ON G-OPEN SET

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Abstract:

A new class of generalized open sets in a topological space, called *G*-open sets, is introduced and studied. This class contains all semi-open, preopen, b-open and semi-preopen sets. It is proved that the topology generated by G-open sets contains the topology generated by preopen,b-open and semi-preopen sets respectively.

1. Introduction

In recent years a number of generalizations of open sets have been considered. Seven of these notions were defined similarly using closure operator (cl) and the interior (int) in the following way.

DEFINITION 1. *A subset S of a space (X,T) is called*:

(1)α-set if S ⊂int(cl(intS)),
(2)Semi-open ifS ⊂cl(intS),
(3)Preopen if S ⊂int(clS),
(4)Regular open if S=int(clS),
(5)Semi-regular if and only if there exists a regular open set u with u ⊂S ⊂clu,
(6)b-open if S ⊂cl(intS) ∪int(clS),
(7)Semi-preopen if S ⊂cl(int(clS).

The first three notions are due to Njastad [10], Levine [1] and Mashhour [11], respectively. The concept of a preopen set was introduced by Corson and Michael [2] who used the term "locally dense". D. Andrijevic was introduce the concept of b-open and semi-preopen in [9], [3] respectively. We denote the classes of α-set semi-open, preopen, b-open and semi-preopen sets in a space (X,T) by Tα, SO(X), PO(X), BO(X) and SPO(X) respectively. All of them are larger than T and closed under forming arbitrary unions. Njastad [10] showed that Tα is a topology on X. In general, SO(X) need not be a topology on X, but the intersection of a semi-open set and an open set is semi-open. The same holds for PO(X), BO(X) and SPO(X) respectively. The complement of a semi-open set is called semi-closed. Thus S is semi-closed if and only if int(clS) ⊂S. Preclosed set is similarly defined. For a subset S of a space X preclosure of S, denoted by pclS is the intersection of all preclosed subsets of X containing S. The preinterior (resp. semi-preinterior)of S, denoted by pintS (resp., spintS) is the union of all preopen (resp. semi-preopen) subsets of X contained in S. The definitions(4),(5) was defined by, Maio and Noiri [5], D.S.Jankovic [4], respectively. By cl_α and int_α we denote the closure and the interior operator in (X,T_α).

An extensive study of these operators was done in [3]. We recollect some of the relations that, together with their duals, we shall use in the sequel.

PROPOSITION 1.1. *Let S be a subset of (X,T). Then:*

(1) pcl*S*=*S* ∪cl(int*S*),
(2) pint*S*=*S* ∩int(cl*S*),
(3) int_α*S*=*S* ∩int(clint*S*),
(4) spint*S*=*S* ∩cl(int(cl*S*),
(5) pcl(pint*S*)=pint*S* ∪cl(int*S*).

2. Some properties of G – open sets.

Now we consider a new class of generalized open sets.

DEFINITION 2. A subset S of a space (X,T) is called G-open if S ⊂cl(int(clS)) ∪int(cl(intS)).The class of all G-open sets in X will be denoted by GO(X).

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It is obvious that PO(X) $\cup$SO(X) $\subset$BO (X) $\subset$SPO(X) $\subset$GO(X) and we shall show that the equality inclusions cannot be replaced with equalities between GO(X) and PO(X), SO(X) respectively.

EXAMPLE. Consider the set R of real numbers with the usual topology, and let S=[0,1] $\cup$((1,2) $\cap$Q) where Q stands for the set of rational numbers. Then S is G-open but neither semi-open nor preopen.

PROPOSITION 2.1. *Let S be a subset of a space (X,T), if S$\subset$pcl(pintS) then S is G-open set.*
Proof. Since S $\subset$pcl(pintS) then by Proposition1. we have:
S $\subset$pintS $\cup$cl(intS) and
S $\subset$int(clS) $\cup$cl(intS)and so
clS $\subset$cl(int(clS) $\cup$cl(int S) and since
cl(intS) $\subset$cl(int(clS)) so
S $\subset$clS $\subset$cl(int(clS) therefore
S $\subset$cl(int(clS)) $\cup$int(cl(intS)) so S is G-open set.■

PROPOSITION 2.2. . *Let S be a subset of a space (X,T), if S is G-open and if int (cl S) ⊂ B ⊂ cl (S) then B is semi-regular*
Proof.since *S*⊂cl(int(cl*S*)) ∪int(cl(int*S*)) Then cl*S*⊂cl(int(cl*S*)) ∪cl(int (cl(int*S*)) so cl*S*⊂cl(int(cl*S*)).Therefore int (cl*S*) ⊂B ⊂cl*S* ⊂cl(int(cl*S*)),then int(cl*S*) ⊂ B ⊂cl (int(cl*S*)) and since int(cl*S*) is regular open. Then B is semi–regular. ■

3. On topology generated by *G*-open sets.

Although none of SO(*X*), BO(*X*),PO(*X*), and GO(*X*) is a topology on *X* , each these classes generates a topology in a natural way. Let *T*(*s*)={*V*⊂ *X*| *V* ∩*S* ∈*s*whenever*S*∈*s*}, where *s* stands for *SO*(*X*), PO(*X*),GO(*X*) SPO(*X*) respectively. Clearly *T*(*s*) is a topology on *X* larger than T. The topology generated by PO(*X*) was studied in [6] and denoted by T_γ.. The closure and the interior of a set *S* in (*X*,T_γ) are denoted by cl_γ *S* and $int_\gamma S$, respectively. In [8] the the topology generated by SPO(*X*) was studied and we will denoted by T_{spo}. In [9] the the topology generated by BO(*X*)was studied and denoted byT_b. The topology generated by *G*-open sets will be denoted by T_G and we shall prove that $T_\gamma \subset T_b \subset T_{spo} \subset T_G$. We first recollect some results.

PROPOSITION 3. *If S is G-open subset of a space (X,T) then: S=spint S ∪ int*$_\alpha$*S.*
Proof. By Proposition 1.1 we have spint *S* ∪int$_\alpha$*S*=[*S* ∩cl(int(cl*S*)] ∪[*S* ∩int (cl(int*S*)]=*S* ∩[cl(int(cl*S*) ∪int(clint*S*)]=*S* ■

PROPOSITION 3.1 *Let S be a G-open subset of a space (X,T) such that intS=ΦThen S is semi-preopen.*
Proof. The proof is very easy and will not be given■

PROPOSITPION 3.2.[6] *Let S be a subset of a space (X,T). Then cl*$_\gamma$ *int S=cl(intS) .*

PROPOSITPION 3.3.[8] *Let S be a subset of a space (X,T). Then S∈T*$_\gamma$ *if and only if S=G∪H with G∈Tα and {h}∈PO(X) for every h∈H*■
The last result has an immediate consequence.

COROLLARY 3.4. [9] *Let v∈T*$_\gamma$ *and G∈* SO(*X*) *then v ∩G∈* BO(*X*).■
By previous corollary,and since BO(*X*) ⊂GO(*X*) we have the following- -ng Lemma

LEMMA 3.5. *Let v∈T*$_\gamma$ *and D∈ SO(X) then v ∩ D ∈ GO(X).*

PROPOSTPION 3.6. [7] *The intersection of semi-open and a preopen set is semi-preopen set.*

Since every semi-preopen set is G-open set. Therefore the previous Proposition tell the following Proposition.

PROPOSTPION 3.7. *The intersection of semi-open and a preopen set is G-open set.*

PRROPOSITION 3.8. [7] *For a space (X,T) and $x\in X$ the following are equivalent:*

(a) $\{X\}\in SPO(X)$

(b) $\{X\}\in PO(X)$.

(c) $\{X\}\in T_\gamma$

Since every semi-preopen set is G-open set. Therefore the previous Proposition tell the following Proposition

PRROPOSITION 3.9. *If $\{X\}\in GO(X)$. Then $\{X\}\in T_\gamma$.*

Now we are in ready to prove our main result.

PRROPOSITION 3.10. *Let (X,T) be a space and $v\in T_G$.Then $S=v\backslash int(cl(int\ v))$is semi-preopen set.*

Proof. Since int(cl(intv)) is semi–closed, then $[int(cl(intv))]^c$ is semi-open and so G-open set, but $v\in T_G$, then $v\backslash int(cl(int\ v))=v\cap [int(cl(intv))]^c=S$ is G-open .On other hand ,int$S=\Phi$ and so *S* is semi-preopen by proposition 3.1.

LEMMA 3.11. *In any space (X,T)we have*: (1) [9] $T_\gamma=T_b$.

(2) [7] $T_\gamma=T_{spo}$.

THEOREM 3.12. *Let (X,T) be a space Then $T_\gamma\subset T_G$.*

Proof. Let $v\in T_\gamma$ and $S\in GO(X)$, Then $S=spintS\cup int_\alpha S$ by proposition 3. Hence $v\cap S=(v\cap spintS)\cup(v\cap int_\alpha S)$ is *G*-open by Lemma 3.5 and so $v\in T_G$.

By Theorem 3.12 and Lemma3.11, it is very easy to proof the following Corollary

COROLLARY 3.13. *In any space (X,T) we have $T_\gamma\subset T_{b.}\subset T_{spo}\subset T_G$.*

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حول المجموعات المفتوحة-G

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الخلاصة:

في هذا البحث تم تقديم ودراسة صنف جديد من تعميمات المجموعات المفتوحة. أن هذا الصنف يحتوي كل الاصناف من المجموعات الشبه مفتوحة و قبل المفتوحة.و المفتوحة-b وشبه قبل المفتوحة وتم أثبات أن صنف المجموعات المفتوحة-G يولد توبولوجي يحوي كل التوبولوجيات المتولدة بوساطة المجموعات الشبه مفتوحة و قبل المفتوحة.و المفتوحة-b وشبه قبل المفتوحة.