2024, 21(3): 1066-1072
https://dx.doi.org/10.21123/bsj.2023.8359
P-ISSN: 2078-8665 - E-ISSN: 2411-7986

Baghdad Science Journal

On Faintly θ- Semi-Continuous and Faintly δ-Semi-Continuous Functions

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Received 11/01/2023, Revised 08/05/2023, Accepted 10/05/2023, Published Online First 20/08/2023, Published 01/03/2024

Abstract

Faintly continuous (FC) functions, entitled faintly θS-continuous and faintly δS-continuous functions have been introduced and investigated via a θ-open and δ-open sets. Several characterizations and properties of faintly θS-continuous and faintly δs-Continuous functions were obtained. In addition, relationships between faintly θs- Continuous and faintly δS-continuous function and other forms of FC function were investigated. Also, it is shown that every faintly θ S-continuous is weakly θS-continuous. The Convers is shown to be satisfied only if the co-domain of the function is almost regular.

Keywords: Faintly θS-continuous, Faintly δS-continuous, θs-open, δs-open, Faint continuity.

Introduction

Faint continuity is a property weaker form of Continuous functions. Throughout this paper, since the introduction of FC functions by Long and Herrington[1], various weak and strong forms of FC functions were studied. Many authors defined and introduced a generalization form of open sets and weak and strong forms of semi-open sets see in [2-6]. The concept FC functions have the attention of many authors see for example [7]. First, a point ҳ∈Ҳ is called an θ -Cluster point of E⊆ Ҳ if E non-trivially intersects the closure of each open set containing ҳ in Ҳ. All θ -Cluster set points of some set is defined to be the θ closure of that set and it's written as Clθ (E). a subset that contains all its θ -Cluster points (i.e., E = Clθ (E)), is θ-closed, and its complement is θ -open. Equivalently, E is θ-open if it has a closed neighborhood of each of its points. Another equivalent definition is that if for all ҳ ∈E, an open set O exists with the property that ҳ ∈ O ⊂Cl(O)⊂ E. The collection, Tθ, of θ -open subsets in Ҳ forms a topology on Ҳ. The intθ(E) is the largest θ-open subset of E. A δ-Cluster point ҳ∈Ҳ of E⊂Ҳ is a point s.t. for every open set U ∋ ҳ its (int (Cl(O) ∩ E ≠∅). The δ-closure of a set E, Clδ(E), is the set of all δ-Cluster points of that set. A δ-closed is one which equals its δ-closure. A δ-open set is one whose complement is δ-closed. Equivalently, E is δ- open if for all ҳ ∈ E there is a regular-open (r-open) subset of E containing ҳ. The collection of all δ-open subsets of Ҳ is a tropology on Ҳ denoted by Tδ. A subset E in Ҳ is proclaimed semi-open denoted by (SO) if ∃ an open set Q s.t Q⊂ E⊂ Cl (Q)[8].

Preliminaries:

The terms (Ҳ, τ) and (Y, σ) pertain to topological spaces where there are no underlying separation axioms. The closure, interior and the complement of a set E are denoted respectively by Cl (E), int(E) and A^c. A point ҳ ∈ Ҳ is called the θ-Cluster point (respectively δ-Cluster point) of the set E if for every open subset Q of Ҳ s.t. ҳ∈ Q, E∩Cl(Q) ≠∅ (respectively E∩int(Cl(Q)) ≠∅). The set of all θ-

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https://dx.doi.org/10.21123/bsj.2023.8359
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Cluster points (respectively δ-Cluster points) of a set, E, is said to be the θ-closure of E denoted by Clθ (E) (respectively δ-closure denoted by Clδ(E)). Also θ-closed (respectively δ-closed) set is one which equals its respective closure. A θ -open set (respectively δ-open) is one whose complement is θ -closed set (respectively δ-closed)[9]. A set E in a topological space (Ӽ, τ) is θ -semi-open if ∃ a θ -open subset Q of Ӽ s.t Q⊂E⊂Cl(Q). Equivalently, if E ⊂Cl(intθ (E)) [10]. A set is θ -semi closed if its complement is θ -semi open. A set E in a topological space (Ӽ, τ) is δ -semi-open if ∃ δ -open set Q of Ӽ s.t Q ⊂ E ⊂ Cl(Q) [11]. Equivalently, if E ⊂Cl(intδ(E)). A δ -semi closed set is one whose complement is δ -semi open, its denote by δs(Ӽ) for the collection consisting of all δ-semi open sets in a space Ӽ. A mapping f: (Ӽ, τ) → (ϒ, σ) is called faintly Continuous (FC) (respectively faintly semi-Continuous (FSC)) if ∀ ӽ∈Ӽ and every δ-open subset Q of ϒ, f(ӽ) ∈Q, there is an open subset (respectively semi-open), O, of Ӽ, containing ӽ s.t f(O)⊂Q. Equivalently, f is FC (respectively FSC) if the pre-image of each θ-open set is open (semi-open) set. A regular-open set Q, is one s.t. Q=int (Cl(Q)). A regular closed (r-closed) is one whose complement is r-open. Equivalently, F is r-closed set if Cl(int(F)) =F. The point ӽ∈Ӽ is a δ-semi-Cluster point of some set E if E∩O≠ ∅ ∀ δ-semi open subset O of Ӽ, ӽ∈ O. The δ-semi closure of a subset E (which denotes sClδ(E)) is the set consisting of its δ-semi-Cluster points. The family consisting of δ-semi open sets (respectively δ-semi closed) will be denoted by δSO(Ӽ, τ) (respectively δSC(Ӽ, τ)). Then, sClδ(E) = ∩ {F: E ⊂ F, F is δ-semi closed}[i]. The concept of θ -semi-closure of some subset E of a space Ӽ, denoted sClθ(E), is the set of all ӽ ∈ Ӽ s.t Cl(O)∩E ≠ ∅ for all semi open subset O of Ӽ s.t. ӽ∈O, sClθ(E) = ∩{F: E⊂F, F is θ -semi closed[12,13].

Results and Discussion

Characterization of faintly θ-semicontinuity:

Definition 1:[14] f: (Ӽ, τ) → (ϒ, σ) is faintly θ-semi-Continuous (θ- S-continuous) if for all point ӽ∈Ӽ and every θ -open set Q of ϒ, f(ӽ) ∈Q, there is an θ -semi open set O of Ӽ, ӽ ∈O s.t f(O) is contained in Q.

Theorem 1:

Given a mapping f: (Ӽ, τ) → (ϒ, σ), such that ϒ is an almost regular space, then the equivalence of the following statements can be established:

i. The mapping f is faintly θ- S-continuous.
ii. The pre-image f^{-1} (Q) is a θ –semi open subset of Ӽ whenever Q is r-open subset of ϒ.
iii. The pre-image f^{-1}(F) is a θ-semi closed set of Ӽ whenever F r-closed is a set in ϒ.
iv. If E ⊂ Ӽ then, f (sClθ(E)) ⊂Clθ (f(E)).
v. If B ⊂ Ӽ sClθ (f^{-1}(B)) ⊂f^{-1}(Clθ (B)).
vi. The pre-image f^{-1} (F) is a θ -semi closed for all θ -closed set F of ϒ.
vii. For every θ -open subset Q of ϒ, f^{-1} (Q) is a θ -semi open subset of Ӽ.

Proof:

(i ⇒ ii) assume that Q is a r-open in ϒ, in addition let ӽ ∈ f^{-1}(Q), then Q =int(Cl(Q)) and f(ӽ)∈ Q, openness of Q, implies θ –the openness Q in ϒ [because ϒ is almost regular] and by application of part (i) and the definition 1, ∃ a θ -semi open set $O_ӽ$ of Ӽ s.t ӽ∈$O_ӽ$ and f($O_ӽ$) ⊂Q. Therefore, ӽ ∈$O_ӽ$ ⊂f^{-1}(f($O_ӽ$)) ⊂ f^{-1}(Q) and ∃ a θ -open set $W_ӽ$ s.t $W_ӽ$⊂$O_ӽ$⊂Cl($W_ӽ$), since $O_ӽ$ is a θ -semi open set. Now, suppose that W=$O_ӽ$ ∈f^{-1}(Q) WӼ. As $\cup_{ӽ\in f\text{-}1(Q)}$ Cl($W_ӽ$) ⊂Cl(W) then O=$\cup_{ӽ\in f\text{-}1(Q)}$ ($O_ӽ$) =f^{-1}(Q) and the proof is concluded.

(ii ⇒ iii) Suppose that F is r-closed in ϒ,so ϒ\F is r-open in ϒ. BY (ii), f^{-1}(ϒ\F) is θ -semi open subset of Ӽ. Since f^{-1} (ϒ\F) = f^{-1} (ϒ)\f^{-1} (F), it is then implied that f^{-1}(F) is a θ -semi closed subset of Ӽ.

(iii ⇒ iv) Suppose that ӽ ∈ sClθ (E) and suppose that f(ӽ)∉Clθ (f(E)). So, ∃ an open set Q_0 s.t f(ӽ) ∈Q_0 and f(E) ∩ Cl(Q_0) = ∅. When taking the r-open set W_0 = int (Cl (Q_0)). Then Cl(W_0) = Cl (Q_0). Thus, f(E) ⊂ ϒ\Cl(W_0). BY part (iii), it got Ӽ\$f^{-1}$($W_0$) is θ -semi closed and E⊂Ӽ\$f^{-1}$($W_0$). Thus, by the definition of sClθ (E), it got ӽ ∈ Ӽ \$f^{-1}$ (W_0) , a

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P-ISSN: 2078-8665 - E-ISSN: 2411-7986
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contradiction with $f(x) \in Q_0 \subset int(Cl(Q_0)) = W_0$.

(iv ⇒ v) Assume that $E = f^{-1}(B) \subset X$. Then, by part (iv) it has $f(sCl\theta(E) \subset Cl\theta(f(E))$. Since $Cl\theta(f(E)) \subset Cl\theta(B)$, it follows that $sCl\theta(E) \subset f^{-1}(Cl\theta(B))$.

(v ⇒ vi) Assume that a set F is an θ -Closed of Y. S0, $F \subset Cl(F) \subset Cl\theta(F) = F$. Taking B = F in part (v), it got $sCl\theta(f^{-1}(F)) \subset f^{-1}(F)$. As $f^{-1}(F) \subset sCl\theta(f^{-1}(F))$ and $sCl\theta(f^{-1}(F)$ is θ -semi closed, it concludes $f^{-1}(F)$ is θ -semi closed subset of X.

(vi ⇒ vii) Let Q be θ-open subset of Y. Taking $F = Q^c$ in part (vi) it got $f^{-1}(Q^c) = (f^{-1}(Q))^c$ is θ -semi closed subset of X,then $f^{-1}(Q)$ is θ -semi open set of X.

(vii ⇒ i) Suppose that $x \in X$ and Q be an θ -open subset of Y, which contains f(x). BY part (vii), if the inverse $f^{-1}(Q)$ is θ -semi open subset in X,then taking $O = f^{-1}(Q)$, it gives $x \in O$ and $f(O) = f(f^{-1}(Q)) \subset Q$, so the map is faintly θ -S-continuous in X.

SO can make another definition of a faintly θ -S-continuous as can see in the following theorem:

Theorem 2:

f: X→Y is faintly θ -S-continuous iff $f^{-1}(B)$ is θ -semi open subset in X ∀ θ -open set B in Y.

Proof: For the necessity, suppose that B θ -open in Y. (to proof) $f^{-1}(B)$ is a θ -semi open subset of X.

If $x \in f^{-1}(B)$ with $f(x) \in B$, then B is θ-open [by the fact that f is faintly θ -S-continuous]. Then ∃ a θ -semi open subset O of X, $x \in O$ s.t $f(O) \subset B$, Then $x \in O \subset f^{-1}(B)$. Therefore, $f^{-1}(B)$ is θ -semi open subset of X (since $f^{-1}(B)$ is a union of θ -semi open set).

For sufficiency, suppose that $x \in X$ and $Q \subset Y$, Q is θ -open subset of Y, then $f^{-1}(Q)$ is θ -semi open set in X. Suppose that $x \in f^{-1}(Q)$ and $f^{-1}(Q) = O$, Then $f(O) = f(f^{-1}(Q)) = Q$, there exists $O = f^{-1}(Q)$ is θ -semi open in X s.t $f(O) \subset Q$, which implies that f is faintly θ -S-continuous.

In definition 1, If we replace the θ-open set by the closure of θ-open set we can define the following definition

Definition 2:

f: X → Y is Called a weakly θ -S-continuous if for all $x \in X$ and all θ -open set $Q \subset Y$ s.t $f(x) \in Q$, there be $O \in \theta SO(x, X)$ s.t $f(O) \subset Cl(Q)$.

Theorem 3:

Every faintly θ S-continuous mapping is weakly θ S-continuous.

Proof: if $x \in X$ and Q is an θ-open subset in Y with $f(x) \in Q$. By faint θ-S-continuity of f, ∃ θ s-open set O, $x \in O$ which is contained in Q, then $f(O) \subset Q \subset Cl(Q)$, consequently $f(O) \subset Cl(Q)$ which implies weak θ S-continuity f.

The converse, however, can only hold if Y is assumed to be almost regular as shown below.

Definition 3:

A space X is almost regular if whenever F r-closed subset in X with $x \notin F$, ∃ disjointed open subsets O and Q of X, s.t $x \in O$ and $F \subset Q$.

Theorem 4:

If $f:(X, \tau) \to (Y, \sigma)$ is weakly θ S-continuous, with Y being almost regular, then f is faintly θ S-continuous.

Proof: Let $x \in X$. Assume further that Q is θ -open set of Y, $f(x) \in Q$. S0, there exists r-open set W in Y s.t $f(x) \in W \subset Cl(w) \subset Q$. [by Theorem 1 in[1]]. Since Y is almost regular, then each r-open set in Y is also θ -open [by Theorem 3 in[1]]. Now, by weak θ -S-continuity of f,∃ θ s-open set O, $x \in O$ s.t $f(O) \subset Cl(W) \subset Q$, ⇒ $f(O) \subset Q$, consequently, faint θ -S-continuity of f is established.

Definition 4:[17]

Suppose that f: X→Y be a function, the function g: X→X×Y is called a graph function of f if g is defined by $g(x) = (x, f(x))$ for each $x \in X$.

Theorem 5:

Given the graph map of f: X→Y to be is faintly θ S-continuous, then so is f.

Proof: Assume that $x \in X$, Q an θ-open subset of Y, $f(x) \in Q$, ⇒ X× Q is θ-open subset of X×Y [By Theorem 5 in[1]] containing $g(x) = (x, f(x))$. Since the graph map g: X →X×Y is faintly θ S-continuous, there exists $O \in \theta SO(X)$ containing X s.t $g(O) \subset X \times Q$, then $f(O) \subset Q$. Hence, faint θ -S-continuity of f is established.

Theorem 6:

Supposing f is faintly θ S-continuous with Y is almost regular. For all $x \in X$ and all θ -open set Q in Y s.t $f(x) \in Q$, ∃ θ s-open subset O of X s.t $f(O) \subset int(Cl(Q))$.

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Proof: If ӽ ∈Ӽ with f (ӽ) ∈ Q, where Q is a θ-open subset in Ƴ, where Ƴ be almost regular, then there exists r-open subset G in Ƴ s.t f (ӽ) ∈ G ⊂ Cl (G) ⊂ Q ⊂ int (Cl (Q)).

[by Theorem 3 in[1]] so, G is r-open, when Ƴ is almost regular then G is θ-open [by theorem 3 in[1]], faint θ S-continuity of f means that it can find a θ s-open set O in Ӽ s.t. ӽ∈O and f(O) ⊂ G ⊂Cl(G) ⊂int (Cl(Q)). Therefore, f(O) ⊂ int (Cl(Q)).

Characterization of faintly δ-semicontinuitƳ:

Definition5:

f: (Ӽ, τ) → (Ƴ, σ) is A faintly δ- S-continuous if ∀ ӽ∈Ӽ and all δ-open subset Q of Ƴ that contains f (ӽ), ∃ a δ-semi open subset O of Ӽ that contains ӽ s.t f(O) ⊆ Q.

Theorem 7:

Given f:(Ӽ, τ) → (Ƴ, σ), then we can establish the equivalence of the following:

i. The map f is faintly δ-S-continuous.
ii. The pre-image f^{-1} (Q) is a δ-semi open subset of Ӽ for all r-open set Q of Ƴ.
iii. The pre-image f^{-1} (F) is a δ-semi closed subset of Ӽ for all r-closed subset F of Ƴ.
iv. f(sClδ(E)) ⊂ Clδ(f(E)) ∀ E ⊂ Ӽ.
v. sClδ (f^{-1} (B)) ⊂ f^{-1} (Clδ(B)) ∀ B ⊂ Ƴ.
vi. The pre-image f^{-1} (F) is a δ-semi closed subset in Ӽ ∀ δ-closed subset F of Ƴ.
vii. The pre-image f^{-1} (Q) is a δ-semi open subset in Ӽ ∀ δ-open subset Q of Ƴ.

Proof:

(i⇒ ii) for an r-open subset Q of Ƴ suppose that ӽ ∈f^{-1} (Q), then Q = int (Cl(Q)) and f(ӽ) ∈ Q, then Q is a δ-open set of Ƴ and by application of part (i) and the definition 5, ∃ a δ-semi open subset set $O_{ӽ}$ of Ӽ s.t ӽ ∈ $O_{ӽ}$ and f($O_{ӽ}$) ⊂ Q. Therefore, ӽ ∈ $O_{ӽ}$ ⊂ f^{-1} (f($O_{ӽ}$)) ⊂ f^{-1} (Q) and ∃ a δ-open set $W_{ӽ}$ s.t $W_{ӽ}$ ⊂ $O_{ӽ}$ ⊂ Cl($W_{ӽ}$), since $O_{ӽ}$ is δ-semi open. Now, suppose that W=$\cup_{ӽ\in f\text{-}1(Q)}$ $W_{ӽ}$. As $\cup_{ӽ\in f\text{-}1(v)}$ Cl($W_{ӽ}$) ⊂Cl(W), then O=$\cup_{ӽ\in f\text{-}1(Q)}$ ($O_{ӽ}$) =f^{-1}(Q) and δ-semi openness is established.

(ii ⇒iii) Suppose that F is an r-closed subset F of Ƴ. ⇒ Ƴ\F is r-open subset of Ƴ. By part (ii), f^{-1} (Ƴ\F) is a δ-semi open subset of Ӽ. Since f^{-1}(Ƴ\ F) = f^{-1}(Ƴ)\ f^{-1} (F), hence f^{-1} (F) is a δ-semi closed subset of Ӽ.

(iii ⇒ iv) Suppose that ӽ ∈ sClδ(E) and suppose that f(ӽ) ∉Clδ(f(E)). Then, ∃ an open set Q_0 s.t f(ӽ) ∈ Q_0 and f(E) ∩ int (Cl (Q 0)) = ∅. Then, take the r-open set W_0 = int (Cl(Q_0)). Hence, f(E) ⊂ Ƴ\W0. BƳ part (iii), it got f^{-1} (Ƴ\$W_0$) is δ-semi closed and E⊂ f^{-1} (Ƴ\$W_0$). Thus, by the definition of sClδ(E), it got ӽ ∈f^{-1}(Ƴ\$W_0$) a contradiction with f(ӽ)∈Q_0⊂int(Cl(Q $_0$)) = W_0.

(iv ⇒ v) Suppose that E = f^{-1} (B) ⊂ Ӽ. Then, by part (iv) take f(sClδ(E)) ⊂ Clδ(f(E)). Since Clδ(f(E)) ⊂ Clδ(B), it follows that sClδ(E) ⊂ f^{-1} (Clδ(B)).

(v ⇒ vi) Suppose F is a δ-closed subset in Ƴ. This means F ⊂Cl(F)⊂Clδ(F) = F. Taking B = F in part (v), and it got sClδ(f^{-1}(F))⊂f^{-1}(F). As f^{-1}(F)⊂sClδ(f^{-1}(F)) and sClδ(f^{-1}(F) is δ-semi closed which concludes f^{-1}(F) is a δ -semiclosed subset of Ӽ.

(vi ⇒ vii) Let Q be an δ-open subset of Ƴ. Taking F = Ƴ\Q c in part (vi) it got f^{-1}(Ƴ\Q) = f^{-1} $(Ƴ\backslash Q)^c$ is δ-semi closed subset of Ӽ. Thus, f^{-1}(Q) is δ-semi open subset of Ӽ.

(vii ⇒i) Assume that ӽ ∈ Ӽ and suppose that Q is δ-open subset of Ƴ, f (ӽ) ∈Q. By part (vii), f^{-1} (Q) is δ-semi open subset of Ӽ. Then, taking O = f^{-1} (Q), it got ӽ ∈ O and f(O) ⊂ f (f^{-1} (Q)) ⊂ Q. Therefore, faint δ -S-continuity of f is established

Theorem 8: For any function between two spaces f: Ӽ → Ƴ. If the graph function g is faintly δS-continuous, then so is f.

Proof Let ӽ∈Ӽ and assume that Q is δ -open set that contains f(ӽ). Then Ӽ×Q is δ -open subset of Ӽ×Ƴ [Theorem5in[1]], it further contains g(ӽ)= (ӽ, f (ӽ)). Therefore, ∃ O ∈δs(Ӽ) containing Ӽ s.t g(O)⊂ Ӽ×Q, which implies f(O)⊂ Q, and faint δ -S-continuity of f is established.

Theorem 9: If f: Ӽ→Ƴ is faintly δ S-continuous with Ƴ almost regular. Then for all ӽ∈Ӽ and δ-open subset Q of Ƴ, s.t f(ӽ)∈ Q, ∃ a δ-open subset O in Ӽ, ӽ ∈ O s.t f(O)⊂ int (Cl (Q)).

Proof. If ӽ∈Ӽ and Q is a δ-open subset in Ƴ with f(ӽ)∈ Q, but Ƴ is almost regular. so ∃ r-open subset G in Ƴ s.t f(ӽ) ∈G ⊂Cl(G)⊂int (Cl(Q)[13] [Theorem 2.2]. Since f is faintly δ S-

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https://dx.doi.org/10.21123/bsj.2023.8359
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
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continuous, since G is r-open, then G is δ-open. It follows ∃ a δs-open subset O of Ҳ, with ҳ∈ O s.t f(O)⊂G⊂ Cl(G) ⊂ int (Cl (Q)).

Remark 1:
Clearly, any union δs-open sets in (Ҳ, t) is δ-open. However, as can be seen in the example below, the result for intersection is generally false.

Example 1:
Suppose R^2 with the usual topology. Suppose that E be the set defined by E= {(Ҳ, Y) ∊ R^2: $Ҳ^2+Y^2$ <1}∪{(cos(α), sin(α)): 0 < α < π/2 }, B= {(Ҳ, Y) ∊R^2: $Ҳ^2+Y^2>1$} ∪ {(cos(α), sin(α)): 0 < α < π/2 }. A is δ-semi open because D is in E and E is in Cl(D), being D= {(Ҳ, Y) ∊R^2: Ҳ^2+Y^2 < 1}, D is a δ-open set. Moreover, D is open and regular. B is δ-semi open because M is in B and B is in Cl (M), being M= {(Ҳ, Y) ∊R^2: Ҳ^2+Y^2>1}. M is a δ−open set. Moreover, M is open and regular. However, E∩B={(cos(α), sin(α)): 0 < α < π/2 } is not δ- semi open because if ∃ δ-open set O s.t O is in E∩B and E∩B is in Cl(O) ,then O is not empty, it follows there exists ҳ in O. For that Ҳ, ∃ an open and regular set W s.t ҳ is in W and W is in O. Therefore, E∩B contains a disk that is a contradiction.

Theorem 10: If f is a mapping of Ҳ into Ү, and Ҳ =Ҳ 1 ∪ Ҳ2, where Ҳ1 and Ҳ2 are δs-open, and $f|_{Ҳ1}$ and $f|_{Ҳ2}$ are faintly δS-continuous, then f is faintly δS-continuous.

Proof. Let ҳ∈Ҳ and suppose that B is a δ-open subset of Ү that contains f(ҳ). If ҳ∈Ҳ1, then there exists δs-open O1 subset of Ҳ1, s.t f(O1) = f|Ҳ1(O1) ⊂ B. Also, if ҳ∈Ҳ2, then there exists δs-open O2 subset of Ҳ2 s.t f (O2) = f|Ҳ2(O2) ⊂ B. If ҳ∈Ҳ1∪ Ҳ2, SO can take O=O1∪ O2: thus, O is δs-open (By Remark 1) and f(O) = f|Ҳ1(O1) ∪ f|Ҳ2(O2) ⊂ B. Therefore, f is faintly δS-continuous.

Lemma 1:[11]
Suppose that G, H ⊂ (Ҳ, τ). And suppose further that G ∈ δSO(Ҳ) and G ∈ δO(Ҳ), then the intersection G ∩ H ∈ δSO(H).

Lemma 2:[11]
Suppose that G,H ⊂ (Ҳ, τ). And suppose further that G ∈ δSO(H) and H ∈ δO(Ҳ), then G∈ δSO(Ҳ).

Theorem 11:
Suppose that f: (Ҳ, τ) → (Ү, σ) is a mapping with {Qi: i ∈ I} an δs-open cover of Ҳ. If the restriction f|Qi: (Qi, τQi) → (Ү, σ) is faintly δ-S-continuous ∀ i ∈ I, so f is faintly δ -S-continuous.

Proof: If O is an δ-open set in (Ү, σ) (By Lemma 1). Therefore, f^{-1}(O) =Ҳ∩ f^{-1} (O) = ∪ {Qi ∩ f^{-1} (O): i ∈ I } = ∪ {$(f |Qi)^{-1}$ (O): i ∈ I}.

But f |Qi is faintly δ-S-continuous ∀ i∈I, $(f |Qi)^{-1}$(O) ∈ δSO(Qi) ∀ i ∈I. (By Lemma 2), for all i ∈ I, $(f |Qi)^{-1}$ (O) is δ-semi open in Ҳ and as f^{-1} (O) is δ-semi open in Ҳ. Therefore, f is faintly δ-S-continuous.

Definition 6:[15]
A mapping f: Ҳ → Ү is An almost δ-semi open if f(Q) ⊂ int (Cl(f(Q))) ∀ δ-semi open subset Q of Ҳ.

Theorem 12:
Given a mapping f: Ҳ → Ү that is faintly δ-S-continuous and almost δ-semi open, then for all ҳ∈Ҳ and all δ-open set O ⊂Ү, s.t f (ҳ)∈ Q, ∃ a δ-semi open set Q∊δSO (Ҳ) s.t f(Q) ⊂ int (Cl (O)).

Proof: Let ҳ∈Ҳ and suppose that O is an δ-open subset of Ү s.t f(ҳ) ∊O. By faint δ -S-continuity of f, then there is Q∊ δSo(Ҳ) s.t f(Q) ⊂O.
but f is almost δ-semi open, which implies that f(Q)⊂int(Cl(f(Q)) ⊂ int(Cl(O)), then f(Q) ⊂ int (Cl(O)).

Note: Many authors defined and introduced a generalization form of semiopen sets and semi closed sets have many applications see for example [16].

Conclusion

In this work, several results on faintly θ S-continuous and faintly δ-S-continuous were obtained. Several properties of these kinds of faint Continuity were considered. Also, the relations between the graph of faintly θ S-continuous and faintly δ-S-continuous functions were obtained. Furthermore, the relation between these types of functions was considered.

Authors' Declaration

- Conflicts of Interest: None.

- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

Authors' Contribution Statement

Sh. H. A. gives some results on faintly θ S-continuous and faintly δs- continuous function. J. H. H. introduces the mapping named "faintly δ-S-continuous" and give some results on it. A. M. Z. give Several properties of faintly θ S-continuous and faintly δ-S-continuous.

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2024, 21(3): 1066-1072
https://dx.doi.org/10.21123/bsj.2023.8359
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

حول الدوال المستمره الضعيفه من النمط θ و δ

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الخلاصة

الدوال المستمرة بشكل ضعيف (FC) والمعنونة باسم الدوال شبه المستمره بشكل ضعيف من النوع θS و الدوال شبه المستمره بشكل ضعيف من النوع δS تم دراستها والتحقق منها بوساطة المجاميع المفتوحة من النوع θو δ. العديد من الخصائص والمميزات للدوال شبه المستمرة بشكل ضعيف من النوع θS والدوال شبه المستمرة بشكل ضعيف من النوع δS تم الحصول عليها. أضافة الى ذلك العلاقات بين الدوال شبه المستمره بشكل ضعيف من النوع θS و الدوال شبه المستمره بشكل ضعيف من النوع δS و انواع اخرى من الدوال المستمرة بشكل ضعيف (FC) تم دراستها والتحقيق بها.أيضا أثبت انه كل دالة شبه مستمرة بشكل ضعيف من النوع θS هي دالة شبه مستمرة ضعيفة من النوع θS. العكس للنتيجة المارة الذكر يتحقق عندما يكون المجال المقابل للدالة من النوع المنتظم تقريبا.

الكلمات المفتاحية: داله من النوع θ ضعيفة شبه مستمرة، داله من النوع δ ضعيفة شبه مستمرة، مجموعة من النوع θ شبه مفتوحة، مجموعة من النوع δ شبه مفتوحة, استمراريه ضعيفه.