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## New Structures of Continuous Functions

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### Abstract:

Continuous functions are novel concepts in topology. Many topologists contributed to the theory of continuous functions in topology. The present authors continued the study on continuous functions by utilizing the concept of  $g\alpha$ -closed sets in topology and introduced the concepts of weakly, subweakly and almost continuous functions. Further, the properties of these functions are established.

**Keywords:** Almost  $g\alpha$ -Continuous Function,  $g\alpha$ -Closed Set,  $g\alpha$ -Open Set, Sub-Weakly  $g\alpha$ -Continuous Function, Weakly  $g\alpha$ -Continuous Function.

### Introduction:

Weak continuity due to Levine<sup>1</sup> is one of the most important weak forms of continuity in topological spaces. The notion of sub-weakly continuous functions is investigated in 1984 and the relationship between weak continuity and sub-weakly continuity is studied. After that some topologists has discovered additional characteristics relating to sub-weakly continuous functions. Noiri demonstrated in<sup>4</sup> that the graph of a function is closed if the range space of a weakly continuous function is Hausdroff. A function  $p:M \rightarrow N$  between any two topological spaces  $M$  and  $N$  is continuous only if it is also both weakly continuous and  $\omega^*$ -continuous, according to Levine. By substituting a strictly weaker condition known as locally weak  $\omega^*$ -continuity for continuity, Levines decomposition of continuity<sup>2</sup> was strengthened.

In the year 2018, Patil<sup>3, 4</sup> studied the concept and properties of  $g\alpha$ -closed sets. The study of  $g\alpha$ -closed sets continued by defining the properties of  $g\alpha$ -closure,  $g\alpha$ -interior,  $g\alpha$ -limit points,  $g\alpha$ -continuous functions and  $g\alpha$ -homeomorphisms. Also, continued in by studying the properties such as,  $g\alpha$ - separation axioms,  $g\alpha$ -regular and  $g\alpha$ -normal spaces<sup>5</sup>.

In this paper, weakly  $g\alpha$ -continuous functions are introduced. Also, subweakly  $g\alpha$ -continuous functions and almost  $g\alpha$ -continuous functions<sup>6</sup> are introduced. Also investigate the functional properties of this type of functions<sup>7</sup>.

### Main Results:

#### Weakly Generalized Pre $\alpha$ - Continuous Functions

**Definition. 1:** Let  $p:M \rightarrow N$  be a map. Then  $p$  is weakly  $g\alpha$ -continuous (briefly  $w.g\alpha$ -c) if for each  $m \in M$  and  $V \in O(N, p(m))$ ,  $\exists U \in g\alpha$ - $O(M, m) \ni p(U) \subseteq g\alpha$ -cl( $V$ ).

**Remark. 1:** Every  $g\alpha$ -continuous function is  $w.g\alpha$ -c but not conversely.

**Example. 1:** Let  $M = N = \{a_1, a_2, a_3\}$  and  $\tau = \{M, \phi, \{a_1, a_2\}\}$ ,  $\sigma = \{N, \phi, \{a_1\}, \{a_2, a_3\}\}$  be topologies on  $M$  and  $N$  respectively. Let  $p$  be the identity function. Then  $p$  is  $w.g\alpha$ -c but not  $g\alpha$ -continuous.

**Theorem. 1:** The following are coincide for a function  $p:M \rightarrow N$

- (i)  $p$  is  $w.g\alpha$ -c.
- (ii) For each  $V \in O(N)$ , then  $p^{-1}(V) \in g\alpha$ -int( $p^{-1}(g\alpha$ -cl( $V$ ))).
- (iii) For each  $F \in C(N)$ , then  $g\alpha$ -cl( $p^{-1}(g\alpha$ -int( $F$ )))  $\subseteq p^{-1}(F)$ .
- (iv)  $B \subseteq N$ , then  $g\alpha$ -cl( $p^{-1}(g\alpha$ -int(cl( $B$ )))  $\subseteq p^{-1}$ (cl( $B$ )).
- (v)  $B \subseteq N$ , then  $p^{-1}$ (int( $B$ ))  $\subseteq g\alpha$ -int( $p^{-1}(g\alpha$ -cl( $B$ ))).
- (vi) If  $V \in O(N)$ , then  $g\alpha$ -cl( $p^{-1}(V) ) \subseteq p^{-1}(g\alpha$ -cl( $V$ )).

**Proof:** (i)  $\rightarrow$  (ii) Let  $m \in M$ ,  $V \in O(N) \ni m_2 p^{-1}(V)$ . So  $p(m) \in V$ . Then,  $\exists U \in \text{gp}\alpha\text{-}O(M, m)$  with  $p(U) \in \text{gp}\alpha\text{-}cl(V)$ . Thus,  $m \in U \subseteq p^{-1}(\text{gp}\alpha\text{-}cl(V))$ . Hence,  $m \in \text{gp}\alpha\text{-}int(p^{-1}(\text{gp}\alpha\text{-}cl(V)))$ , so  $p^{-1}(V) \in \text{gp}\alpha\text{-}int(p^{-1}(\text{gp}\alpha\text{-}cl(V)))$ .

(ii)  $\rightarrow$  (iii) Let  $F \in C(N)$  and so  $N \setminus F \in O(N)$ . From (ii),  $\text{gp}\alpha\text{-}cl(p^{-1}(\text{gp}\alpha\text{-}int(F))) \subseteq p^{-1}(F)$ . Thus

(iii) holds.

(iii)  $\rightarrow$  (iv) Let  $B \subset N$  and so  $cl(B) \subset C(N)$ . From (iii),  $\text{gp}\alpha\text{-}cl(p^{-1}(\text{gp}\alpha\text{-}int(cl(B)))) \subseteq p^{-1}(cl(B))$ . Thus (iv) holds.

(iv)  $\rightarrow$  (v) Let  $V \in O(N)$ . Assume,  $m \in p^{-1}(\text{gp}\alpha\text{-}cl(V))$ . Then  $p(m) \in \text{gp}\alpha\text{-}cl(V_1)$ . Thus,  $\exists U_1 \in \text{gp}\alpha\text{-}O(N, p(m))$  with  $U \cap V = \emptyset$ . Hence  $\text{gp}\alpha\text{-}cl(U) \cap V = \emptyset$ . From (v),  $m \in p(U) \subset \text{gp}\alpha\text{-}int(p^{-1}(\text{gp}\alpha\text{-}cl(U)))$ . So,  $\exists W \in \text{gp}\alpha\text{-}O(M, m) \ni W \subset p^{-1}(\text{gp}\alpha\text{-}cl(U))$ . But,  $\text{gp}\alpha\text{-}cl(U) \cap V = \emptyset$ , that is  $W \cap p^{-1}(V) = \emptyset$ . So,  $m \notin \text{gp}\alpha\text{-}cl(p^{-1}(V))$ . Thus  $\text{gp}\alpha\text{-}cl(p^{-1}(V)) \subseteq p^{-1}(\text{gp}\alpha\text{-}cl(V))$ .

(v)  $\rightarrow$  (i) Let  $m \in M$  and  $V \in O(N, p(m))$ . From (v),  $m \in p^{-1}(V) \subseteq p^{-1}(\text{int}(\text{gp}\alpha\text{-}cl(V))) \subseteq p^{-1}(\text{gp}\alpha\text{-}int(\text{gp}\alpha\text{-}cl(V))) \subseteq \text{gp}\alpha\text{-}cl(p^{-1}(\text{Ngp}\alpha\text{-}cl(V))) = \text{gp}\alpha\text{-}int(p^{-1}(\text{gp}\alpha\text{-}cl(V)))$ . Thus,  $\exists U \in \text{gp}\alpha\text{-}O(M, m)$  with  $U \subset \text{gp}\alpha\text{-}cl(V)$ .

**Lemma. 1:** A function  $p: M \rightarrow N$  is having a graph  $G(p)$  is  $\text{gp}\alpha$ -closed in  $M \times N$  iff  $\forall (m, n) \in (M \times N) \setminus G(p)$ ,  $\exists U \in \text{gp}\alpha\text{-}O(M, m)$  and  $V \in O(N, n)$  such that  $p(U) \cap V = \emptyset$ .

**Theorem. 2:** Let  $p: M \rightarrow N$  be w.  $\text{gp}\alpha$ -c function and  $N$  is  $T_2$ -space. Then the graph  $G(p)$  is  $\text{gp}\alpha$ -closed in  $M \times N$ .

**Definition. 2:** A function  $p: M \rightarrow N$  is called w.  $\text{gp}\alpha$ -c retraction, if  $p$  is w.  $\text{gp}\alpha$ -c with  $A \subset M$  and  $p|_A$  is the identity function on set  $A$ .

**Theorem. 3:** Let  $A \subset M$  and  $p: M \rightarrow N$  be w.  $\text{gp}\alpha$ -c retraction of  $M$  onto the set  $A$ . If  $M$  is  $T_2$ -space, then  $A$  is  $\text{gp}\alpha$ -closed.

**Definition. 3:** A space  $M$  is  $\text{gp}\alpha$ -connected if it cannot be described as the disjoint union of two non-empty  $\text{gp}\alpha$ -open sets.

**Example. 2:** Let  $M = \{a_1, a_2, a_3\}$  and  $\tau = \{M, \emptyset, \{a_1\}, \{a_2, a_3\}\}$  be a topology on  $M$ . Here  $\text{gp}\alpha$ -open sets are  $P(M)$ . Hence  $M$  is  $\text{gp}\alpha$ -connected.

**Theorem. 4:** If  $p: M \rightarrow N$  is w.  $\text{gp}\alpha$ -c surjective function with  $M$  is  $\text{gp}\alpha$ -connected, then  $N$  is connected.

**Proof:** On the contrary assume that  $N$  is not connected. Then  $\exists U, V \in O(N)$  with  $U \cap V = \emptyset \ni U \cap V = N$ . By surjectiveness of  $p$ ,  $p^{-1}(U)$ ,  $p^{-1}(V) \in O(M)$  with  $p^{-1}(U) \cap p^{-1}(V) = \emptyset$ ,  $\exists p^{-1}(U) \cup p^{-1}(V) = M$ . But from Theorem 1,  $p^{-1}(U) \subseteq \text{gp}\alpha\text{-}int(p^{-1}(\text{gp}\alpha\text{-}cl(U)))$  and  $p^{-1}(V) \subseteq \text{gp}\alpha\text{-}int(p^{-1}(\text{gp}\alpha\text{-}cl(V)))$ . As  $U, V \in O(M)$ , implies  $U, V \in \text{gp}\alpha\text{-}O(M)$ . Thus,  $p^{-1}(U) \subseteq \text{gp}\alpha\text{-}int(p^{-1}(U))$  and  $p^{-1}(V) \subseteq \text{gp}\alpha\text{-}int(p^{-1}(V))$ . Thus  $p^{-1}(U)$  and  $p^{-1}(V)$  are  $\text{gp}\alpha$ -open sets in  $M$  which is contradiction to the hypothesis. Thus,  $N$  must be connected.

**Definition. 4:** If each  $\text{gp}\alpha$ -open cover in a space  $M$  has a finite sub-cover, then the space is said to be a  $\text{gp}\alpha$ -compact.

**Example. 3:** If  $M = N = \{a_1, a_2, a_3\}$  and  $\tau = \{M, \emptyset, \{a_1, a_2\}\}$  be a topology on  $M$ , then  $M$  is  $\text{gp}\alpha$ -compact.

**Definition. 5:** A space  $M$  is called

(i)  $\text{gp}\alpha$ -closed compact if every cover of  $M$  by  $\text{gp}\alpha$ -open sets has a finite sub-cover whose  $\text{gp}\alpha$ -closure covers  $M$ .

(ii) Let  $A \subseteq M$ . Then  $A$  is said to be  $\text{gp}\alpha$ -closed relative to  $M$ , if every cover  $\{V_\alpha : \alpha \in \lambda\}$  of  $A$  by  $\text{gp}\alpha$ -open sets,  $\exists$  a finite sub-cover  $\lambda_0$  of  $\wedge \ni A \subseteq \{\text{gp}\alpha\text{-}cl(V_\alpha), \alpha \in \wedge\}$ .

**Theorem. 5:** Let  $p: M \rightarrow N$  be w.  $\text{gp}\alpha$ -c and  $A$  is  $\text{gp}\alpha$ -compact subset of  $M$ . Then  $p(A)$  is  $\text{gp}\alpha$ -closed relative to  $N$ .

**Proof:** Let  $\{V_i : i \in I\}$  be any cover of  $p(K)$ ,  $K \subset M$  by  $\text{gp}\alpha$ -open sets in  $N$ . Then,  $\forall m \in M$ ,  $\exists \alpha(m) \in I \ni p(m) \in V_{\alpha(m)}$ . By w.  $\text{gp}\alpha$ -c,  $\exists U(m) \in \text{gp}\alpha\text{-}O(M, m)$  such that  $p(U(m)) \subset \text{gp}\alpha\text{-}cl(V_{\alpha(m)})$ . So  $\{U(m) : m \in A\}$  is a cover of  $A$  by  $\text{gp}\alpha$ -open sets. As  $A$  is  $\text{gp}\alpha$ -compact,  $\exists$  finite number of points, say  $m_1, m_2, m_3, \dots, m_n \ni A \subseteq \cup \{U_{m_k} : m_k \in A; 1 \leq k \leq n\}$ . Thus,  $p(A) \subseteq \{p(U_{m_k}) : m_k \in A, 1 \leq k \leq n\} \subset \cup \{\text{gp}\alpha\text{-}cl(V_{\alpha(m_k)}) : m_k \in A; 1 \leq k \leq n\}$ . Hence  $p(A)$  is  $\text{gp}\alpha$ -closed relative to  $N$ .

**Theorem. 6:** The point  $m \in M$  at which the function  $p: M \rightarrow N$  is not w.  $\text{gp}\alpha$ -c iff the union of  $\text{gp}\alpha$ -frontier of the inverse images of the closure of open sets containing  $p(m)$ .

**Proof:** On the contrary assume that  $p$  is not w.  $\text{gp}\alpha$ -c at each point  $m \in M$ . Then,  $V \in O(N, p(m)) \ni p(U) \not\subset \text{gp}\alpha\text{-}cl(V)$  holds  $\forall U \in \text{gp}\alpha\text{-}O(M, m)$ . Then  $U \cap M \setminus p^{-1}(\text{gp}\alpha\text{-}cl(V)) \neq \emptyset$  for every  $V \in \text{gp}\alpha\text{-}O(N, p(m))$ . Thus,  $m \in \text{gp}\alpha\text{-}cl(M \setminus p^{-1}(\text{gp}\alpha\text{-}cl(V)))$ . On the other hand, let  $m \in p^{-1}(V) \subset \text{gp}\alpha\text{-}cl(p^{-1}(\text{gp}\alpha\text{-}cl(V)))$ . Thus,  $m \in \text{gp}\alpha\text{-}Fr(p^{-1}(\text{gp}\alpha\text{-}cl(V)))$ .

Converse follows from the Theorem 1.

**Theorem. 7:** Let  $p: M \rightarrow N$  be  $w.g\alpha$ -c and  $q: N \rightarrow S$  is continuous. Then  $q \circ p: M \rightarrow S$  is  $w.g\alpha$ -c.

**Theorem. 8:** Let  $p_\alpha: M \rightarrow N$  is weakly  $g\alpha$ -continuous  $\forall \alpha \in \lambda$ . Then  $p: M \rightarrow \prod N_\alpha$  defined as  $p(m) = (p_\alpha(m))_\alpha$  is weakly  $g\alpha$ -continuous.

**Theorem. 9:** Let  $p: M \rightarrow N$  be  $w.g\alpha$ -c and  $K$  is closed subset of  $M \times N$ . Suppose  $g\alpha$ -O(M) is closed under the finite intersections with  $P_1$  is the projection of  $M \times N$  onto  $M$ , then  $P_1(K \setminus G(p)) \in g\alpha$ -C(M).

### Sub-Weakly Generalized Pre $\alpha$ - Continuous Functions

**Definition. 6:** A function  $p: M \rightarrow N$  is said to have sub-weakly  $g\alpha$ -continuous (briefly  $sw.g\alpha$ -c) if  $\exists$  a base  $\beta$  for the topology on  $N$  for which  $g\alpha$ -cl( $p^{-1}(V)$ )  $\subset p^{-1}(cl(V)) \forall V \in \beta$ .

**Theorem. 10:** If  $p: M \rightarrow N$  is  $sw.g\alpha$ -c with  $N$  is  $T_2$ -space, then the graph  $G(p)$  is  $g\alpha$ -closed in  $M \times N$ .

**Proof:** Let  $(m, n) \in (M \times N) \setminus G(p)$ . Then  $n \in p(m)$ . Let  $\beta$  be the base for the topology on  $N$  with  $g\alpha$ -cl( $p^{-1}(V_1)$ )  $\subset p^{-1}(cl(V_1))$ ,  $\forall V \in \beta$ . By  $T_2$ -ness of  $N$ ,  $\exists V, W \in O(N)$  with  $n \in V$ ,  $p(m) \in W$  and  $V \cap W = \emptyset$  holds for all  $V \in \beta$ . Then  $p(m) \notin cl(V_1)$  and so  $m \notin p^{-1}(cl(V))$ . By sub-weakly  $g\alpha$ -continuity of  $p$ ,  $g\alpha$ -cl( $p^{-1}(V)$ )  $\subset p^{-1}(cl(V))$  and hence  $m \notin g\alpha$ -cl( $p^{-1}(V)$ ). Thus,  $(m, n) \in (M \setminus g\alpha$ -cl( $p^{-1}(V)$ ))  $\times V \subset (M \times N) \setminus G(p)$ . Thus,  $G(p)$  is  $g\alpha$ -closed

**Theorem. 11:** If  $p: M \rightarrow N$  is  $sw.g\alpha$ -c then the graph  $g: M \rightarrow M \times N$  is  $sw.g\alpha$ -c.

**Theorem. 12:** Let  $p: M \rightarrow N$  is sub-weakly continuous injective function with  $N$  is  $T_2$ . Then  $M$  is  $g\alpha$ - $T_2$  space.

**Theorem. 13:** Let  $p: M \rightarrow N$  be  $sw.g\alpha$ -c and  $A \in O(M)$ . Then  $p|_A: A \rightarrow N$  is sub-weakly continuous.

**Theorem. 14:** Let  $p: M \rightarrow N$  is  $sw.g\alpha$ -c and  $A \in O(M)$  with  $p(M) \subset A$ . Then  $p: M \rightarrow A$  is  $sw.g\alpha$ -c.

**Theorem. 15:** Let  $p: M \rightarrow N$  is continuous and  $g: M \rightarrow N$  be  $sw.g\alpha$ -c with  $N$  is  $T_2$ -space. Then  $A = \{m \in M : p(m) = g(m)\}$  is  $g\alpha$ -closed.

**Proof:** Let  $m \in M \setminus A$ , then  $p(m) \neq g(m)$ . By sub-weakly  $g\alpha$ -continuity,  $\exists$  a base  $\beta$  for the topology on  $N$  with  $g\alpha$ -cl( $p^{-1}(V)$ )  $\subset p^{-1}(cl(V))$  holds  $\forall V \in \beta$ . By  $T_2$ -ness of  $N$ ,  $\exists V, W \in \beta \ni p(m) \in V, g(m) \in W$  and  $V \cap W = \emptyset$ . Then,  $p(m) \notin cl(V)$  and

so  $m \notin g^{-1}(cl(V))$ . Then,  $m \in M \setminus g\alpha$ -cl( $g^{-1}(V)$ ). So,  $m \in p^{-1}(V) \cap (M \setminus g\alpha$ -cl( $g^{-1}(V)$ ))  $\subset (M \setminus A)$ . But,  $M \setminus g\alpha$ -cl( $g^{-1}(V)$ ) which is  $g\alpha$ -open in  $M$ . Since  $p$  is continuous,  $p^{-1}(V)$  is open in  $M$ . So,  $p^{-1}(V) \cap (M \setminus g\alpha$ -cl( $g^{-1}(V)$ )) is  $g\alpha$ -open in  $M$ . Thus,  $A$  is  $g\alpha$ -closed.

**Corollary. 1:** Let  $p: M \rightarrow N$  is continuous and  $g: M \rightarrow N$  is  $sw.g\alpha$ -c with  $N$  is  $T_2$ -space. If  $p$  and  $g$  are open and  $g\alpha$ -dense then  $p = g$ .

**Theorem. 16:** Let  $p_\alpha: M \rightarrow N$  is  $sw.g\alpha$ -c  $\forall \alpha \in \lambda$ . Then  $p: M \rightarrow \prod N_\alpha$  defined as  $p(m) = (p_\alpha(m))_\alpha$  is  $sw.g\alpha$ -c.

**Theorem. 17:** If  $p: M \rightarrow N$  is  $sw.g\alpha$ -c, then the graph of the function  $p$  is  $sw.g\alpha$ -c.

### Almost Generalized Pre $\alpha$ - Continuous Functions

**Definition. 7:** A function  $p: M \rightarrow N$  is called almost  $g\alpha$ -continuous (briefly  $a.g\alpha$ -c) at a point  $m \in M$  and  $\forall V \in O(N, p(m))$ ,  $\exists U \in g\alpha$ -O(M,  $m$ ) with  $p(U) \subseteq int(cl(V))$ . If  $p$  is  $a.g\alpha$ -c at every point of  $M$ , then it is  $a.g\alpha$ -c.

**Remark. 2:** Every  $a.g\alpha$ -c function is  $g\alpha$ -continuous, but not conversely.

**Example. 4:** Let  $M = N = \{a_1, a_2, a_3\}$ ,  $\tau = \{M, \emptyset, \{a_1\}, \{a_1, a_2\}\}$  and  $\sigma = \{N, \emptyset, \{a_1\}, \{a_1, a_2\}, \{a_1, a_3\}\}$ . Let  $p$  be the identity function from  $M$  to  $N$ . Then  $p$  is  $g\alpha$ -c but not  $a.g\alpha$ -c.

**Theorem. 18:** For  $p: M \rightarrow N$ , the following coincide:

- (i)  $p$  is  $a.g\alpha$ -c.,  $\forall V \in RO(N)$ .
- (ii)  $p^{-1}(F) \in g\alpha$ -C(M),  $\forall F \in RC(N)$ .
- (iii)  $\forall A \subset M, p(g\alpha$ -cl(A))  $\subset cl_\delta(p(A))$ .
- (iv)  $g\alpha$ -cl( $p^{-1}(B)$ )  $\subset p^{-1}(cl_\delta(B))$ ,  $\forall B \subset N$ .
- (v)  $p^{-1}(U) \in g\alpha$ -C(M),  $\forall U \in \delta C(N)$ .
- (vi)  $p^{-1}(V) \in g\alpha$ -O(M),  $\forall V \in \delta O(N)$ .

**Theorem. 19:** Every  $a.g\alpha$ -c function is  $w.g\alpha$ -c, but not conversely.

**Example. 5:** Let  $M = N = \{a_1, a_2, a_3\}$ ,  $\tau = \{M, \emptyset, \{a_1, a_2\}\}$  and  $\sigma = \{N, \emptyset, \{a_1\}, \{a_1, a_2\}, \{a_1, a_3\}\}$ . Let  $p$  be the identity function from  $M$  to  $N$ . Then  $p$  is  $w.g\alpha$ -c, but not  $a.g\alpha$ -c.

**Theorem. 20:** Following are coinciding for  $p: M \rightarrow N$ :

- (i)  $p$  is  $a.g\alpha$ -c.
- (ii)  $P^{-1}(int(cl(V))) \in g\alpha$ -O(M), for each  $V \in$

$O(N)$ .

(iii)  $p^{-1}(\text{cl}(\text{int}(F))) \in \text{gp}\alpha\text{-C}(M)$ , for each  $F \in C(N)$ .

**Theorem. 21:** Let  $p : M \rightarrow N$  be an a.gp $\alpha$ -c function and  $V \in O(N)$ . If  $m \in \text{gp}\alpha\text{-cl}(p^{-1}(V)) \setminus p^{-1}(V)$  then  $p(m) \in \text{gp}\alpha\text{-cl}(V)$ .

**Proof:** Let  $m \in M$  with  $m \in \text{gp}\alpha\text{-cl}(p^{-1}(V)) \setminus p^{-1}(V)$ . Let  $p(m) \notin \text{gp}\alpha\text{-cl}(V)$ . Then  $\exists H \in \text{gp}\alpha\text{-O}(N, p(m)) \ni H \cap V_1 = \emptyset$ . Then  $\text{cl}(H) \cap V = \emptyset$ , that is  $\text{int}(\text{cl}(H)) \cap V = \emptyset$  where  $\text{int}(\text{cl}(H)) \in \text{RO}(m)$ . By almost gp $\alpha$ -continuity of  $p$ ,  $\exists U \in \text{gp}\alpha\text{-O}(M, m) \ni p(U) \subset \text{int}(\text{cl}(H))$ . Thus  $p(U) \cap V = \emptyset$ . Since  $m \in \text{gp}\alpha\text{-cl}(p^{-1}(V))$ ,  $U \cap p^{-1}(V) = \emptyset$ ,  $\forall U \in \text{gp}\alpha\text{-O}(M, m)$ . So  $p(U) \cap V \neq \emptyset$ . which is contradiction. Thus,  $p(m) \in \text{gp}\alpha\text{-cl}(V)$ .

**Definition. 8:** Let  $(M, \tau)$  be a topological space and a filter base  $\Lambda$  is called,

- (i) gp $\alpha$ -convergent to a point  $m \in M$ , if  $\forall U \in \text{gp}\alpha\text{-O}(M, m) \exists B \in \Lambda \ni B \subset U$ .
- (ii) r-convergent to a point  $m \in M$ , if for every  $U \in \text{RO}(M, m) \exists B \in \Lambda \ni B \subset U$ .

**Theorem. 22:** Let  $p : M \rightarrow N$  be an almost continuous and  $q : M \rightarrow N$  is a.gp $\alpha$ -c function, with  $N$  is  $T_2$ -space. Then the set  $R = \{m \in M : p(m) = q(m)\}$  is gp $\alpha$ -closed in  $M$ .

**Proof:** Let  $m \in M \setminus R$  then  $p(m) \neq q(m)$ . As  $N$  is  $T_2$ -space,  $\exists$  disjoint  $V, W \in O(N)$  with  $p(m) \in V$  and  $q(m) \in W$ . As  $p$  is almost continuous and  $q$  is a.gp $\alpha$ -c  $p^{-1}(V) \in O(M)$ ,  $q^{-1}(W) \in \text{gp}\alpha\text{-O}(M)$  with  $m \in p^{-1}(V)$  and  $m \in q^{-1}(W)$ . Put  $A = p^{-1}(V) \cap q^{-1}(W)$  and so  $A \in \text{gp}\alpha\text{-O}(M)$ . Thus  $p(A) \cap q(A) = \emptyset$  and so  $m \notin \text{gp}\alpha\text{-cl}(R)$ . Hence  $R$  is gp $\alpha$ -closed in  $M$ .

**Theorem. 23:** Let  $p : M \rightarrow N$  be a.gp $\alpha$ -c,  $q : M \rightarrow N$  is w.gp $\alpha$ -c, with  $N$  is  $T_2$ -space. Then the set  $\{m \in M : p(m) = q(m)\}$  is gp $\alpha$ -closed in  $M$ .

**Proof:** Let  $A = \{m \in M : p(m) = q(m)\}$  and  $m \in M \setminus A$ , then  $p(m) \neq q(m)$ . As  $N$  is  $T_2$ -space,  $\exists$  disjoint  $V, W \in O(N)$  with  $p(m) \in V$  and  $q(m) \in W$  and  $U \cap N = \emptyset$ , and so  $\text{int}(\text{cl}(V)) \cap \text{cl}(W) = \emptyset$ . By almost gp $\alpha$ -continuity of  $m$ ,  $\exists G \in \text{gp}\alpha\text{-O}(M, m)$  with  $p(G) \subset \text{int}(\text{cl}(V))$ . As  $q$  is w.gp $\alpha$ -c  $\exists H \in \text{gp}\alpha\text{-O}(M)$  with  $q(H) \subset \text{cl}(W)$ . Put  $U = G \cap H$  then  $U \in \text{gp}\alpha\text{-O}(M, m)$  and  $p(U) \cap q(U) \subset \text{int}(\text{cl}(V)) \cap \text{cl}(W) = \emptyset$  and so  $U \cap A = \emptyset$ . Hence  $A$  is gp $\alpha$ -closed in  $M$ .

**Theorem. 24:** Assume that the product of two gp $\alpha$ -open sets is gp $\alpha$ -open. If  $p_1 : (M, \tau) \rightarrow (N, \sigma)$  is w.gp $\alpha$ -c,  $p_2 : (M, \tau) \rightarrow (N, \sigma)$  is a.gp $\alpha$ -c with  $N$  is  $T_2$ -space. Then the set  $\{(m_1, m_2) \in M_1 \times M_2 : p_1(m_1) = p_2(m_2)\}$  is gp $\alpha$ -closed in  $M_1 \times M_2$ .

**Theorem. 25:** For each  $m_1, m_2 \in M$ ,  $\exists$  a function  $p$

on  $M$  into a  $T_2$ -space  $N$  such that  $p(m_1) \neq p(m_2)$ ,  $p$  is w.gp $\alpha$ -c at  $m_1$  and  $p$  is a.gp $\alpha$ -c at  $m_2$ , then  $M$  is gp $\alpha$ - $T_2$ .

**Proof:** As  $N$  is  $T_2$ -space, for each  $m_1, m_2 \in M$ ,  $\exists V_1, V_2 \in O(N)$  containing  $p(m_1)$  and  $p(m_2)$  respectively with  $V_1 \cap V_2 = \emptyset$ . Then  $\text{cl}(V_1) \cap \text{int}(\text{cl}(V_2)) = \emptyset$ . By weakly gp $\alpha$ -continuity at  $p$ ,  $\exists U_1 \in \text{gp}\alpha\text{-O}(M, m_1)$  with  $p(U_1) \subset \text{cl}(V_1)$ . As  $p$  is a.gp $\alpha$ -c at the point  $m_2$ ,  $\exists U_2 \in \text{gp}\alpha\text{-O}(M, m_2)$  with  $p(U_2) \subset \text{int}(\text{cl}(V_2))$ . Thus  $U_1 \cap U_2 = \emptyset$ . Hence  $M$  is gp $\alpha$ - $T_2$ -space.

**Definition. 9:** A function  $p: M \rightarrow N$  is called gp $\alpha$ -strongly closed graph if for each  $\{(m, n) \in M \times N \setminus G(p)\}$ ,  $\exists U \in \text{gp}\alpha\text{-O}(M, m)$  and  $V \in O(M, n)$  with  $(U \times \text{cl}(V)) \cap G(p) = \emptyset$ .

**Lemma. 2:** A function  $p : M \rightarrow N$  is gp $\alpha$ -strongly closed graph  $G(p)$  iff for every  $\{(m, n) \in M \times N \setminus G(p)\} \exists U_1 \in \text{gp}\alpha\text{-O}(M)$  and  $V_1 \in O(N)$  respectively  $\exists p(U) \cap \text{cl}(V) = \emptyset$ .

**Example. 6:** Let  $M = N = \{a_1, a_2, a_3\}$  and  $\tau = \{M, \emptyset, \{a_1\}\}$  and  $\sigma = \{N, \emptyset, \{a_1\}, \{a_2, a_3\}\}$  be topologies on  $M$  and  $N$  respectively. Let  $p$  be the identity function from  $M$  to  $N$ . Then  $p$  is gp $\alpha$ -strongly closed graph.

### Conclusion:

In this present work, analysed new weaker form of some types of continuous functions namely weakly gp $\alpha$ -continuous functions and subweakly gp $\alpha$ -continuous functions. Also established the properties and some preservation theorems of weakly gp $\alpha$ -continuous functions and subweakly gp $\alpha$ -continuous functions. Further, almost gp $\alpha$ -continuous functions are studied here. There is a scope to study and extend these newly defined concepts in topological spaces.

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- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Besides, the Figures and images, which are not ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Karnatak University.



### Author's contributions:

This work was carried out in collaboration between the authors. P. G. P (Corresponding Author) gives the idea about the concept, also reviewed revised and proof reading the manuscript. And B. R. Pi has done literature survey and drafting the manuscript. All authors read and approved the final manuscript.

### References:

1. Levine N. A decomposition of continuity in topological spaces. Am Math Mon. 1961; 68(1): 44-46. <http://dx.doi.org/10.2307/2311363>
2. Noiri T. Between Continuity and Weak Continuity. Bull Un Mat Ital. 1974; 9(4): 647-654.
3. Mershkhan SM. Almost Pp-continuous functions. New Trend Math Sci. 2019; 4(7): 413-420. <http://dx.doi.org/10.20852/ntmsci.2019.382>
4. Praveen P, Patil PG. Generalized pre  $\alpha$ -closed sets in topological spaces. J New Theory. 2018; 20: 48-56. <https://dergipark.org.tr/tr/download/article-file/413174>
5. Patil PG, Pattanashetti BR.  $g\alpha$ -Kuratowski closure operators in topological space. Ratio Mathematica. 2021; 40: 139-149. <http://dx.doi.org/10.23755/rm.v40i1.578>
6. Mohammedali MN. Fuzzy Real Pre-Hilbert Space and Some of Their Properties. Baghdad Sci J. 2022; 19(2): 315-320. <http://dx.doi.org/10.21123/bsj.2022.19.2.0313>
7. Matar SF, Hijab AA. Some Properties of Fuzzy Neutrosophic Generalized Semi Continuous Mapping and Alpha Generalized Continuous Mapping. Baghdad Sci J. 2022; 19(3): 536-541. <http://dx.doi.org/10.21123/bsj.2022.19.3.0536>

## الهيكل الجديدة للوظائف المستمرة

بي جي باتيل بي آر باتاناشيتي

قسم الرياضيات ، جامعة كارناتاك ، Dharwad-580 003 ، كارناتاك ، الهند.

### الخلاصة:

الدوال المستمرة هي مفاهيم جديدة في الطوبولوجيا. ساهم العديد من الطوبولوجيين في نظرية الوظائف المستمرة في الطوبولوجيا. واصل المؤلفون الحاليون الدراسة حول الوظائف المستمرة من خلال استخدام مفهوم مجموعات  $g\alpha$  المغلقة في الطوبولوجيا وقدموا مفاهيم الوظائف الضعيفة والضعيفة والمتواصلة تقريباً. علاوة على ذلك، يتم إنشاء خصائص هذه الوظائف.

**الكلمات المفتاحية:** وظيفة  $g\alpha$  المستمرة تقريباً، مجموعة  $g\alpha$  المغلقة، مجموعة  $g\alpha$ -open، وظيفة  $g\alpha$  المستمرة شبه الضعيفة، وظيفة  $g\alpha$  المستمرة الضعيفة.