition 3:¹¹ (σ – cut) is the crisp set of elements that belong to $\tilde{\mathcal{A}}$ and satisfy

$$\tilde{\mathcal{A}}_{\sigma} = \{x \in X \mid \mathcal{M}_{\tilde{\mathcal{A}}}(x) \geq \sigma, \sigma \in [0, 1]\}.$$

Proposed Decagonal Membership Function

In this section, propose a membership function $\mathcal{M}_{\tilde{\mathcal{A}}_{Dec}}(x)$ of a Decagonal fuzzy number $\tilde{\mathcal{A}}_{Dec} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10})$, whereas $\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4 \leq \alpha_5 \leq \alpha_6 \leq \alpha_7 \leq \alpha_8 \leq \alpha_9 \leq \alpha_{10} \in \mathcal{R}$, and the graph function as shown in Fig.1:

$$\mathcal{M}_{\tilde{\mathcal{A}}_{Dec}}(x) = \begin{cases} 0 & \alpha_1 < 0 \\ \frac{(x-\alpha_1)}{3(\alpha_2-\alpha_1)} & \alpha_1 \leq x < \alpha_2 \\ \left(\frac{1}{3}\right) \left(1 + \frac{(x-\alpha_2)}{\alpha_3-\alpha_2}\right) & \alpha_2 \leq x < \alpha_3 \\ \frac{2}{3} & \alpha_3 \leq x < \alpha_4 \\ \left(\frac{1}{3}\right) \left(2 + \frac{(x-\alpha_4)}{\alpha_5-\alpha_4}\right) & \alpha_4 \leq x < \alpha_5 \\ 1 & \alpha_5 \leq x < \alpha_6 \\ 1 - \frac{(x-\alpha_6)}{3(\alpha_7-\alpha_6)} & \alpha_6 \leq x < \alpha_7 \\ \frac{2}{3} & \alpha_7 \leq x < \alpha_8 \\ \left(\frac{1}{3}\right) \left(2 - \frac{(x-\alpha_8)}{\alpha_9-\alpha_8}\right) & \alpha_8 \leq x < \alpha_9 \\ \left(\frac{1}{3}\right) \left(1 - \frac{(x-\alpha_9)}{\alpha_{10}-\alpha_9}\right) & \alpha_9 \leq x \leq \alpha_{10} \\ 0 & x > \alpha_{10} \end{cases}$$

$$(\text{inf}_1 \tilde{\mathcal{A}}_{Dec_{\sigma}} + \text{sup}_3 \tilde{\mathcal{A}}_{Dec_{\sigma}}) = ([\alpha_1 + 3\sigma(\alpha_2 - \alpha_1)] + [\alpha_9 + (1 - 3\sigma)(\alpha_{10} - \alpha_9)]) \quad \sigma \in \left[0, \frac{1}{3}\right]$$

$$(\text{inf}_2 \tilde{\mathcal{A}}_{Dec_{\sigma}} + \text{sup}_2 \tilde{\mathcal{A}}_{Dec_{\sigma}}) = ([\alpha_2 + (3\sigma - 1)(\alpha_3 - \alpha_2)] + [\alpha_8 + (2 - 3\sigma)(\alpha_9 - \alpha_8)]) \quad \sigma \in \left[\frac{1}{3}, \frac{2}{3}\right]$$

$$(\text{inf}_3 \tilde{\mathcal{A}}_{Dec_{\sigma}} + \text{sup}_1 \tilde{\mathcal{A}}_{Dec_{\sigma}}) = ([\alpha_4 + (3\sigma - 2)(\alpha_5 - \alpha_4)] + [\alpha_6 + 3(1 - \sigma)(\alpha_7 - \alpha_6)]) \quad \sigma \in \left[\frac{2}{3}, 1\right]$$

Ranking Function

A ranking function $\mathfrak{R}(\tilde{\mathcal{A}})$ is a mathematical function in fuzzy logic that determines a membership or truth value for a set based on the degree to which its elements are comparable to a given fuzzy set. In

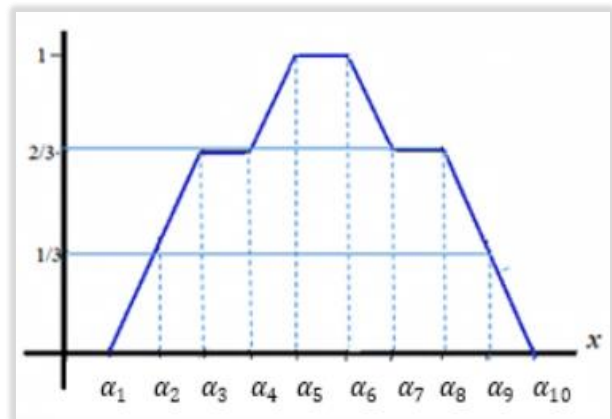


Figure 1. Decagonal membership function.

The (σ – cut) function is defined as:

$$\tilde{\mathcal{A}}_{Dec_{\sigma}} = \begin{cases} \alpha_1 + 3\sigma(\alpha_2 - \alpha_1) & \sigma \in \left[0, \frac{1}{3}\right] \\ \alpha_2 + (3\sigma - 1)(\alpha_3 - \alpha_2) & \sigma \in \left[\frac{1}{3}, \frac{2}{3}\right] \\ \alpha_4 + (3\sigma - 2)(\alpha_5 - \alpha_4) & \sigma \in \left[\frac{2}{3}, 1\right] \\ \alpha_6 + 3(1 - \sigma)(\alpha_7 - \alpha_6) & \sigma \in \left[\frac{2}{3}, 1\right] \\ \alpha_8 + (2 - 3\sigma)(\alpha_9 - \alpha_8) & \sigma \in \left[\frac{1}{3}, \frac{2}{3}\right] \\ \alpha_9 + (1 - 3\sigma)(\alpha_{10} - \alpha_9) & \sigma \in \left[0, \frac{1}{3}\right] \end{cases}$$

Where,

order to make informed decisions and better organize data, a ranking function might help prioritize certain characteristics. A novel ranking function, based on the suggested decagonal membership function, is presented in this section.

$$\mathfrak{R}(\widetilde{\mathcal{A}}_{Dec}) = \left(\frac{1}{2}\right) \int_0^1 \left(\frac{1}{3}\right) (\inf_i \widetilde{\mathcal{A}}_{Dec\sigma} + \sup_j \widetilde{\mathcal{A}}_{Dec\sigma}) d\sigma$$

$$i = 1, 2, 3 \quad j = \begin{cases} i + 2 & i = 1 \\ i & i = 2 \\ i - 2 & i = 3 \end{cases}$$

$$\begin{aligned} \mathfrak{R}(\widetilde{\mathcal{A}}_{Dec}) &= \left(\frac{1}{2}\right) \left[\int_0^{\frac{1}{3}} \left(\frac{1}{3}\right) (\alpha_1 + 3\sigma(\alpha_2 - \alpha_1)) d\sigma \right. \\ &+ (\alpha_9 + (1 - 3\sigma)(\alpha_{10} - \alpha_9)) d\sigma \\ &+ \int_{\frac{1}{3}}^{\frac{2}{3}} \left(\frac{1}{3}\right) (\alpha_2 + (3\sigma - 1)(\alpha_3 - \alpha_2)) d\sigma \\ &+ (\alpha_8 + (2 - 3\sigma)(\alpha_9 - \alpha_8)) d\sigma \\ &+ \int_{\frac{2}{3}}^1 \left(\frac{1}{3}\right) (\alpha_4 + (3\sigma - 2)(\alpha_5 - \alpha_4)) d\sigma \\ &\left. + (\alpha_6 + 3(1 - \sigma)(\alpha_7 - \alpha_6)) d\sigma \right] \end{aligned}$$

$$\mathfrak{R}(\widetilde{\mathcal{A}}_{Dec}) = \frac{1}{36} [\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + 2\alpha_9 + \alpha_{10}]$$

Fuzzy Mathematical Operations of Decagonal Fuzzy Numbers^{12, 13}

Let $\widetilde{\mathcal{A}}_{Dec}$ and $\widetilde{\mathcal{B}}_{Dec}$ be two arbitrary Decagonal fuzzy numbers, such that

$$\begin{aligned} \widetilde{\mathcal{A}}_{Dec} &= (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)_{(\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10})}, \widetilde{\mathcal{B}}_{Dec} \\ &= (b_1, b_2, b_3, b_4, b_5)_{(b_6, b_7, b_8, b_9, b_{10})} \end{aligned}$$

Define the Addition, subtraction, and multiplication operations as follows:

- $\widetilde{\mathcal{A}}_{Dec} \oplus \widetilde{\mathcal{B}}_{Dec} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)_{(\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10})} \oplus (b_1, b_2, b_3, b_4, b_5)_{(b_6, b_7, b_8, b_9, b_{10})} = (\alpha_1 + b_1, \alpha_2 + b_2, \alpha_3 + b_3, \alpha_4 + b_4, \alpha_5 + b_5)_{(\alpha_6 + b_6, \alpha_7 + b_7, \alpha_8 + b_8, \alpha_9 + b_9, \alpha_{10} + b_{10})}$
- $\widetilde{\mathcal{A}}_{Dec} \ominus \widetilde{\mathcal{B}}_{Dec} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)_{(\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10})} \ominus (b_1, b_2, b_3, b_4, b_5)_{(b_6, b_7, b_8, b_9, b_{10})} = (\alpha_1 - b_1, \alpha_2 - b_2, \alpha_3 - b_3, \alpha_4 - b_4, \alpha_5 - b_5)_{(\alpha_6 - b_6, \alpha_7 - b_7, \alpha_8 - b_8, \alpha_9 - b_9, \alpha_{10} - b_{10})}$

- $\widetilde{\mathcal{A}}_{Dec} \otimes \widetilde{\mathcal{B}}_{Dec} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)_{(\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10})} \otimes (b_1, b_2, b_3, b_4, b_5)_{(b_6, b_7, b_8, b_9, b_{10})} = (\alpha_1 * b_1, \alpha_2 * b_2, \alpha_3 * b_3, \alpha_4 * b_4, \alpha_5 * b_5)_{(\alpha_6 * b_6, \alpha_7 * b_7, \alpha_8 * b_8, \alpha_9 * b_9, \alpha_{10} * b_{10})}$
- $\lambda \otimes \widetilde{\mathcal{A}}_{Dec} = \begin{cases} (\lambda\alpha_1, \lambda\alpha_2, \lambda\alpha_3, \lambda\alpha_4, \lambda\alpha_5)_{(\lambda\alpha_6, \lambda\alpha_7, \lambda\alpha_8, \lambda\alpha_9, \lambda\alpha_{10})} & \text{if } \lambda > 0 \\ 0 & \\ (\lambda\alpha_{10}, \lambda\alpha_9, \lambda\alpha_8, \lambda\alpha_7, \lambda\alpha_6)_{(\lambda\alpha_5, \lambda\alpha_4, \lambda\alpha_3, \lambda\alpha_2, \lambda\alpha_1)} & \text{if } \lambda < 0 \end{cases}$
- if $0 \notin (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)_{(\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10})}$ then $\frac{(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)_{(\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10})}}{(b_1, b_2, b_3, b_4, b_5)_{(b_6, b_7, b_8, b_9, b_{10})}} = (\frac{\alpha_1}{b_1}, \frac{\alpha_2}{b_2}, \frac{\alpha_3}{b_3}, \dots, \frac{\alpha_{10}}{b_{10}})$.

Fully Fuzzy Fractional Linear Programming Problem (FFFLPP)¹⁴

Consider the following FFFLPP problem having m fuzzy constraints and n fuzzy variables:

$$\text{Max } \tilde{\omega} \cong \frac{\tilde{n}^T \tilde{x} \oplus \tilde{\beta}}{\tilde{d}^T \tilde{x} \oplus \tilde{\delta}} = \frac{\tilde{N}(x)}{\tilde{D}(x)}$$

S. to

$$\tilde{A} \otimes \tilde{x} \leq \tilde{B}$$

$$\tilde{x} \geq \tilde{0}$$

Where, $\tilde{n}^T = (\tilde{n}_j)_{1*n}$, $\tilde{d}^T = (\tilde{d}_j)_{1*n}$, $\tilde{x} = (\tilde{x}_j)_{n*1}$, $\tilde{\beta}, \tilde{\delta} \in$ decagonal fuzzy numbers.

$$\tilde{A} = (\tilde{a}_{ij})_{m*n}, \tilde{B} = (\tilde{b}_i)_{m*1}$$

$\tilde{n}^T, \tilde{d}^T, \tilde{x}, \tilde{A}, \tilde{B}$ are decagonal fuzzy numbers $\forall 1 \leq j \leq n, 1 \leq i \leq m$

The Fully Fuzzy Simplex Method¹⁵

In order to reach the optimal fuzzy solution for FFFLP problem, we first need to convert the FFFLP problem into FFLP problem utilizing the development of complementary method¹⁶, then adding fuzzy slack variables $\tilde{S}_i, i = 1, 2, \dots, m$, with coefficients equal to zero in the objective function, therefore transforming all the inequalities of the constraints into equations and construct the fully fuzzy simplex tableau as shown in Table 1:

Table 1. Fully fuzzy Simplex Tableau

Basic var.	\tilde{x}_1	\tilde{x}_2	... \tilde{x}_n	\tilde{S}_1	\tilde{S}_2	...	\tilde{S}_n	R.H.S	$\Re(R.H.S)$
$(\tilde{\omega} \ominus \tilde{c}_j)$	$-\tilde{c}_1$	$-\tilde{c}_2$... $-\tilde{c}_n$	$(0,0,\dots,0)$	$(0,0,\dots,0)$...	$(0,0,\dots,0)$	$\tilde{\beta}_n$	$\Re(\tilde{\beta}_n)$
\tilde{S}_1	\tilde{a}_{11}	\tilde{a}_{12}	... \tilde{a}_{1n}	$(0,0,\dots,1)$	$(0,0,\dots,0)$...	$(0,0,\dots,0)$	\tilde{b}_1	$\Re(\tilde{b}_1)$
\tilde{S}_2	\tilde{a}_{21}	\tilde{a}_{22}	... \tilde{a}_{2n}	$(0,0,\dots,0)$	$(0,0,\dots,1)$...	$(0,0,\dots,0)$	\tilde{b}_2	$\Re(\tilde{b}_2)$
\vdots	\vdots	\vdots	...	\vdots	\vdots	...	\vdots	\vdots	\vdots
\tilde{S}_m	\tilde{a}_{m1}	\tilde{a}_{m2}	... \tilde{a}_{mn}	$(0,0,\dots,0)$	$(0,0,\dots,0)$...	$(0,0,\dots,1)$	\tilde{b}_m	$\Re(\tilde{b}_m)$

Using the fully fuzzy Simplex method to select the entering and leaving fuzzy variables. The arithmetic operations of the decagonal fuzzy numbers facilitate access to a new fuzzy tableau (new iteration) of a fully fuzzy simplex table. The optimal fuzzy solution

of the maximum problem is reached, when $(\Re(\tilde{\omega} \ominus \tilde{c}_j) \geq 0)$, in the maximization objective function and at the minimum $(\Re(\tilde{\omega} \ominus \tilde{c}_j) \leq 0)$.

Results and Discussion

Numerical Example.

The following fractional linear programming problem is:

$$Max \omega = \frac{6x_1 + 5x_2}{2x_1 + 7}$$

s. t

$$\begin{aligned} x_1 + 2x_2 &\leq 3 \\ 3x_1 + 2x_2 &\leq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

First, the crisp optimal solution of this problem is:

$$x_1 = \frac{3}{2}, x_2 = \frac{3}{4}, \omega = 1.28, \text{ Example 1 in the paper}^{17}$$

Second, taking the above example with all the variables are decagonal fuzzy numbers:

$$Max \tilde{\omega} = \frac{\begin{pmatrix} 5, 6, 7, 8, 9 \\ 10, 10.5, 11, 12, 13 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 3, 5, 6, 6.5, 7 \\ 9, 9.5, 10.1, 11, 12 \end{pmatrix} \otimes \tilde{x}_2}{\begin{pmatrix} 1, 2, 4, 4, 5 \\ 6, 7, 8, 9, 10 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 5.5, 7, 8, 9, 9.5 \\ 10, 11, 12, 13, 14 \end{pmatrix}}$$

s. t

$$\begin{aligned} &\begin{pmatrix} 0.5, 1, 1.5, 2, 3 \\ 4, 4.5, 4.6, 5, 7 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 1.5, 2, 2.5, 3, 4 \\ 5, 7, 7.1, 9, 10 \end{pmatrix} \otimes \tilde{x}_2 \leq \\ &\begin{pmatrix} 2, 3, 4, 5, 7 \\ 8, 9, 11, 11.5, 12 \end{pmatrix} \\ &\begin{pmatrix} 2, 3, 4, 5, 6 \\ 7.1, 7.5, 12, 13, 14 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 1, 2, 3, 4, 5 \\ 6, 7, 8, 9, 10 \end{pmatrix} \otimes \tilde{x}_2 \leq \\ &\begin{pmatrix} 5, 6, 7, 8, 9 \\ 10, 10.5, 11, 12, 13 \end{pmatrix} \\ &\tilde{x}_1, \tilde{x}_2 \geq 0 \end{aligned}$$

Applying the proposed algorithm, the first step uses the development complementary method to convert the problem to a fully fuzzy linear programming problem (FFLPP).

Let $Max \tilde{\omega} = \frac{\tilde{\omega}_1}{\tilde{\omega}_2}$ where, $\tilde{\omega}_1 =$

$$\begin{pmatrix} 5, 6, 7, 8, 9 \\ 10, 10.5, 11, 12, 13 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 3, 5, 6, 6.5, 7, 8 \\ 9, 9.5, 10.1, 11, 12 \end{pmatrix} \otimes \tilde{x}_2$$

$$\tilde{\omega}_2 = \begin{pmatrix} 1, 2, 4, 4, 5 \\ 6, 7, 8, 9, 10 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 5.5, 7, 8, 9, 9.5 \\ 10, 11, 12, 13, 14 \end{pmatrix}$$

$$\therefore \ominus \tilde{\omega}_2 = \ominus \begin{pmatrix} 1, 2, 4, 4, 5 \\ 6, 7, 8, 9, 10 \end{pmatrix} \otimes \tilde{x}_1$$

$$\ominus \begin{pmatrix} 5.5, 7, 8, 9, 9.5 \\ 10, 11, 12, 13, 14 \end{pmatrix}$$

The new form of the fuzzy objective function is:

$$Max \omega' = \tilde{\omega}_1 - \tilde{\omega}_2$$

$$Max \omega' = \begin{pmatrix} 5, 6, 7, 8, 9 \\ 10, 10.5, 11, 12, 13 \end{pmatrix} \otimes \tilde{x}_1 \ominus$$

$$\begin{pmatrix} 1, 2, 4, 4, 5 \\ 6, 7, 8, 9, 10 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 3, 5, 6, 6.5, 7 \\ 9, 9.5, 10.1, 11, 12 \end{pmatrix} \otimes \tilde{x}_2 \ominus$$

$$\begin{pmatrix} 5.5, 7, 8, 9, 9.5 \\ 10, 11, 12, 13, 14 \end{pmatrix}$$

$$=$$

$$\begin{pmatrix} 5 - 10, 6 - 9, 7 - 8, 8 - 7, 9 - 6 \\ 10 - 5, 10.5 - 4, 11 - 4, 12 - 2, 13 - 1 \end{pmatrix} \otimes \tilde{x}_1 \oplus$$

$$\begin{pmatrix} 3, 5, 6, 6.5, 7 \\ 9, 9.5, 10.1, 11, 12 \end{pmatrix} \otimes \tilde{x}_2 \ominus \begin{pmatrix} 5.5, 7, 8, 9, 9.5 \\ 10, 11, 12, 13, 14 \end{pmatrix}$$

$$= \begin{pmatrix} -5, -3, -1, 1, 3 \\ 5, 6.5, 7, 10, 12 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 3, 5, 6, 6.5, 7 \\ 9, 9.5, 10.1, 11, 12 \end{pmatrix} \otimes \tilde{x}_2 \oplus$$

$$\begin{pmatrix} -14, -13, -12, -11, -10 \\ -9.5, -9, -8, -7, -5.5 \end{pmatrix}$$

Then the new problem becomes as follows:

$$Max \omega' =$$

$$\begin{pmatrix} -5, -3, -1, 1, 3 \\ 5, 6.5, 7, 10, 12 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 3, 5, 6, 6.5, 7 \\ 9, 9.5, 10.1, 11, 12 \end{pmatrix} \otimes \tilde{x}_2 \oplus$$

$$\begin{pmatrix} -14, -13, -12, -11, -10 \\ -9.5, -9, -8, -7, -5.5 \end{pmatrix}$$

s. t

$$\begin{aligned} & \begin{pmatrix} 0.5, 1, 1.5, 2, 3 \\ 4, 4.5, 4.6, 5, 7 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 1.5, 2, 2.5, 3, 4 \\ 5, 7, 7.1, 9, 10 \end{pmatrix} \otimes \tilde{x}_2 \\ & \leq \begin{pmatrix} 2, 3, 4, 5, 7 \\ 8, 9, 11, 11.5, 12 \end{pmatrix} \\ & \begin{pmatrix} 2, 3, 4, 5, 6 \\ 7.1, 7.5, 12, 13, 14 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 1, 2, 3, 4, 5 \\ 6, 7, 8, 9, 10 \end{pmatrix} \otimes \tilde{x}_2 \\ & \leq \begin{pmatrix} 5, 6, 7, 8, 9 \\ 10, 10.5, 11, 12, 13 \end{pmatrix} \\ & \tilde{x}_1, \tilde{x}_2 \geq 0 \end{aligned}$$

The second step is to convert the problem to the standard form by adding the fuzzy slack variables,

$$\begin{aligned} & \text{Max } \tilde{\omega}' = \\ & \begin{pmatrix} -5, -3, -1, 1, 3 \\ 5, 6.5, 7, 10, 12 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 3, 5, 6, 6.5, 7 \\ 9, 9.5, 10.1, 11, 12 \end{pmatrix} \otimes \tilde{x}_2 \oplus \end{aligned}$$

$$\begin{aligned} & \begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix} \otimes \tilde{s}_1 \oplus \begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix} \otimes \tilde{s}_2 \\ & \oplus \begin{pmatrix} -14, -13, -12, -11, -10 \\ -9.5, -9, -8, -7, -5.5 \end{pmatrix} \end{aligned}$$

s. t

$$\begin{aligned} & \begin{pmatrix} 0.5, 1, 1.5, 2, 3 \\ 4, 4.5, 4.6, 5, 7 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 1.5, 2, 2.5, 3, 4 \\ 5, 7, 7.1, 9, 10 \end{pmatrix} \otimes \tilde{x}_2 \oplus \\ & \begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 1 \end{pmatrix} \otimes \tilde{s}_1 = \begin{pmatrix} 2, 3, 4, 5, 7 \\ 8, 9, 11, 11.5, 12 \end{pmatrix} \\ & \begin{pmatrix} 2, 3, 4, 5, 6 \\ 7.1, 7.5, 12, 13, 14 \end{pmatrix} \otimes \tilde{x}_1 \oplus \begin{pmatrix} 1, 2, 3, 4, 5 \\ 6, 7, 8, 9, 10 \end{pmatrix} \otimes \tilde{x}_2 \oplus \\ & \begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 1 \end{pmatrix} \otimes \tilde{s}_2 = \begin{pmatrix} 5, 6, 7, 8, 9 \\ 10, 10.5, 11, 12, 13 \end{pmatrix} \\ & \tilde{x}_1, \tilde{x}_2, \tilde{s}_1, \tilde{s}_2 \geq \tilde{0} \end{aligned}$$

The first tableau of the fully fuzzy simplex is shown in Table 2 below:

Table 2. Primary table of the fully fuzzy simplex method

B.V	\tilde{x}_1	\tilde{x}_2	\tilde{s}_1	\tilde{s}_2	R.H.S	θ
$A_1 \quad \tilde{\omega}' \ominus \tilde{c}_j$	$\begin{pmatrix} -12, -10, -7, -6.5, -5 \\ -3, -1, 1, 3, 5 \end{pmatrix}$	$\begin{pmatrix} -12, -11, -10.1, -9.5, -9 \\ -7, -6.5, -6, -5, -3 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix}$	$\begin{pmatrix} -14, -13, -12, -11, -10 \\ -9.5, -9, -8, -7, -5.5 \end{pmatrix}$	
$A_2 \rightarrow \tilde{s}_1$	$\begin{pmatrix} 0.5, 1, 1.5, 2, 3 \\ 4, 4.5, 4.6, 5, 7 \end{pmatrix}$	$\begin{pmatrix} 1.5, 2, 2.5, 3, 4 \\ 5, 7, 7.1, 9, 10 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 1 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix}$	$\begin{pmatrix} 2, 3, 4, 5, 7 \\ 8, 9, 11, 11.5, 12 \end{pmatrix}$	$\frac{5.34}{1.7}$
$A_3 \quad \tilde{s}_2$	$\begin{pmatrix} 2, 3, 4, 5, 6 \\ 7.1, 7.5, 12, 13, 14 \end{pmatrix}$	$\begin{pmatrix} 1, 2, 3, 4, 5 \\ 6, 7, 8, 9, 10 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 1 \end{pmatrix}$	$\begin{pmatrix} 5, 6, 7, 8, 9 \\ 10, 10.5, 11, 12, 13 \end{pmatrix}$	$\frac{3.04}{1.83}$

Determining the entering and leaving variables by the ranking function technique, as follows:

$$\text{The entering variable} = \min\{\mathfrak{R}(\tilde{x}_1), \mathfrak{R}(\tilde{x}_2)\} = \min\{-1.15, -2.64\} = -2.64$$

$$\text{The leaving variable} = \theta =$$

$$\min\left\{\frac{\mathfrak{R}\left(\begin{pmatrix} 2, 3, 4, 5, 7 \\ 8, 9, 11, 11.5, 12 \end{pmatrix}\right)}{\mathfrak{R}\left(\begin{pmatrix} 1.5, 2, 2.5, 3, 4 \\ 5, 7, 7.1, 9, 10 \end{pmatrix}\right)}, \frac{\mathfrak{R}\left(\begin{pmatrix} 5, 6, 7, 8, 9 \\ 10, 10.5, 11, 12, 13 \end{pmatrix}\right)}{\mathfrak{R}\left(\begin{pmatrix} 1, 2, 3, 4, 5 \\ 6, 7, 8, 9, 10 \end{pmatrix}\right)}\right\} =$$

$$\min\left\{\frac{2.36}{1.7}, \frac{3.013}{1.8}\right\} = \min\{1.37, 1.66\}$$

Therefore, the entering variable is \tilde{x}_2 and the leaving variable is \tilde{s}_1 .

The pivot element of decagonal fuzzy numbers is:

$$\begin{pmatrix} 1.5, 2, 2.5, 3, 4 \\ 5, 7, 7.1, 9, 10 \end{pmatrix}$$

Now, using the operation of the decagonal fuzzy number is as following:

$$\text{The pivot row } A_2' = \left(\frac{1}{2}\right) \otimes A_2, \text{ and the other rows: } A_1' = (11 \otimes A_2' \oplus A_1), A_3' = ((-2/9) \otimes A_2) \oplus A_3$$

So, the new alteration of the fully fuzzy simplex method is shown in Table 3:

Table 3. The optimal table solution for the fully fuzzy simplex method

Row	B.V	\tilde{x}_1	\tilde{x}_2	\tilde{s}_1	\tilde{s}_2	R.H.S
A_1'	$\tilde{\omega}' \ominus \tilde{c}_j$	$\begin{pmatrix} -9.2, -4.5, 1.25, 4.5, 11.5 \\ 19, 23.7, 26.3, 30.5, 43.5 \end{pmatrix}$	$\begin{pmatrix} -3.7, 0, 3.6, 7, 13 \\ 20.5, 32, 33.05, 44.5, 52 \end{pmatrix}$	$\begin{pmatrix} 5.5, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix}$	$\begin{pmatrix} -3.3, 5, 10, 16.5, 28.5 \\ 34.5, 40.5, 52.5, 56.2, 60.5 \end{pmatrix}$
A_2'	\tilde{x}_2	$\begin{pmatrix} 0.25, 0.5, 0.75, 1, 1.5 \\ 2.2, 2.5, 2.3, 2.5, 3.5 \end{pmatrix}$	$\begin{pmatrix} 0.75, 1, 1.25, 1.5, 2 \\ 2.5, 3.5, 3.55, 4.5, 5 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0.5 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix}$	$\begin{pmatrix} 1, 1.5, 2, 2.5, 3.5 \\ 4.4, 5.5, 5.5, 7.5, 6 \end{pmatrix}$
A_3'	\tilde{s}_2	$\begin{pmatrix} -1.1, 0.1, 1.3, 3.3, 4.4 \\ 5.7, 6.3, 11.1, 12.3, 13.5 \end{pmatrix}$	$\begin{pmatrix} -1.2, 0.1, 4.2, 5.3, 9 \\ 5.2, 6.4, 7.5, 8.6, 9.7 \end{pmatrix}$	$\begin{pmatrix} -0.2, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{pmatrix}$	$\begin{pmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 1 \end{pmatrix}$	$\begin{pmatrix} 2.3, 3.4, 4.5, 6.7, 2 \\ 8.4, 9.3, 10.1, 11.3, 12.5 \end{pmatrix}$

Since $\Re(\tilde{\omega}' \ominus \tilde{c}_j) \geq 0$, and $\Re(R.H.S) \geq 0$, the optimal solution is reached.

$$\begin{aligned} \therefore \text{Max } \tilde{\omega} &= \frac{\begin{pmatrix} 3, 5, 6, 6.5, 7 \\ 9, 9.5, 10.1, 11, 12 \end{pmatrix} \otimes \begin{pmatrix} 1, 1.5, 2, 2.5, 3.5 \\ 4, 4.5, 5.5, 5.7, 6 \end{pmatrix}}{\begin{pmatrix} 5.5, 7, 8, 9, 9.5 \\ 10, 11, 12, 13, 14 \end{pmatrix}} \\ &= \begin{pmatrix} 0.21, 2.5, 1, 1.47, 2.4 \\ 3.78, 4.7, 6.9, 8.9, 13.09 \end{pmatrix} \end{aligned}$$

Conclusion

In this paper, we have addressed the fully fuzzy fractional linear programming problem using decagonal fuzziness numbers. This paper is helpful in finding solutions to these types of problems. The proposed new ranking function technique has been effective in selecting the entering and leaving variables of decagonal fuzzy numbers in the simplex table. Through utilizing the arithmetic operations of decagonal fuzzy numbers, a new iteration of the fully fuzzy simplex table has been found. The

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Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been

Authors' Contribution Statement

I. H. H. contributed in developing and carrying out the study, as well as in obtaining the best possible results. I.H. A. took an active role in creating the

The crisp optimal solution using the proposed ranking function is $\Re(\tilde{x}_2) = 1.18$, $\Re(\tilde{x}_1) = 0$, $\Re(\tilde{\omega}) = 1.564$ is the best solution compared with the crisp optimal solution ($\omega = 1.28$) for the crisp problem in example 1 paper¹⁷.

effectiveness of the procedure has been ensured by the proximity of the fuzzy optimal solution to the crisp optimal solution of the same problem. This work can be considered a good start to making steps to solve various types of fully fuzzy programming problems. Since these problems are still in need of development, new algorithms that can process large problems and return optimal or near-optimal solutions in a reasonable amount of computational time should be the focus of future work.

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study design, editing the paper, and reviewing the final results. The authors improved the overall quality of their work by examining their results.

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استخدام دالة رتبية جديدة لحل مسائل البرمجة الخطية الكسرية الضبابية التامة

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الخلاصة

البرمجة الضبابية مفيدة بشكل خاص في الحالات التي تكون فيها المعاملات غامضة. بسبب هذه الميزة ، في السنوات الأخيرة ، ظهرت العديد من التقنيات لمعالجة عدم اليقين. يقترح هذا البحث تقنية دالة ترتيب جديدة مع متغيرات من نوع الأرقام الضبابية العشرية لحل مشاكل البرمجة الخطية الكسرية الضبابية التامة (FFFLP). تعتمد هذه التقنية على إدخال دالة رتبية جديدة لرقم ضبابي عشري ، واستخدام الطريقة المبسطة الضبابية التامة. بعد تحويل مشكلة FFFLP إلى مشكلة البرمجة الخطية الضبابية التامة (FFLP) باستخدام الطريقة التكميلية ، ثم يتم حلها باستخدام الجداول المبسطة الضبابية التامة ، والتي تكون فيها جميع القيم أرقامًا عشرية ضبابية. بمساعدة العمليات الحسابية للأرقام العشرية ، يتم الوصول إلى التكرار الجديد للجدول البسيط. يتم تكرار الخطوات لحين الوصول إلى الحل الضبابي الأمثل. ولبيان الطريقة المقترحة، يتم تقديم مثال عددي يوضح خطوات إيجاد الحل الضبابي الأمثل للمشكلة.

الكلمات المفتاحية: العمليات الحسابية العشرية، دالة الانتماء العشرية، البرمجة الكسرية الخطية الضبابية التامة ، الطريقة المبسطة الضبابية التامة ، الدالة الرتبية.