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Traveling Wave Solutions of Fractional Differential Equations Arising in Warm Plasma

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Abstract:

This paper aims to study the fractional differential systems arising in warm plasma, which exhibits traveling wave-type solutions. Time-fractional Korteweg-De Vries (KdV) and time-fractional Kawahara equations are used to analyze cold collision-free plasma, which exhibits magnet-acoustic waves and shock wave formation respectively. The decomposition method is used to solve the proposed equations. Also, the convergence and uniqueness of the obtained solution are discussed. To illuminate the effectiveness of the presented method, the solutions of these equations are obtained and compared with the exact solution. Furthermore, solutions are obtained for different values of time-fractional order and represented graphically.

Keywords: Caputo fractional derivative, Fractional Adomian decomposition method, Fractional Kawahara equation, Fractional Korteweg–De Vries equation, Riemann–Liouville fractional integral.

Introduction:

The study of nonlinear fractional systems becomes crucial in all fields of mathematics, engineering, physics, etc. Due to their nonlinear behavior, numerous applications of such fractional systems can be found in fluid dynamics, plasma physics, nonlinear biological systems, viscoelasticity, solid mechanics, quantum field theory, etc[1-4]. Finding the exact solutions to such differential equations is not a straightforward task so; researchers prefer the best-estimated solutions[5]. Tools like series solution methods and numerical methods are widely used for such determination[6-9]. The existence of traveling wave behavior occurs in many physical phenomena such as plasma physics, fluid mechanics, waves in shallow water, etc. The Adomian decomposition method which is invented by George Adomian is extensively used for solving nonlinear PDEs[10-13]. Moreover, solutions obtained by this method are convergent.

The KdV equation narrates the appearance of collision-free shock wave[14-17]. The behavior of weak non-linear dispersive waves arising in gravity waves, plasma waves, and lattice waves can be described by the KdV equation. The time-fractional KdV model for the potential $\phi(\xi,\tau)$ can express as follows[18]

Time-fractional Kawahara equation is given by

$$\frac{\partial^\gamma \phi}{\partial \tau^\gamma} + Af(u)\frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} = 0,\ 0 < \gamma \le 1 \qquad \mathbf{1}$$

$$L^\gamma z + azz_x + bz_{xxx} - cz_{xxxxx} = 0,\ 0 < \gamma \le 1 \qquad \mathbf{2}$$

which is the fifth-order KdV equation. Due to the fifth-order term u_{xxxxx}, it is used for analyzing the cold collision-free plasma having magneto-acoustic waves.

The Adomian decomposition method is explored to solve these equations. Many facts can be explored by incorporating time-fractional derivatives. The arrangement of the paper is as follows: Definitions related to fractional derivatives and integrals are given in section 2. A description of the fractional Adomian decomposition method is narrated in Section 3. The convergence and uniqueness of the solution are proved in section 4. To check the performance of the proposed methods, numerical illustrations are given in the last section.

Basic Preliminaries:

Some basic definitions related to fractional ordered calculus are given in this section.

Definition. 1:

The Riemann-Liouville fractional integral of a function f is defined as[19]

$$J^{\gamma} f(t) = \frac{1}{\Gamma(\gamma)} \int_0^t (t-\tau)^{\gamma-1} f(\tau) d\tau, \quad \text{for } 0 < \gamma \leq 1$$

Definition. 2:

The Caputo fractional derivative of a function f is defined as[19]

$$L^{\beta} f(x) = \frac{1}{\Gamma(r-\beta)} \int_0^x (x-\tau)^{r-\beta-1} f^{(r)}(\tau) d\tau, \quad \text{for } r-1 < \beta \leq r$$

where $r = \lceil \beta \rceil$.

Remark. 1:

(i) $J^{\gamma}(t^s) = \frac{\Gamma(s+1)}{\Gamma(\gamma+s+1)} t^{\gamma+s}$ for $s > -1, \gamma \geq 0$

(ii) $J^{\gamma} L^{\gamma}(f(x)) = f(x) - \sum_{k=0}^{r-1} f^{(k)}(0^+) \frac{x^k}{k!}, x > 0, \gamma \geq 0$

(iii) $J^{\gamma} J^{\beta}(f(t)) = J^{\gamma+\beta}(f(t))$, for $\gamma, \beta \geq 0$

Fractional Adomian Decomposition Method:

To demonstrate the proposed method, consider the non-linear PDE:

$$L^{\gamma} z + Mz + Nz = 0, \; r-1 < \gamma \leq r \qquad 3$$

where $z = z(x,t), r = \lceil \gamma \rceil \in \mathbb{N}$, the differential operator L^{γ} is the γth order fractional derivative, N is a nonlinear operator and M is a linear differential operator. Applying J^{γ} to Eq.3 and from the above remarks, Eq.3 becomes,

$$z(x,t) = \sum_{k=0}^{r-1} z^{(k)}(0^+) \frac{x^k}{k!} - J^{\gamma}(Mz + Nz)$$

The method decomposes $z(x,t)$ into a sum $z = \sum_{n=0}^{\infty} z_n$ and the nonlinear term Nz can be expressed as $Nz = \sum_{n=0}^{\infty} A_n(z_0, z_1, z_2, \dots, z_n)$, where Adomian polynomials A_n 's are determined as follows[20]:

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N\left(\sum_{i=0}^{n} \lambda^i z_i \right) \right]_{\lambda=0}, n = 0,1,2,\dots \qquad 4$$

Substituting decomposition series for z and Nz in Eq.4,

$$\sum_{n=0}^{\infty} z_n = \sum_{k=0}^{r-1} z^{(k)}(0^+) \frac{x^k}{k!} - J^{\gamma}\left[M\left(\sum_{n=0}^{\infty} z_n \right) + \sum_{n=0}^{\infty} A_n \right]$$

The values $z_n(x,t)$ are determined by the recurrence relation:

$$\begin{aligned} z_0 &= \sum_{k=0}^{r-1} z^{(k)}(0^+) \frac{x^k}{k!} \\ z_{n+1} &= -J^{\gamma}[Mz_n + A_n], \quad \text{for } n = 0,1,2,\dots \end{aligned}$$

Therefore, the required solution $z(x,t)$ can be obtained by calculating the values of $z_n(x,t)$, for $n \geq 1$. Eventually, the solution $z(x,t)$ is as follows

$$z(x,t) = \lim_{N\to\infty} \phi_N(x,t), \text{ where } \phi_N(x,t) = \sum_{n=0}^{N-1} z_n(x,t)$$

Uniqueness and Convergence:

Theorem 1:

Time-fractional partial differential equation

$$L^{\gamma} z + Mz + Nz = 0, \; r-1 < \gamma \leq r \qquad 5$$

with $z(x,0) = z_0$ has a unique solution for $0 < \frac{Ct^{\gamma}}{\Gamma(\gamma+1)} < 1$, for some constant C.

Proof: Let an interval $I = [0,T]$ and Banach space of all continuous functions $X = (C(I), \|.\|)$. Define norm $\| z(t) \| = \max_{t\in I} |z(t)|$. Consider a function $H : X \to X$, such that $H(z(t)) = z_0 - J^{\gamma}(Mz) - J^{\gamma}(Nz)$. If the nonlinear term Nz is Lipschitzian, then

$$|N(z) - N(p)| \leq C_1 |z - p|$$

where $p \in X$ and C_1 is Lipschitz constant. Let $z, z' \in X$. Consider

$$\begin{aligned} \|H(z) - H(z')\| &= \max_{t\in I} |-J^{\gamma}N(z(t)) - J^{\gamma}M(z(t)) + J^{\gamma}N(z'(t)) + J^{\gamma}M(z'(t))| \\ &= \max_{t\in I} |-J^{\gamma}(Nz - Nz') - J^{\gamma}(Mz - Mz')| \\ &= \max_{t\in I} |J^{\gamma}(Nz - Nz') + J^{\gamma}(Mz - Mz')| \\ &\leq \max_{t\in I} |J^{\gamma}(Nz - Nz')| + |J^{\gamma}(Mz - Mz')| \end{aligned}$$

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Also, if $M(z(t))$ is Lipschitzian, then $|M(z)-M(p)| \leq C_2|z-p|$ where C_2 is the Lipschitz constant. Therefore,

$$\begin{aligned}\|H(z)-H(z')\| &\leq \max_{t\in I}(C_1 J^\gamma|z-z'| \\ &+ C_2 J^\gamma|z-z'|) \\ &\leq (C_1+C_2)\|z-z'\|\frac{t^\gamma}{\Gamma(\gamma+1)} \leq \zeta\|z-z'\|\end{aligned}$$

where $\zeta = \frac{(C_1+C_2)t^\gamma}{\zeta(\gamma+1)}$. Hence, whenever $0<\zeta<1, H$ would be a contraction mapping. Therefore, by the Banach fixed point theorem, the time-fractional partial differential Eq.5 has a unique solution.

To prove the convergence, consider the following theorem.

Theorem. 2:

Let $\phi_n = \sum_{i=0}^{n} z_i(x,t)$ be the n^{th} the partial sum then the sequence $\{\phi_n\}$ is a Cauchy sequence in Banach space X.

Proof: For $p \in \mathbb{N}$, consider

$$\begin{aligned}\|\phi_{n+p}-\phi_n\| &= \max_{1\in I}|\phi_{n+p}-\phi_n| \\ &= \max_{t\in l}\left|\sum_{i=n+1}^{n+p} z_i(x,t)\right| \\ &= \max_{t\in l}\left|-J^\gamma\sum_{i=n+1}^{n+p} Mz_{i-1}(x,t) - J^\gamma\sum_{i=n+1}^{n+\rho} Nz_{i-1}(x,t)\right| \\ &= \max_{t\in I}\left|J^\gamma M\phi_{n+p-1} - M\phi_{n-1} + J^\gamma N\phi_{n+p-1} - N\phi_{n-1}\right| \\ &\leq \max_{t\in I} J^\gamma\left|M\phi_{n+p-1}-M\phi_{n-1}\right| + \max_{t\in I} J^\gamma\left|N\phi_{n+p-1}-N\phi_{n-1}\right| \\ &\leq C_2\max_{t\in I} J^\gamma\left(|\,(\phi_{n+p-1}-\phi_{n-1}\,|\right) + C_1\max_{|\in I} J^\gamma\left(|\,(\phi_{n+p-1}-\phi_{n-1}\,|\right) \\ &\leq (C_1+C_2)\frac{t^\gamma}{\Gamma(\gamma+1)}\|\phi_{n+p-1}-\phi_{n-1}\| \leq \zeta\|\phi_{n+p-1}-\phi_{n-1}\|\end{aligned}$$

where $\zeta = (C_1+C_2)\frac{t^\gamma}{\Gamma(\gamma+1)}$. The following inequality can be obtained similarly,

$$\begin{aligned}\|\phi_{n+p}-\phi_n\| &\leq \zeta^2\|\phi_{n+p-2}-\phi_{n-2}\| \leq \cdots \leq \zeta^n\|\phi_p-\phi_0\| \leq \zeta^n\|\phi_1-\phi_0\|, \text{ for } p=1 \\ &\leq \zeta^n\|u_1\|\end{aligned}$$

Assume $n>m$, for $n,m \in N$. Consider

$$\begin{aligned}\|\phi_n-\phi_m\| &\leq \|\phi_{m+1}-\phi_m\| + \|\phi_{m+2}-\phi_{m+1}\| + \cdots + \|\phi_n-\phi_{n-1}\| \\ &\leq (\zeta^m+\zeta^{m+1}+\cdots+\zeta^{n-1})\|u_1\| \leq \zeta^m\left[\frac{1-\zeta^{n-m}}{1-\zeta}\right]\|u_1\|\end{aligned}$$

Since $0<\zeta<1,\ 1-\zeta^{n-m}<1$, therefore $\|\phi_n-\phi_m\| \leq \frac{\zeta^m}{1-\zeta}\|u_1\|$. As $z(t)$ is bounded, $\|z_1\|<\infty$.

This implies, $\lim_{n\to\infty}\|\phi_n-\phi_m\| \to 0$. Therefore, ϕ_n is a Cauchy sequence in X. Hence, the solution of the given equation is convergent.

Numerical Examples:

Example 1: Consider the time fractional KdV equation

$$L_t^\gamma z + 6zz_x + z_{xxx} = 0, \text{ for } 0<\gamma\leq 1$$

6

with

$$z(x,0) = \frac{1}{2}\operatorname{sech}^2\left(\frac{x}{2}\right)$$

7

The exact solution to Eq.6 for $\gamma=1$ is $z(x,t) = \frac{1}{2}\operatorname{sech}^2\left(\frac{x-t}{2}\right)$. To solve Eq.6 by the Adomian decomposition method, apply the integral operator J^γ on both sides. Using initial condition Eq.7,

$$z = z(x,0) + J^\gamma(-6zz_x - z_{xxx}) = 0$$

8

Now, inserting $z=\sum_{n=0}^{\infty} z_n$ and the polynomial representation for the non-linear term $zz_x = \sum_{n=0}^{\infty} A_n$ into Eq.8, the recurrence relation can obtain to estimate the values of u_n as follows:

$$z_0 = z(x,0)$$

9

$$z_{n+1} = J^{\gamma}\left(-6A_n - \frac{\partial^3}{\partial x^3} z_n(x,0)\right), n \geq 0 \qquad 10$$

Thus, by solving Eq.9 and Eq.10,

$$z_0 = \frac{1}{2}\operatorname{sech}^2\left(\frac{x}{2}\right)$$

$$z_1 = \frac{\sinh\left(\frac{x}{2}\right)}{2\cosh^3\left(\frac{x}{2}\right)} \frac{t^{\gamma}}{\Gamma(\gamma+1)}$$

$$z_2 = \frac{2\sinh^6\left(\frac{x}{2}\right)+3\sinh^4\left(\frac{x}{2}\right)-1}{4\cosh^8\left(\frac{x}{2}\right)} \frac{t^{2\gamma}}{\Gamma(2\gamma+1)}$$

$$z_3 = \frac{\left[2\cosh^4\left(\frac{x}{2}\right)\Gamma(\gamma+1)^2 - 6\left(3\Gamma(\gamma+1)^2-\Gamma(2\gamma+1)\right)\cosh^2\left(\frac{x}{2}\right)+9\left(2\Gamma(\gamma+1)^2-\Gamma(2\gamma+1)\right)\right]\sinh\left(\frac{x}{2}\right)}{4\cosh^7\left(\frac{x}{2}\right)} \frac{t^{3\gamma}}{\Gamma(3\gamma+1)\Gamma(\gamma+1)^2}$$

The required solution $u(x,t)$ is given by

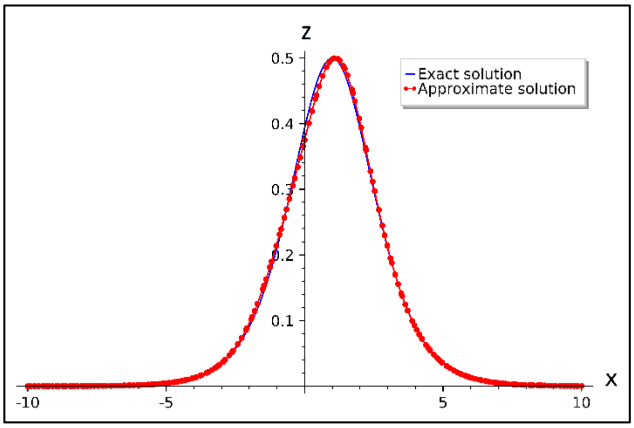
$$z(x,t) = \frac{1}{2}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{\sinh\left(\frac{x}{2}\right)}{2\cosh^3\left(\frac{x}{2}\right)} \frac{t^{\gamma}}{\Gamma(\gamma+1)} + \frac{2\sinh^6\left(\frac{x}{2}\right)+3\sinh^4\left(\frac{x}{2}\right)-1}{4\cosh^8\left(\frac{x}{2}\right)} \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{\left[2\cosh^4\left(\frac{x}{2}\right)\Gamma(\gamma+1)^2 - 6\left(3\Gamma(\gamma+1)^2-\Gamma(2\gamma+1)\right)\cosh^2\left(\frac{x}{2}\right)+9\left(2\Gamma(\gamma+1)^2-\Gamma(2\gamma+1)\right)\right]\sinh\left(\frac{x}{2}\right)}{4\cosh^7\left(\frac{x}{2}\right)} \frac{t^{3\gamma}}{\Gamma(3\gamma+1)\Gamma(\gamma+1)^2} + \cdots$$

The numerical solution obtained using the fractional Adomian decomposition method is compared with the exact solution in Table 1, which shows the efficiency and effectiveness of the method.

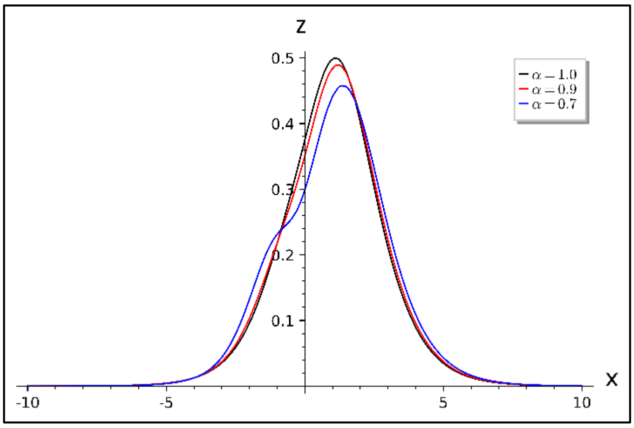
Table 1. Comparison of approximate solution with an exact solution at $t = 1$.

x	Approximate solution	Exact solution	Absolute error
0	0.3750000	0.3932238	0.0182238
1	0.4984448	0.5000000	0.0015551
2	0.4018358	0.3932238	0.0086119
3	0.2113086	0.2099871	0.0013215
4	0.0896437	0.0903533	0.0007095
5	0.0348090	0.0353254	0.0005163
6	0.0130655	0.0132961	0.0002305
7	0.0048423	0.0049330	0.0000906
8	0.0017862	0.0018204	0.0000341
9	0.0006577	0.0006704	0.0000126
10	0.0002420	0.0002467	0.0000046

A comparison of the exact and approximate solutions is given in Fig. 1(a), and observe that the approximate solution is enormously agreed with the exact solution. In Fig. 1(b), the behavior of solutions for $\gamma = 1.0, 0.9, 0.7$ and observed that the obtained solutions are stable and sufficiently approximate to the exact solution.



(a)



(b)

Figure 1. (a) Comparison of the exact and approximate solutions at $t = 1$. (b) Comparison of the approximate solutions at $t = 1$.

In Fig. 2, the obtained solution $z(x,t)$ is presented for the parameters $\gamma = 1, 0 \le t \le 1, -10 \le x \le 10$, and observe the soliton solution exists in plasma waves which has infinite support or infinite tails.

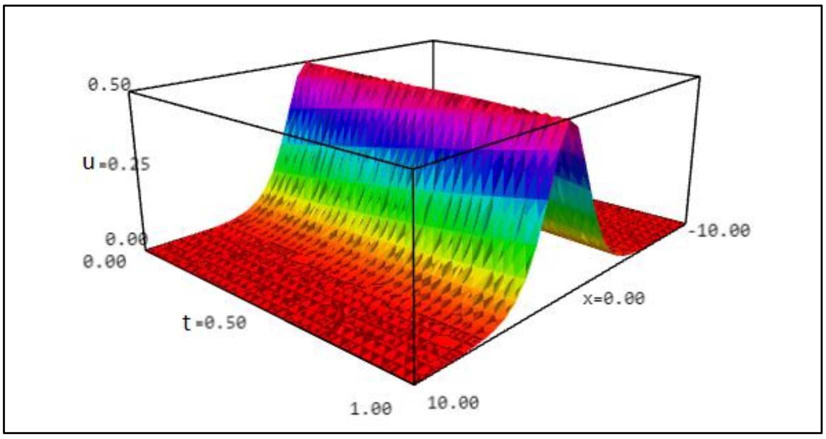


Figure 2. Graph of soliton solution of KdV equation in plasma waves for $\gamma = 1$.

Example. 2: Consider the time-fractional Kawahara equation

$$L_t^{\gamma} z + 6zz_x + z_{xxx} - z_{xxxxx} = 0, \text{ for } 0 < \gamma \le 1 \quad \mathbf{11}$$

with

$$z(x,0) = \frac{35}{338}\operatorname{sech}^4\left(\frac{x}{2\sqrt{13}}\right) \quad \mathbf{12}$$

The exact solution to Eq.11 for $\gamma = 1$ is

$$z(x,t) = \frac{35}{338}\operatorname{sech}^4\left[\frac{1}{2\sqrt{13}}\left(x - \frac{36}{169}t\right)\right]$$

Apply the integral operator J^{γ} to Eq.11 inserting $z = \sum_{n=0}^{\infty} z_n$ and $z_x = \sum_{n=0}^{\infty} A_n$, the following recurrence relation is obtained to estimate values of z_n.

$$z_0 = u(x,0)$$

$$z_{n+1} = J^{\gamma}\left(-6A_n - \frac{\partial^3}{\partial x^3} z_n(x,0) + \frac{\partial^5}{\partial x^5} z_n(x,0)\right), n \ge 0 \quad \mathbf{13}$$

Thus, by solving Eqs.13, the solution $u(x,t)$ is obtained in a series form as follows

$$z(x,t) = \frac{35}{338}\operatorname{sech}^4\left(\frac{x}{2\sqrt{13}}\right) + \frac{1260\sqrt{13}}{371293}\operatorname{sech}^4\left(\frac{x}{2\sqrt{13}}\right)\tanh\left(\frac{x}{2\sqrt{13}}\right)\frac{t^{\gamma}}{\Gamma(\gamma+1)} + \frac{22680}{62748517}\left[4\operatorname{sech}^4\left(\frac{x}{2\sqrt{13}}\right) - 5\operatorname{sech}^6\left(\frac{x}{2\sqrt{13}}\right)\right]\frac{t^{2\gamma}}{\Gamma(2\gamma+1)}$$

$$+ \frac{136080}{137858491849}\Big[48\sqrt{13}\operatorname{sech}^5\left(\frac{x}{2\sqrt{13}}\right)\Gamma(\gamma+1)^2 - 90\sqrt{13}\operatorname{sech}^7\left(\frac{x}{2\sqrt{13}}\right)\Gamma(\gamma+1)^2 + -140\left(2\sqrt{13}\Gamma(\gamma+1)^2 - \sqrt{13}\Gamma(2\gamma+1)\right)\operatorname{sech}^9\left(\frac{x}{2\sqrt{13}}\right) + +175\left(2\sqrt{13}\Gamma(\gamma+1)^2 - \sqrt{13}\Gamma(2\gamma+1)\right)\tanh^2\left(\frac{x}{2\sqrt{13}}\right)\operatorname{sech}^9\left(\frac{x}{2\sqrt{13}}\right)\Big]\frac{t^{3\gamma}}{\Gamma(3\gamma+1)\Gamma(\gamma+1)^2}$$

The estimation obtained by the fractional Adomian decomposition method and the exact solution are compared in Table 2, which shows the efficiency of the method. The Kawahara equation is the key model for to study of magnet-acoustic waves.

Table 2. Comparison of approximate solution with the exact solution at $t = 1$.

x	Approximate solution	Exact solution	Absolute error
0	0.1033697	0.1033695	1.838423e-7
1	0.1011175	0.1011174	1.430337e-7
2	0.0916956	0.0916955	4.319975e-8
3	0.0773600	0.0773600	5.084331e-8
4	0.0610750	0.0610751	9.508468e-8
5	0.0454548	0.0454549	8.915362e-8
6	0.0321486	0.0321486	5.803359e-8
7	0.0217826	0.0217826	2.567129e-8
8	0.0142464	0.0142464	3.490274e-9
9	0.0090547	0.0090547	7.503175e-9
10	0.0056251	0.0056250	1.074931e-8

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In Fig. 3, a comparison of the exact and approximate solutions is given, and observed that the approximate solution is enormously agreed with the exact solution.

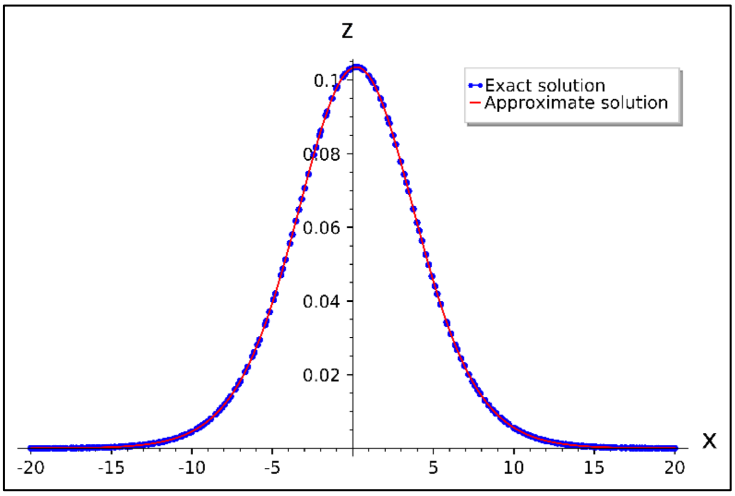

Figure 3. Comparison of the exact and approximate solutions at $t = 1$.

In Fig. 4, the obtained solution $z(x, t)$ is presented for the parameters $\gamma = 0.9, 0 \leq t \leq 1, -20 \leq x \leq 20$, and observed the soliton-type solution in plasma waves which has infinite support or infinite tails.

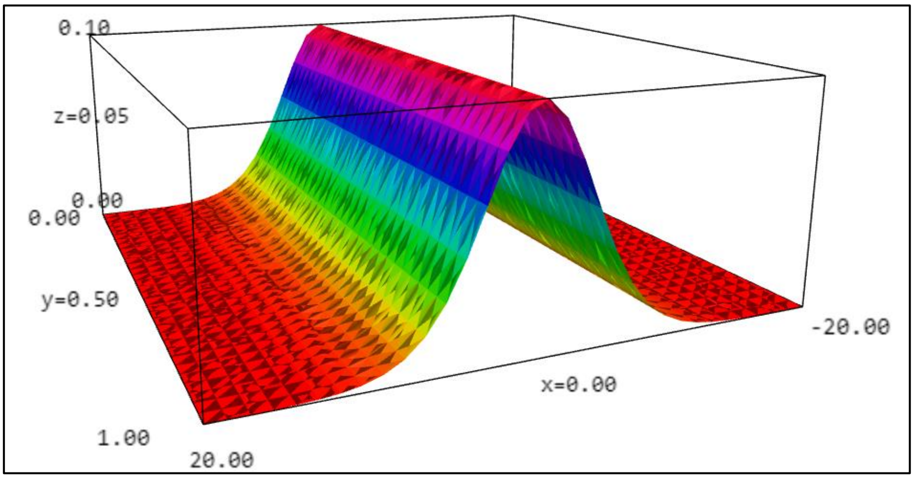

Figure 4. Graph of soliton solution of time-fractional Kawahara equation in plasma waves for $\gamma = 0.9$

Conclusion:

The fractional Adomian decomposition method is executed to solve the time-fractional Korteweg-De Vries equation and the time-fractional Kawahara equation. Furthermore, the solutions obtained by the proposed method converge to exact solutions and uniquely exist in Banach space. Also, an absolute error is obtained, which shows that the approximate solutions are well close to the exact solution with high precision. The obtained results will support to study of traveling wave solutions in an unmagnetized collisionless plasma as well as magnetacoustic waves in a cold collision-free plasma. This work conveys that the Adomian decomposition method is effective and suitable to obtain the traveling wave solutions for nonlinear fractional ordered partial differential equations.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Besides, the Figures and images, which are not ours, have been given permission for re-publication and attached to the manuscript.
- Authors sign on ethical consideration's approval.
- Ethical Clearance: The project was approved by the local ethical committee at the B. K. Birla College of Arts.

Authors' Contributions Statement:

K. G. proposed the idea and developed the method. K. T. collected parameter values and designed a graph. S. G. discussed convergence and stability. K. G., K. T., and S. G. wrote the paper with input from all authors.

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حلول الموجة المتنقلة للمعادلات التفاضلية الكسرية الناشئة في البلازما الدافئة

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الخلاصة:
تهدف هذه الورقة إلى دراسة الأنظمة التفاضلية الكسرية الناشئة في البلازما الدافئة ، والتي تعرض حلولا من النوع الموجي المتنقل. تستخدم معادلات الوقت-كسور كورتويغ-دي فريس (كدف) ومعادلات الوقت-كسور كاواهارا لتحليل البلازما الباردة الخالية من الاصطدام ، والتي تعرض موجات المغناطيس الصوتية وتشكيل موجات الصدمة على التوالي. لحل المعادلات المقترحة ، تم استخدام طريقة التحلل لحل المسألة المقترحة. كذلك تم مناقشة التقارب والوحدانية للحل الذي تم الحصول عليه. لإلقاء الضوء على فعالية الطريقة المقدمة ، تم الحصول على حلول هذه المعادلات وتم مقارنتها مع الحل الدقيق. علاوة على ذلك ، تم الحصول على حلول لقيم مختلفة من الوقت-ترتيب كسري وتم تمثيلها بيانيا.

الكلمات المفتاحية: مشتقة كابوتو الكسرية، طريقة تحلل أدومين الكسرية، معادلة كاواهارا الكسرية، معادلة كورتويغ دي فريس الكسرية، تكامل ريمان-ليوفيل الكسري .