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On Hyper ρ/δ-Algebra

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Abstract

In this work, new concepts of algebra structures such as hyper $\rho-$algebra, Hyper $\delta-$algebra were defined. These concepts are introduced by using a hyper operation * on $¥ \neq \emptyset$ is a mapping from $¥ \times ¥$ to the non-empty power set $P^*(¥) = P(¥)/\emptyset$ i.e. $*: ¥ \times ¥ \rightarrow P^*(¥)$, $(\sigma_1, \sigma_2) \mapsto \sigma_1 * \sigma_2 \subseteq ¥$, $\forall\, \sigma_1,\sigma_2 \in ¥$ and $\gamma_1, \gamma_2 \subseteq ¥$ defined as $\gamma_1 * \gamma_2 = \bigcup_{\sigma_1 \in \gamma_1, \sigma_2 \in \gamma_2} \sigma_1 * \sigma_2$ and $\gamma_1 * \sigma_2 = \gamma_1 * \{\sigma_2\}$, $\sigma_1 * \gamma_2 = \{\sigma_1\} * \gamma_2$. Let $(\Omega,*)$ be a hyper structure such that $1 \in \Omega$, in hyper structure $(\Omega,*)$ a hyper order is a relation was defined by $(\forall \gamma_1, \gamma_2 \in P^*(¥))$ $(\gamma_1 \ll \gamma_2 \leftrightarrow (\forall a \in \gamma_1)\ \ \exists b \in \gamma_{2)})(1 \in a * b)$, with some properties. Also, some examples to illustrate our notions are given. And the relationship between these concepts is also discussed.

Keywords: Hyper $\rho-$algebra, Hyper $\delta-$algebra, Hyper $\rho/\delta-$ideal, Hyper $\bar{\rho}$-ideal, Hyper $\rho/\delta-$subalgebra.

Introduction

Abstract algebra was used in many fields of science, notions of Abstract algebra are discussed by many researchers see[1,2] . Many applications were used as hyper-structures in both pure and applied sciences. In 2014 Redfar A et al, used the hyperstructures to define the concept of hyper *BE*-algebra[3]. In[4] the hyperstructure applied by Jun Y et al. to BCK-algebra and he defined the notion hyper BCK-algebra that generalization of BCK-algebra. In 2018 Surdive A et al, used hyper structures to define the concept of BCK-algebra[5]. Also, Uzay D and Firat A introduced the notion of the multiplier of a hyper BCI-algebra[6].

A new concept of a hyper BZ-algebra generalization of BZ-algebra was introduced by DU Y and Zhang X H[7]. Also, the concept of hyperstructure KU-algebra was defined and some properties were investigated by Moustafa S et al.[8].

In 2019 the concept of hyper UP-algebras was discussed by Romano D [9]. Also, khan M and another researcher introduced the notion of BCH-algebra[10]. In 2020 Tawfeeq A et al. defined the concept of Hyper AT- ideal on AT-algebra[11].

A hyper operation * on $¥ \neq \emptyset$ is a mapping from $¥ \times ¥$ to the non-empty power set $P^*(¥) = P(¥)/\emptyset$ i.e. $*: ¥ \times ¥ \rightarrow P^*(¥)$, $(\sigma_1, \sigma_2) \mapsto \sigma_1 * \sigma_2 \subseteq ¥$, $\forall\, \sigma_1,\sigma_2 \in ¥$ and for all $\emptyset \neq \gamma_1, \gamma_2 \subseteq ¥$ then $\gamma_1 * \gamma_2$ defined as $\gamma_1 * \gamma_2 = \bigcup_{\sigma_1 \in \gamma_1, \sigma_2 \in \gamma_2} \sigma_1 * \sigma_2$ and $\gamma_1 * \sigma_2 = \gamma_1 * \{\sigma_2\}$, $\sigma_1 * \gamma_2 = \{\sigma_1\} * \gamma_2$.

Let $(¥,*)$ be a hyper structure such that $1 \in ¥$ in hyper structure $(¥,*)$ a hyper order is a relation was defined by $\forall \gamma_1, \gamma_2 \in P^*(¥))$ $(\gamma_1 \ll \gamma_2 \leftrightarrow$

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$(\forall a \in \gamma_1)\ \exists b \in \gamma_{2)})(1 \in a * b)$. This relationship is called hyper-order. Let $\sigma_1 \ll \sigma_2$ be instead of $\{\sigma_1\} \ll \{\sigma_2\}$. Then for every $\sigma_1,\sigma_2 \in ¥ \rightarrow (\sigma_1 \ll \sigma_2 \leftrightarrow 1 \in \sigma_1 * \sigma_2)$.

The concept of ρ −algebra was introduced and discussed by Mahmood S. and Alredha M. [12]. The constructions of δ −algebra were proposed by Khalil S and Hassan A [13]

In this work the concept of hyper ρ −algebra and some new concepts like hyper ρ −subalgebra, hyper ρ −ideal, hyper $\overline{\rho}$ −ideal and hyper δ −algebra, hyper δ −subalgebra, hyper δ −ideal were introduced. And the relationship between them was studied.

Preliminaries

Definition 1[10]: A ρ −algebra $(\Omega,*)$ is a non-empty set Ω with a constant $1 \in \Omega$ and a binary operation * that satisfying the following for every $\sigma_1, \sigma_2 \in \Omega$:

(1) $\sigma_1 * \sigma_1 = 1$,

(2) $1 * \sigma = 1$,

(3) $\sigma_1 * \sigma_2 = 1 = \sigma_2 * \sigma_1$ imply that $\sigma_1 = \sigma_2$,

(4) $For\ \sigma_1 \neq \sigma_2$, imply $\sigma_1 * \sigma_2 = \sigma_2 * \sigma_1 \neq 1$.

Definition 2 [10]: Let $(¥,*,1)$ be a ρ −algebra and $\emptyset \neq \mu \subseteq ¥$ then μ is called ρ −ideal of ρ −algebra if:

(1) $\sigma_1,\sigma_2 \in \mu$ imply $\sigma_1 * \sigma_2 \in \mu$,

(2) $\sigma_1 * \sigma_2 \in \mu$ and $\sigma_2 \in \mu$ imply $\sigma_1 \in \mu$ for all $\sigma_1,\sigma_2 \in ¥$.

Definition 3 [10]: Let $(¥,*,1)$ be a ρ −algebra and I be a subset of ¥. Then I is called $\overline{\rho}$ −ideal of ρ −algebra ¥ if:

(1) $1 \in I$,

(2) $\sigma_1 \in I$ and $\sigma_2 \in ¥ \rightarrow \sigma_1 * \sigma_2 \in I$, for all $\sigma_1, \sigma_2 \in ¥$.

Definition 4 [10]: Let $(\Omega,*)$ be a $\rho-$ algebra and let $\emptyset \neq H \subseteq \Omega$. H is called a ρ −subalgebra of $(\Omega,*)$ if $x * y \in H$ whenever $x \in H$ and $y \in H$.

Definition 5 [5]: An algebra$(¥,*,1)$ of type (2,0) is called a hyper BCK-algebra if it satisfies the following hold:

(1) $(\sigma_1 * \sigma_2) * (\sigma_2 * \sigma_1) \ll \sigma_1 * \sigma_2$,

(2) $(\sigma_1 * \sigma_2) * \sigma_3 = (\sigma_1 * \sigma_3) * \sigma_2$,

(3) $\sigma_1 * ¥ \ll \{\sigma_1\}$,

(4) $\sigma_1 \ll \sigma_2$ and $\sigma_2 \ll \sigma_1 \ \ll \sigma_1 \rightarrow \sigma_1 = \sigma_2$,

Definition 6 [8]: Let $\emptyset \neq \Psi$ with a constant 1 and * be a hyper operation defined on ¥. Then $(H,*,1)$ is called a hyper BCH-algebra:

(1) $\sigma_1 \ll \sigma_2$,

(2) $(\sigma_1 * \sigma_2) * \sigma_3 = (\sigma_1 * \sigma_3) * \sigma_2$,

(3)$\sigma_1 \ll \sigma_2$ and $\sigma_2 \ll \sigma_1 \rightarrow \sigma_1 = \sigma_2$ for all $\sigma_1, \sigma_2, \sigma_3 \in ¥$

And $\sigma_1 \ll \sigma_2$ is defined by $1 \in \sigma_1 * \sigma_2\ \forall\ \gamma_1, \gamma_2 \subseteq ¥, \gamma_1 \ll \gamma_2$ is defined by: for all $a \in \gamma_1 \ \exists\ b \in \gamma_2$ such that $a \ll b$.

Proposition 1 [8]: Any hyper *BCK*-algebra is a hyper *BCH*-algebra.

Definition 7 [7]: Let ¥ be a non-empty set such that $1 \in ¥$ and $(¥,*,\ll,1)$ be a hyper-structure. Then $(¥,*,\ll,1)$ is called a hyper UP-algebra if:

(1) $(\forall\ \sigma_1, \sigma_2, \sigma_3 \in ¥)(\sigma_2 * \sigma_3 \ll (\sigma_1 * \sigma_2) * (\sigma_1 * \sigma_3))$

(2) $(\forall\ \sigma_1 \in ¥)(\sigma_1 * 1 = \{1\}$,

(3) $(\forall\ \sigma_1 \in ¥)(1 * \sigma_1 = \{\sigma_1\}$,

(4) $(\forall \sigma_1, \sigma_2 \in ¥)\ ((\sigma_1 \ll \sigma_2 \wedge \sigma_2 \ll \sigma_1$

Definition 8 [11]: Algebra system $(\Omega,*,\ll,f)$ is a δ −algebra if $f \in \Omega$ and the following hold:

(1) $\sigma * \sigma = f$,

(2) $\sigma_1 * \sigma_2 = f$,

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(3) $\sigma_1 * \sigma_2 = f$ and $\sigma_2 * \sigma_1 = f \rightarrow \sigma_1 = \sigma_2$, for all $\sigma_1, \sigma_2 \in \Omega$,

(4) For all $\sigma_1 \neq \sigma_2 \in \Omega - \{f\} \rightarrow \sigma_1 * \sigma_2 = \sigma_2 * \sigma_1 \neq f$,

(5) For all $\sigma_1 \neq \sigma_2 \in \Omega - \{f\} \rightarrow (\sigma_1 * (\sigma_2 * \sigma_3)) * (\sigma_3 * \sigma_2) = f$.

The Concept of Hyper ρ −Algebra:

The concept of hyper ρ −algebra, hyper ρ −subalgebra, hyper ρ −ideal and hyper $\bar{\rho}-$ ideal are discussed.

Definition 9: Let Ω be a non-empty set such that $1 \in \Omega$ and $(\Omega, *, \ll, 1)$ be a hyper structure. Then, $(\Omega, *, \ll, 1)$ is called a hyper ρ −algebra if the following hold:

(1) $\sigma_1 \ll \sigma_2$,

(2) $1 * \sigma_1 = \{1\}$,

(3) $\sigma_1 * 1 = \{\sigma_1\}$,

(4) $(\forall \sigma_1 \neq \sigma_2 \in \Omega - \{1\} \rightarrow \sigma_1 * \sigma_2 = \sigma_2 * \sigma_1 \neq \{1\}$,

(5) $(\forall \sigma_1, \sigma_2 \in \Omega)(\sigma_1 \ll \sigma_2 \wedge \sigma_2 \ll \sigma_1) \rightarrow \sigma_1 = \sigma_2$.

Example 1: Let $\Omega = \{1,2,3,4,5\}$ be a set, define a hyper operation (*) on Ω as follows in Table 1:

Table 1. $(\Omega, *, \ll, 1)$ **is a hyper** ρ **−algebra with** $\gamma = \{1,2,3,4,5\}$

*	1	2	3	4	5
1	{1}	{1}	{1}	{1}	{1}
2	{2}	{1,2}	{3}	{4}	{5}
3	{3}	{3}	{1,3}	{4}	{5}
4	{4}	{4}	{4}	{1,4}	{5}
5	{5}	{5}	{5}	{5}	{1,5}

Then $(\Omega, *, \ll, 1)$ is a hyper ρ −algebra.

Proposition 2: Let $(\Omega, *, \ll, 1)$ be a hyper ρ −algebra then for every $\sigma_1, \sigma_2 \in \Omega$:

(1) $\sigma_1 \in \sigma_1 * 1$,

(2) $1 \ll \sigma_1$,

(3) $\sigma_1 \ll \sigma_2, \forall \sigma_1, \sigma_2 \in \Omega\ with\ \ \sigma_1 = \sigma_2$

(4) $\sigma_1 * \sigma_2 = \sigma_2 * \sigma_1\ \forall\ \sigma_1, \sigma_2 \in \Omega/\{1\}\ with\ \ \sigma_1 \neq \sigma_2$,

(5) $\sigma_1 * \sigma_2 = \{\sigma_1\} \leftrightarrow \sigma_2 = 1$,

(6) $\sigma_1 * \sigma_2 = \{1\} \leftrightarrow \sigma_1 = 1, \sigma_2 = 1$,

(7) $\gamma_2 * 1 = \gamma_2, \quad \forall \gamma_2 \subseteq \Omega$,

(8) $1 * \gamma_1 = 1, \quad \forall\ \gamma_1 \subseteq \Omega$,

(9) $(1 * \sigma) * 1 = \{1\}, \quad \forall\ \sigma \in \Omega$,

(10) $\sigma * (1 * 1) = \{\sigma\},\ \forall \sigma \in \Omega$,

(11) $\sigma * \sigma = \{\sigma\}, \quad \leftrightarrow \sigma = 1$,

Proof: **(1)** It is clear (by condition 3 definition 9.). **(2)** by Definition 3.1 condition 2 $\{1\} \subseteq 1 * \sigma$, then $1 \in 1 * \sigma$ that mean $1 \ll \sigma$. **(3)** Since $\sigma_1 = \sigma_2$ that means $1 \in \sigma_1 * \sigma_2$ and $1 \in \sigma_2 * \sigma_1$ then $\sigma_1 \ll \sigma_2$. **(4)** It is verifier (by condition 4 of Definition 9). **(5)** Suppose that $\sigma_1 * \sigma_2 = \{\sigma_1\}$; wanted to prove that $\sigma_2 = 1$, from definition 3.1 condition 3 then $\sigma_2 = 1$. Now, suppose that $\sigma_2 = 1$, wanted to show that $\sigma_1 * \sigma_2 = \{\sigma_1\}$; since $\sigma_2 = 1$, then $\sigma_1 * 1 = \{\sigma_1\}$.

(6) Suppose that $\sigma_1 * \sigma_2 = \{1\}$, wanted to prove that $\sigma_1 = 1 = \sigma_2$ or $\sigma_1 = 1$. Now, suppose that $\sigma_1 \neq 1 \neq \sigma_2$ and $\sigma_1 \neq 1$, then if $\sigma_1 \neq 1$ then $\sigma_1 * \sigma_2 = \{\sigma_1 * \sigma_2\}$, and that contradiction with $\sigma_1 * \sigma_2 = 1$, then $\sigma_1 = 1 = \sigma_2$ or $\sigma_1 = 1$. Now, suppose that $\sigma_1 * \sigma_2 = 1$, then $\sigma_1 = 1 = \sigma_2$ or $\sigma_1 = 1$, we wanted $\sigma_1 * \sigma_2 = \{1\}$, by Definition 3.1 condition 5 deduces that $1 \in 1 * \sigma_1 \wedge 1 \in \sigma_{21\text{condition 3}} * \sigma_1$, and since $\sigma_1 = 1 = \sigma_2$, then $1 \in 1 * \sigma_2 \wedge 1 \in 1 * \sigma_1$, that mean $\sigma_1 * \sigma_2 = \sigma_2 * \sigma_1 = \{1\}$, when $\sigma_1 = 1 = \sigma_2$, or $\sigma_1 = 1$.

(7) Since $\sigma_1 * 1 = \{\sigma_1\}$, that is clear, take any $\emptyset \neq \gamma_2 \subseteq \Omega$, then $\gamma_2 * 1 = \gamma_2$. **(8)** Since $1 * \sigma_1 = \{1\}$, then $1 * \gamma_1 = \{1\}, \forall \emptyset \neq \gamma_1 \subseteq \Omega$. **(9)** Since $1 * \sigma_1 = \{1\}$, then $\{1\} * 1 = \{1\}$. **(10)** Since $1 * 1 = \{1\}$, then $\sigma_1 * \{1\} = \{\sigma_1\},\ \forall\ \sigma_1 \in \Omega$. **(11)** Suppose that $\sigma_1 *$

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$\sigma_1 = \{\sigma_1\}$, wanted $\sigma_1 = 1$. Since $\sigma_1 * \sigma_1 = \{\sigma_1\}$, that means $\sigma_1 = 1$. Now, suppose $\sigma_1 = 1$, wanted to prove that $\sigma_1 * \sigma_1 = \{\sigma_1\}$, since $\sigma_1 * \sigma_1 = 1 * 1 = \{1\}$, then $\sigma_1 * \sigma_2 = \{\sigma_1\}$.

Definition 10: Let $(\Omega,*,\ll,1)$ be a hyper ρ −algebra and let $\gamma \subseteq \Omega$, be a proper subset of Ω. Then $(\gamma,*,\ll,1)$ is called a hyper ρ −subalgebra if it satisfies the following:

$$\forall\, \sigma_1,\sigma_2 \in \gamma \rightarrow \sigma_1 * \sigma_2 \subseteq \gamma$$

Example 2: Take a hyper ρ −algebra $(\Omega,*,\ll,1)$ in Example **1** and let $\gamma \subseteq \Omega$. where $\gamma = \{1,2,3\}$. Table 2 will be:

Table 2. $(\gamma,*,\ll,1)$ **is a hyper** ρ **−subalgebra with** $\gamma = \{1,2,3\}$

*	1	2	3
1	{1}	{1}	{1}
2	{2}	{1,2}	{3}
3	{3}	{3}	{1,3}

Then $(\gamma,*,\ll,1)$ is a hyper $\rho-$ subalgebra since $\forall\, \sigma_1,\sigma_2 \in \gamma$, then $\sigma_1 * \sigma_2 \subseteq \gamma$.

Remark 1: There is no relation between hyper ρ −algebra and hyper *UP/BCK/BCH* –algebra.

Example 3: Let $\Omega = \{1,2,3,4,5,6\}$, and * be a hyper operation defined on Ω as in Table 3:

Table 3. $(\Omega,*,\ll,1)$ **is a hyper** ρ **−algebra with** $\Omega = \{1,2,3,4,5,6\}$.

*	1	2	3	4	5	6
1	{1}	{1}	{1}	{1}	{1}	{1}
2	{2}	{1,2}	{3}	{4}	{5}	{6}
3	{3}	{3}	{1,3}	{4}	{5}	{6}
4	{4}	{4}	{4]	{1,4}	{5}	{6}
5	{5}	{5}	{5}	{5}	{1,5}	{6}
6	{6}	{6}	{6}	{6}	{6}	{1,6}

Then $(\Omega,*,\ll,1)$ is a hyper ρ −algebra. But it is not hyper *UP*-algebra since if take $\sigma_1 = 2$, and$\sigma_2 = 1$, then $2 * 1 = \{2\} \neq \{1\}$. Also, the system is not hyper *BCK*-algebra to verify that if $\sigma_1 = 2$, $\sigma_2 = 3$, $\sigma_3 = 4$, then $(2 * 3) * (3 * 4) \ll 2 * 3 \rightarrow \{3\} * \{4\} \ll \{3\} \rightarrow \{4\} \ll \{3\}$, but $1 \notin \{4\} * \{3\}$, and by Proposition 2.7 then the system is not *BCH*-algebra.

Definition 11: Let $(\Omega,*,\ll,1)$be a hyper ρ −algebra and let a non-empty set $\gamma \subseteq \Omega$ then γ is called a hyper ρ −ideal of a hyper ρ −algebra if:

(1) $\forall\, \sigma_1,\sigma_2 \in \gamma$ imply $\sigma_1 * \sigma_2 \subseteq \gamma$,

(2) if $\sigma_1 * \sigma_2 \subseteq \gamma$, and $\sigma_2 \in \Omega$ then $\sigma_1 \in \Omega$ $\forall \sigma_1,\sigma_2 \in \Omega$.

Example 4: Let $\Omega = \{1,2,3,4\}$ and $*$ be a hyper operation defined on Ω as in Table 4:

Table 4. $(\Omega,*,\ll,1)$ **is a hyper** ρ **−algebra with** $\Omega = \{1,2,3,4\}$

*	1	2	3	4
1	{1}	{1}	{1}	{1}
2	{2}	{1,2}	{1}	{2}
3	{3}	{1}	{1,3}	{3}
4	{4}	{2}	{3}	{1,4}

and let $\gamma = \{1,2\}$, then Table 5 will be:

Table 5. $(\gamma,*,\ll,1)$ **is a hyper** ρ **−ideal of hyper** ρ **−algebra**

*	1	2
1	{1}	{1}
2	{2}	{1,2}

Note that, $\forall\, \sigma_1,\sigma_2 \in \gamma$, then $\sigma_1 * \sigma_2 \subseteq \gamma$ and take any $\sigma_1 * \sigma_2 \subseteq \gamma$ and $\sigma_2 \in \Omega$ since $\sigma_1 \in \gamma$, then $(\gamma,*,\ll,1)$ is a hyper ρ −ideal of hyper ρ −algebra.

Remark 2: Every hyper ρ −ideal is a hyper ρ −subalgebra. But the converse is not true in fact.

Example 5: Take $\gamma = \{1,3,5\}$ in Example **3** Table 6 will be:

Table 6. $(\gamma,*,\ll,1)$ **is not hyper** ρ **−ideal**

*	1	3	5
1	{1}	{1}	{1}
3	{3}	{1,3}	{5}
5	{5}	{5}	{1,5}

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It is clear that γ is a hyper $\rho-$subalgebra, but it is not hyper $\rho-$ideal since if $\sigma_1 * \sigma_2 = \{5\} \subseteq \gamma$ and $\sigma_2 = 5$, then $\sigma_1 = 4 \notin \gamma$.

Proposition 3: The intersection of hyper $\rho-$ideals is a hyper $\rho-$ideal.

Proof: Suppose that $\gamma_i, i \in I$ be a hyper $\rho-$ideal of a hyper $\rho-$algebra Ω and let $\sigma_1, \sigma_2 \in \cap_{i\in I}\gamma_i$, then $\sigma_1 * \sigma_2 \subseteq \gamma_i, \quad \forall\, i \in I$ (since γ_i is a hyper $\rho-$ideal), so $\sigma_1 * \sigma_2 \subseteq I_{i\in I}\gamma_i$.

Now, let $\sigma_1 * \sigma_2 \subseteq\cap_{i\in I}\gamma_i$. And $\sigma_2 \in \gamma_i$, so, since $\sigma_1 * \sigma_2 \subseteq \gamma_i$ and $\sigma_2 \subseteq \gamma_i, \forall\, i \in I$ (since γ_i is a hyper $\rho-$ideal in $\Omega \quad \forall\, i \in I$) then $\sigma_1 \subseteq \gamma_i\ \forall\, i \in I$, thus $\sigma_1 \subseteq\cap_{i\in I}\gamma_i$.

Remark 3: The union of two hyper $\rho-$ideals of hyper $\rho-$algebra is not necessary to be hyper $\rho-$ideal.

Example 6*:* Let Ω a hyper $\rho-$algebra where $\Omega = \{1,2,3,4,5\}$ with Table 7:

Table 7. The union of two hyper $\rho-$ideals of hyper $\rho-$algebra is not necessary to be hyper $\rho-$ideal

*	1	2	3	4	5
1	{1}	{1}	{1}	{1}	{1}
2	{2}	{1,2}	{4}	{5}	{4}
3	{3}	{4}	{1,3}	{5}	{4}
4	{4}	{5}	{5]	{1,4}	{4}
5	{5}	{4}	{4}	{4}	{1,5}

And let $\gamma_1 = \{1,2\},\ \gamma_2 = \{1,3\}$ be a hyper $\rho-$ideal in a hyper $\rho-$algebra Ω, but $\gamma_1 \cup \gamma_2 = \{1,2,3\}$ is not hyper $\rho-$ideal since $2 * 3 = \{4\} \not\subset \gamma_1 \cup \gamma_2$.

Definition 12: Let $(\Omega, *, \ll, 1)$ be a hyper $\rho-$algebra and let λ be a subset of Ω. Then λ called a hyper $\overline{\rho}-$ ideal of a hyper $\rho-$algebra if:

(1) $1 \in \lambda$,

(2) $\sigma_1 \in \lambda,\ \sigma_2 \in \Omega \to \sigma_1 * \sigma_2 \subseteq \lambda, \sigma_1, \sigma_2 \in \Omega$.

Example 7: Let Ω= {1,2,3,4} be a hyper $\rho-$ algebra with Table 8:

Table 8. $(\Omega, *, \ll, \mathbf{1})$ **a hyper $\rho-$ algebra with** Ω={1,2,3,4}

*	1	2	3	4
1	{1}	{1}	{1}	{1}
2	{2}	{1,2}	{3}	{3}
3	{3}	{3}	{1,3}	{2}
4	{4}	{3}	{2}	{1,4}

Then γ_1 is a hyper $\overline{\rho}-$ideal with Table 9:

Table 9.$(\gamma_1, *, \ll, \mathbf{1})$ is a hyper $\overline{\rho}-$ideal

*	1	2	3
1	{1}	{1}	{1}
2	{2}	{1,2}	{3}
3	{3}	{3}	{1,3}

But $\gamma_2 = \{1,2\}$ is not hyper $\overline{\rho}-$ideal of Ω since $2 \in \gamma_2$ and $3 \in \Omega$, but $2 * 3 \notin \gamma_2$.

Lemma 1: Every hyper $\overline{\rho}-$ideal is a hyper $\rho-$subalgebra.

Proof: Suppose that γ is hyper $\overline{\rho}-$ideal of hyper $\rho-$algebra; wanted to show that γ is hyper $\rho-$subalgebra. Now, since γ is hyper $\overline{\rho}-$ideal then $\forall\sigma_1, \sigma_2 \in \Omega$, then $\sigma_1 \in \gamma,\ \sigma_2 \in \Omega,\ \to \sigma_1 * \sigma_2 \subseteq \gamma$, Put $\sigma_2 \in \gamma$, then $\forall\sigma_1, \sigma_2 \in \gamma,\ \sigma_1 * \sigma_2 \subseteq \gamma$ then γ is hyper $\rho-$subalgebra.

Remark 4: The converse of the above lemma is not true in fact.

Example 8: Let $\gamma = \{1,3\}$ in Example **6** then γ is hyper $\rho-$subalgebra, but it is not hyper $\overline{\rho}-$ideal, take $\sigma_1 = 3$, and $\sigma_2 = 4$, show that $\sigma_1 * \sigma_2 = \{2\} \not\subset \gamma$.

Proposition 4: The intersection of hyper $\overline{\rho}-$ideals is a a hyper $\overline{\rho}-$ideal.

Proof: Suppose that $\gamma_i, i \in I$ be a hyper $\overline{\rho}-$ideal of hyper $\rho-$algebra then $1 \in \gamma_i, i \in I$, that mean $1 \in \cap_{i\in I}\gamma_i$, Now, let $\sigma_1 \in \gamma_i, i \in I$, and $\sigma_2 \in \Omega$, and since $\gamma_i, i \in I$ are hyper $\overline{\rho}-$ideal then $\sigma_1 * \sigma_2 \subseteq \gamma_i$, $\forall i \in I$ that mean $\sigma_1 * \sigma_2 \subseteq\cap_{i\in I}\gamma_i$, is a hyper $\overline{\rho}-$ideal.

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The Concept of Hyper δ −Algebra:

The concept of hyper δ −algebra, hyper $\delta -$ subalgebra, hyper δ −ideal and the relation between them with the conceptions hyper ρ −algebra, hyper $\rho - sub$algebra, hyper ρ −ideal and hyper $\bar{\rho}$ −ideal are discussed in this section.

Definition 13: Let $(\Omega,*,\ll,1)$ be a hyper ρ −algebra. Then $(\Omega,*,\ll,1)$ is called a hyper δ −algebra if the following hold:

$$\big(\sigma_1 * (\sigma_2 * \sigma_3)\big) * (\sigma_3 * \sigma_2) = \{1\}$$

Theorem 1: Every hyper δ −algebra is a hyper ρ −algebra.

Remark 5: The converse of the above theorem is not true in general.

Example 9: Suppose that $\Omega = \{1, v, w, \sigma\}$ and the binary operation $*$ is described as in a Table 10:

Table 10. $(\Omega,*,\ll,f)$ **is a hyper** $\boldsymbol{\rho}$ **−algebra but it is not hyper** $\boldsymbol{\delta}$ **−algebra**

*	F	V	W	σ
F	{f}	{f}	{f}	{f}
V	{v}	{f}	{v}	{w}
W	{w}	{v}	{f}	{v}
σ	$\{\sigma\}$	{w}	{v}	{f}

Hence $(\Omega,*,\ll,f)$ is a hyper ρ −algebra. However, it is not hyper δ −algebra, since $v \neq \sigma \in \Omega - \{f\}$ and $\big(v * (v * \sigma)\big) * (\sigma * v) = (v * \{w\}) * w = \{v\} * w \neq \{f\}$.

Definition 14: Assume that $\emptyset \neq H \subseteq \Omega$, where $(\Omega,*,\ll,f)$ is a hyper δ −algebra, then H is a hyper δ −subalgebra of Ω if:

$\sigma_1 * \sigma_2 \subseteq H$, whenever $\sigma_1 \in H$ and $\sigma_2 \in H$.

Example 10: In Example 9 let $\gamma = \{f, v\}$ then $(\gamma,*,\ll,f)$ is a hyper $\delta - sub$algebra of Ω since $\forall\, \sigma_1, \sigma_2 \in \gamma \rightarrow \sigma_1 * \sigma_2 \subseteq \gamma$.

Theorem 2: Let $(\Omega,*,\ll,f)$ is a hyper δ −algebra and let $\emptyset \neq \beta \subseteq \Omega$ is a hyper $\delta - sub$algebra of Ω, then β is a hyper ρ −subalgebra of Ω.

Proof: Assume $\emptyset \neq \beta \subseteq \Omega$ is a hyper $\delta - sub$algebra of Ω. Then Ω is a hyper ρ −subalgebra and β satisfies that $\sigma_1 * \sigma_2 \subseteq \beta$ whenever $\sigma_1 \in \beta$ and $\sigma_2 \in \beta$. Thus β is a hyper ρ −subalgebra of hyper ρ −algebra Ω.

Definition 15: Assume $(\Omega,*,\ll,f)$ a hyper δ −algebra and $\emptyset \neq \mu \subseteq \Omega$. Then μ is said to be hyper δ −ideal of a hyper δ −algebra Ω if:

(1) $\sigma_1, \sigma_2 \in \mu \rightarrow \sigma_1 * \sigma_2 \subseteq \mu$,

(2) $\sigma_1 * \sigma_2 \subseteq \mu$ and $\sigma_2 \in \mu \rightarrow \sigma_1 \in \mu\ \forall \sigma_1, \sigma_2 \in \mu$

Lemma 2: In a hyper δ −algebra, every hyper δ −ideal is a hyper ρ −ideal.

Lemma 3: The intersection of family of hyper δ −ideals in a hyper δ −algebra Ω is a hyper δ −ideal in Ω.

Remark 6: The Diagram1 shows the results:

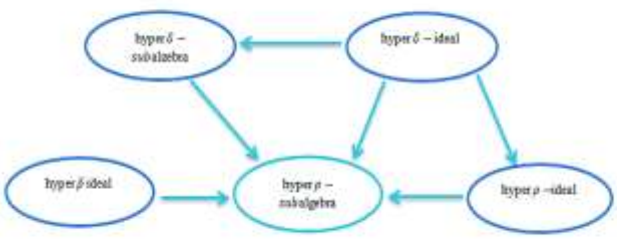

Diagram 1

Conclusion

In this paper new concepts of algebra structures such as hyper ρ −algebra, Hyper δ −algebra were defined. And the concepts of a hyper $\rho/\delta - sub$algebra, hyper ρ/δ −ideal and hyper $\bar{\rho}$— ideal were studied. In this work, we extracted the following:

There is no relationship between hyper ρ −algebra and hyper $BCH/UP/$

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BCK −algebra. If $(\Omega, *, \ll, 1)$ is a hyper δ −algebra then it is hyper ρ −algebra but the converse is not true. If $(F, *, \ll, 1)$ is a hyper $\rho/\delta/\overline{\rho}$ −ideal then it a hyper $\rho/\delta - sub$algebra but the converse is not true. And the relationship between these concepts was discussed and illustrated in diagram 1.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Ministry of Education, General Directorate of Basrah Education.

Authors' Contribution Statement

This work was carried out in collaboration between authors. M.A.K and Z.A.H contributed to the design and implementation of the research, to the analysis of results and to the writing of the manuscript.

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حول الجبر- ρ\δ المفرط

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الخلاصة

في هذا العمل تم تعريف مفاهيم جديدة للبنية الجبرية مثل الجبر- ρ\δ المفرط و درست مفاهيم الجبر الجزئي و المثالي مثل الجبر الجزئي- ρ\δ المفرط و الجبر المثالي- ρ\δ المفرط و الجبر المثالي-ρ̄ المفرط. وقد تم تقديم هذه المفاهيم باستخدام العملية المفرطة * على المجموعة غير الخالية ¥. مع اعطاء بعض المبرهنات و الامثلة لتوضيح هذه المفاهيم بالإضافة الى مناقشة العلاقة بينها.

الكلمات المفتاحية: الجبر-ρ المفرط، الجبر-δ المفرط, الجبر-ρ\δ المثالي المفرط، الجبر- ρ̄ المثالي المفرط، الجبر الجزئي-ρ\δ المفرط.