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Minimum Neighborhood Domination of Split Graph of Graphs

ANJALINE. W *   *A.STANIS ARUL MARY*

Department of Mathematics, Nirmala College for Women, Coimbatore, India.
*Corresponding author: anjaline97@gmail.com
E-mail address: stanisarulmary@gmail.com
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Abstract:

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a non-trivial simple graph. A dominating set in a graph is a set of vertices such that every vertex not in the set is adjacent to at least one vertex in the set. A subset $\mathcal{D} \subseteq \mathcal{V}(\mathcal{G})$ is a minimum neighborhood dominating set if $\mathcal{D}$ is a dominating set and if for every $\mathfrak{v}_i \in \mathcal{D}, |\cap_{i=1}^{n} N(\mathfrak{v}_i)| < \delta(\mathcal{G})$ holds. The minimum cardinality of the minimum neighborhood dominating set of a graph $\mathcal{G}$ is called as minimum neighborhood dominating number and it is denoted by $\gamma_{mN}(\mathcal{G})$. A minimum neighborhood dominating set is a dominating set where the intersection of the neighborhoods of all vertices in the set is as small as possible, (i.e., $\delta(\mathcal{G})$). The minimum neighborhood dominating number, denoted by $\gamma_{mN}(\mathcal{G})$, is the minimum cardinality of a minimum neighborhood dominating set. In other words, it is the smallest number of vertices needed to form a minimum neighborhood-dominating set. The concept of minimum neighborhood dominating set is related to the study of the structure and properties of graphs and is used in various fields such as computer science, operations research, and network design. A minimum neighborhood dominating set is also useful in the study of graph theory and has applications in areas such as network design and control theory. This concept is a variation of the traditional dominating set problem and adds an extra constraint on the intersection of the neighborhoods of the vertices in the set. It is also an NP-hard problem. The main aim of this paper is to study the minimum neighborhood domination number of the split graph of some of the graphs.

Keywords: Dominating set, Domination number, Minimum neighborhood dominating set, Minimum neighborhood dominating number, Split graph.

Introduction:

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a non-trivial simple graph with $|\mathcal{V}| = p, |\mathcal{E}| = q$.[1] Let D be the subset of $\mathcal{V}$ is a dominating set if every vertex in $\mathcal{V} - \mathcal{D}$ is adjacent to at least one element in $\mathcal{D}$.[2-4] The minimum cardinality of dominating set is called the domination number and it is denoted by $\gamma(\mathcal{G})$.[5-7] The concept of splitting graphs of families of the graph was studied to implement the new concept of minimum neighborhood dominating set over it.[8,9] On a small scale, there was a need for a minimum neighborhood dominating set in the case that, management has to pass information to each and every student of the institution. For that, management uses the concept of dominating set and chooses a set of students as dominating set. But it left some students unaware of information because some students were dominated by many students and those dominant students lavishly thought other will pass on the information. To reduce this problem a new concept called minimum neighborhood domination is introduced by imposing a condition on dominating set.

Main Results:

Definition. 1:

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a non-trivial simple graph with no isolated vertices. A subset $\mathcal{D} \subseteq \mathcal{V}(\mathcal{G})$ is a minimum neighborhood dominating set if $\mathcal{D}$ is a dominating set and if for every $\mathfrak{v}_i \in \mathcal{D}, |\cap_{i=1}^{n} N(\mathfrak{v}_i)| < \delta(\mathcal{G})$ holds. The minimum cardinality of the minimum neighborhood dominating set of a graph $\mathcal{G}$ is called as minimum neighborhood dominating number and it is denoted by $\gamma_{mN}(\mathcal{G})$.

Example. 1:

Consider $\mathcal{C}_6$ graph as in Fig. 1.

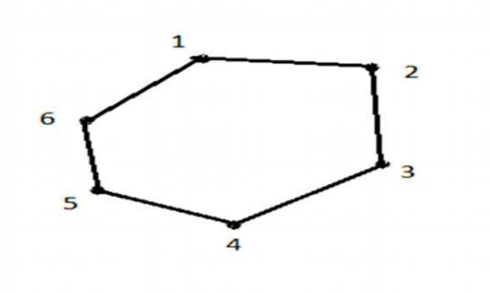

Figure 1. Cycle graph $\mathcal{C}_6$

Here $\mathcal{V}(\mathcal{C}_6) = \{1,2,3,4,5,6\}$
Thus minimum neighborhood dominating set $\mathcal{D} = \{2,5\}$. Therefore $\gamma_{mN}(\mathcal{C}_6) = 2$.

Theorem. 1:
For a split graph of the path graph $P_{\mathfrak{n}}$,

$$\gamma_{mN}(S(P_{\mathfrak{n}})) = \begin{cases} 2; & if\ \ \mathfrak{n} < 4, \\ \mathfrak{n}-2; & if\ \ \mathfrak{n} \geq 4 \end{cases}$$

Proof:
Let $S(P_{\mathfrak{n}})$ be a split graph of path graph with vertex set $\mathcal{V} = \{\mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_n, \mathfrak{w}_1, \mathfrak{w}_2, \dots \mathfrak{w}_n\}$. Let $\mathcal{D} \subseteq \mathcal{V}(\mathcal{G})$ be a minimum neighborhood-dominating set of a split graph of the path graph. To compute the minimum neighborhood dominating number, the following cases are taken:

Case 1: $\mathfrak{n} < 4$

Let $\mathfrak{n}$=2 and the vertex set of $S(P_2)$ be $\{\mathfrak{v}_1, \mathfrak{v}_2, w_1, w_2\}$. Here $D = \{\mathfrak{v}_1, \mathfrak{v}_2\}$ is a minimum neighborhood-dominating set. Therefore $\gamma_{mN}(S(P_2)) = 2$.

Let n=3 and the vertex set of $S(P_3)$ be $\{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3, \mathfrak{w}_1, \mathfrak{w}_2, \mathfrak{w}_3\}$. Here every vertex not in the set $D = \{\mathfrak{v}_2, \mathfrak{v}_3\}$ is adjacent to at least one element of D and $|N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3)| < \delta(S(P_3))$. So that $\mathcal{D} = \{\mathfrak{v}_2, \mathfrak{v}_3\}$ is a minimum neighborhood dominating set of $S(P_3)$. Therefore $\gamma_{mN}(S(P_3)) = 2$.

Case 2: $\mathfrak{n} \geq 4$

Let $\mathfrak{n}$=4 and the vertex set of $S(P_4)$ be $\{\mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_4, w_1, w_2, \dots w_4\}$. Here every vertex not in the set $D = \{\mathfrak{v}_2, \mathfrak{v}_3\}$ is adjacent to at least one element of D and $|N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3)| < \delta(S(P_4))$. So that $D = \{\mathfrak{v}_2, \mathfrak{v}_3\}$ is a minimum neighborhood dominating set of $S(P_4)$. Therefore $\gamma_{mN}(S(P_4)) = 2$.
As proceeding for any $\mathfrak{n} \geq 4$, every vertex not in the set $D = \{\mathfrak{v}_2, \mathfrak{v}_3, \mathfrak{v}_4, \dots \mathfrak{v}_{n-1}\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3) \cap N(\mathfrak{v}_4) \dots \cap N(\mathfrak{v}_{n-1})| < \delta(S(P_{\mathfrak{n}}))$. So that $\mathcal{D} = \{\mathfrak{v}_2, \mathfrak{v}_3, \mathfrak{v}_4, \dots \mathfrak{v}_{n-1}\}$ is a minimum neighborhood dominating set of $S(P_{\mathfrak{n}})$. Therefore $\gamma_{mN}(S(P_{\mathfrak{n}})) = \mathfrak{n} - 2$.

Theorem. 2:
For a split graph of a cycle graph $S(C_{\mathfrak{n}})$,

$$\gamma_{mN}(S(C_{\mathfrak{n}})) = \begin{cases} \frac{\mathfrak{n}}{2}; & if\ \ \mathfrak{n} = 4k, k = 1,2,3\dots \\ \lceil\frac{\mathfrak{n}}{2}\rceil; & if\ \ \mathfrak{n}\ is\ odd, \\ \lceil\frac{\mathfrak{n}}{2}\rceil + 1; & otherwise \end{cases}$$

Proof:
Let $S(C_{\mathfrak{n}})$ be a split graph of a cycle graph with vertex set $\mathcal{V} = \{\mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_n\}$.
Let $D \subseteq \mathcal{V}(\mathcal{G})$ be a minimum neighborhood-dominating set of a split graph of the cycle graph
To compute the minimum neighborhood dominating number, the following cases are taken:

Case 1: $\mathfrak{n} = 4k, k = 1,2,\dots,$

Let $\mathfrak{n}$=4 and the vertex set of $S(C_4)$ be $\{\mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_4, \mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_4\}$. Here every vertex not in the set $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2)| < \delta(S(C_4))$. So that $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2\}$ is a minimum neighborhood dominating set of $S(C_4)$. Therefore $\gamma_{mN}(S(C_4)) = 2$.
As proceeding for any $\mathfrak{n} = 4k, k = 1,2,3..$,
every vertex not in the set $D = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_5, \mathfrak{v}_6, ...\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3) \dots| < \delta(S(C_{4k}))$. So that $D = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_5, \mathfrak{v}_6, \dots\}$ is a minimum neighborhood dominating set of $S(C_{4k})$. Therefore $\gamma_{mN}(S(C_{\mathfrak{n}=4k})) = \frac{\mathfrak{n}}{2}$.

Case 2: $\mathfrak{n}$ is odd, $\mathfrak{n} \neq 3$

If $\mathfrak{n}$ is odd and $\mathfrak{n} \neq 3$, then every vertex not in the set $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_5, \mathfrak{v}_6, \dots\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3) \dots| < \delta(S(C_{\mathfrak{n}}))$. So that $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_5, \mathfrak{v}_6, \dots\}$ is a minimum neighborhood dominating set of $S(C_{\mathfrak{n}})$. Therefore $\gamma_{mN}(S(C_{\mathfrak{n}})) = \lceil\frac{\mathfrak{n}}{2}\rceil$.

Case 3: For $\mathfrak{n}$ =3 and when $\mathfrak{n}$ is even & $\mathfrak{n} \neq 4k, k = 1,2, ...$

If $\mathfrak{n}$=3,6,10,.., then every vertex not in the set $D = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_5, \mathfrak{v}_6, \dots\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3) \dots| < \delta(S(C_{\mathfrak{n}}))$. So that $D = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_5, \mathfrak{v}_6, \dots\}$ is a minimum neighborhood dominating set of $S(C_{\mathfrak{n}})$. Therefore $\gamma_{mN}(S(C_{\mathfrak{n}})) = \lceil\frac{\mathfrak{n}}{2}\rceil + 1$.

Theorem. 3:
Let $S(\mathcal{G})$ be a split graph of graph G, then $\gamma_{mN}(S(\mathcal{G})) = \mathfrak{n}+1$, when $\mathcal{G}$ is (i) Helm graph $H_{1,\mathfrak{n},\mathfrak{n}}$ (ii) Sunflower graph $sf_{1,\mathfrak{n},\mathfrak{n}}$.
Proof:
Let $\mathcal{D} \subseteq \mathcal{V}(\mathcal{G})$ be a minimum neighborhood dominating set of a split graph of the graph.
Let $S(H_{1,\mathfrak{n},\mathfrak{n}})$ be a split graph of Helm graph with vertex set

$\mathcal{V} = \{\mathfrak{u}_1, \mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_n, \mathfrak{w}_1, \mathfrak{w}_2, \dots \mathfrak{w}_n, \mathbf{x}_1, \mathbf{y}_1, \mathbf{y}_2, \dots \mathbf{y}_n, \mathbf{z}_1, \mathbf{z}_2, \dots \mathbf{z}_n\}$.

Let $\mathfrak{n}$=3 and the vertex set of $S(H_{1,3,3})$ be $\{\mathfrak{u}_1, \mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3, \mathfrak{w}_1, \mathfrak{w}_2, \mathfrak{w}_3, \mathbf{x}_1, \mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3, \mathbf{z}_1, \dots \mathbf{z}_3\}$.
Here every vertex not in the set $\mathcal{D} = \{\mathfrak{u}_1, \mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{u}_1) \cap N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3)| < \delta(S(H_{1,3,3}))$. So that $\mathcal{D} = \{\mathfrak{u}_1, \mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3\}$ is a minimum neighborhood dominating set of $S(H_{1,3,3})$. Therefore $\gamma_{mN}(S(H_{1,3,3})) = 4$.

As proceeding for any $\mathfrak{n}$, every vertex not in the set $\mathcal{D} = \{\mathfrak{u}_1, \mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_n\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{u}_1) \cap N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3) \cap \dots \cap N(\mathfrak{v}_n)| < \delta(S(H_{1,\mathfrak{n},\mathfrak{n}}))$. So that $\mathcal{D} = \{\mathfrak{u}_1, \mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_n\}$ is a minimum neighborhood dominating set of $S(H_{1,\mathfrak{n},\mathfrak{n}})$. Therefore $\gamma_{mN}(S(H_{1,\mathfrak{n},\mathfrak{n}})) = \mathfrak{n}+1$.

Similarly, $\gamma_{mN}(S(\mathcal{G})) = \mathfrak{n}+1$ for split graph of Sunflower graph $S(sf_{1,\mathfrak{n},\mathfrak{n}})$.

Theorem. 4:
Let $S(\mathcal{G})$ be a split graph of graph $\mathcal{G}$, then $\gamma_{mN}(S(\mathcal{G})) = \mathfrak{n}$, when $\mathcal{G}$ is (i) Closed Helm graph $CH_{1,\mathfrak{n},\mathfrak{n}}$ (ii) Gear graph $G_\mathfrak{n}$ (iii) Prism graph $Y_\mathfrak{n}$ (iv) Sunlet graph $C_\mathfrak{n} \odot K_1$.
Proof:
Let $\mathcal{D} \subseteq \mathcal{V}(\mathcal{G})$ be a minimum neighborhood dominating set of the split graph of a graph.

(i)Let $S(CH_{1,\mathfrak{n},\mathfrak{n}})$ be a split graph of a Closed Helm graph with vertex set $\mathcal{V} = \{\mathfrak{u}_1, \mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_n, \mathfrak{w}_1, \mathfrak{w}_2, \dots \mathfrak{w}_n, \mathbf{x}_1, \mathbf{y}_1, \mathbf{y}_2, \dots \mathbf{y}_n, \mathbf{z}_1, \mathbf{z}_2, \dots \mathbf{z}_n\}$.

Let $\mathfrak{n}$ =3

Let the vertex set of $S(CH_{1,3,3})$ be $\{\mathfrak{u}_1, \mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3, \mathfrak{w}_1, \mathfrak{w}_2, \mathfrak{w}_3, \mathbf{x}_1, \mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3, \mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3\}$.
Here every vertex not in the set $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3)| < \delta(S(CH_{1,3,3}))$. So that $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3\}$ is a minimum neighborhood-dominating set of $S(CH_{1,3,3})$. Therefore $\gamma_{mN}(S(CH_{1,3,3})) = 2$.

As proceeding for any n, every vertex not in the set $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_n\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2) \cap \dots \cap N(\mathfrak{v}_n)| < \delta(S(CH_{1,\mathfrak{n},\mathfrak{n}}))$. So that $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_n\}$ is a minimum neighborhood-dominating set of $S(CH_{1,n,n})$. Therefore $\gamma_{mN}(S(CH_{1,n,n})) = n$.

Proceeding similarly for a split graph of a (ii) Gear graph $\mathcal{G}_n$ (iii) Prism graph $Y_\mathfrak{n}$ (iv) Sunlet graph $C_\mathfrak{n} \odot K_1$, $\gamma_{mN}(S(\mathcal{G})) = \mathfrak{n}$.

Theorem. 5:
Let $S(\mathcal{G})$ be a split graph of graph G, then $\gamma_{mN}(S(\mathcal{G})) = \mathfrak{n}-1$, when $\mathcal{G}$ is (i) Sun graph $S_{\mathfrak{n},\mathfrak{n}}, \mathfrak{n} > 3$ (ii) Closed Sun graph $CS_{\mathfrak{n},\mathfrak{n}}$ (iii) Anti-prism graph $A_{\mathfrak{n},\mathfrak{n}}$.
Proof:
Let $\mathcal{D} \subseteq \mathcal{V}(\mathcal{G})$ be a minimum neighborhood dominating set of a split graph of a graph.

(i)Let $S(S_{\mathfrak{n},\mathfrak{n}})$ be a Split graph of the Sun graph with vertex set $\mathcal{V} = \{\mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_n, \mathfrak{w}_1, \mathfrak{w}_2, \dots \mathfrak{w}_n, \mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_n, \mathbf{y}_1, \mathbf{y}_2, \dots \mathbf{y}_n\}$.

Let $\mathfrak{n}$=4 and the vertex set of $S(S_{4,4})$ be $\{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3, \mathfrak{v}_4, W_1, \mathfrak{w}_2, \mathfrak{w}_3, \mathfrak{w}_4, \mathbf{x}_1, \dots, \mathbf{x}_4, \mathbf{y}_1, \dots, \mathbf{y}_4\}$.
Here every vertex not in the set $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3)| < \delta(S(S_{4,4}))$. So that $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3\}$ is a minimum neighborhood dominating set of $S(S_{4,4})$. Therefore $\gamma_{mN}(S(S_{4,4})) = 3$.

As proceeding for any $\mathfrak{n}$, every vertex not in the set $\mathcal{D} = \{\mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_{n-1}\}$ is adjacent to at least one element of $\mathcal{D}$ and $|N(\mathfrak{v}_1) \cap N(\mathfrak{v}_2) \cap N(\mathfrak{v}_3) \cap \dots \cap N(\mathfrak{v}_{n-1})| < \delta(S(S_{\mathfrak{n},\mathfrak{n}}))$. So that $D = \{\mathfrak{v}_1, \mathfrak{v}_2, \dots \mathfrak{v}_{n-1}\}$is a minimum neighborhood dominating set of $S(S_{\mathfrak{n},\mathfrak{n}})$. Therefore $\gamma_{mN}(S(S_{\mathfrak{n},\mathfrak{n}})) = \mathfrak{n}-1$.

Similarly for a split graph of a (i) Sun graph $S_{\mathfrak{n},\mathfrak{n}}, \mathfrak{n} > 3$ (ii) Closed Sun graph $CS_{\mathfrak{n},\mathfrak{n}}$ (iii) Anti-prism graph $A_{\mathfrak{n},\mathfrak{n}}$, $\gamma_{mN}(S(\mathcal{G})) = \mathfrak{n}-1$.

Remark. 1: For a split graph of Sun graph $S(S_{3,3})$, $\gamma_{mN}(S(S_{3,3})) = 3$.

Conclusion:

The minimum neighborhood dominating the number of the split graph of some class of graphs is computed.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Besides, the Figures and images, which are not ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Nirmala College for Women.

Authors' Contribution:

A W and A. S A Mcontributed to the design and implementation of the research, the analysis of the results, and the writing of the manuscript.

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الحد الأدنى من هيمنة الجوار على الرسم البياني المقسم للرسوم البيانية

أنجال. دبليو *　　　　أستانيس أرول ماري

قسم الرياضيات ، كلية نيرمالا للبنات ، كويمباتور، الهند.

الخلاصة:

لنفترض أن $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ رسم بياني بسيط غير تافه. المجموعة المسيطرة في الرسم البياني هي مجموعة من الرؤوس بحيث تكون كل قمة ليست في المجموعة مجاورة لرأس واحد على الأقل في المجموعة. المجموعة الفرعية D⊆V (G) هي مجموعة تهيمن على الجوار الأدنى إذا كانت D هي مجموعة مهيمنة وإذا كانت لكل $\mathfrak{v}_i \in \mathcal{D}, |\cap_{i=1}^{n} N(\mathfrak{v}_i)| < \delta(\mathcal{G})$ تتحقق. يُطلق على الحد الأدنى من عدد العناصر لأدنى حد يهيمن على مجموعة الرسم البياني G على أنه الحد الأدنى من العدد المسيطر على الجوار ويُشار إليه بالرمز $\gamma_{mN}(\mathcal{G})$. الحد الأدنى للمجموعة المسيطرة على الجوار هو مجموعة مهيمنة حيث يكون تقاطع اللجوارات لجميع الرؤوس في المجموعة صغيرًا قدر الإمكان. الحد الأدنى للرقم المسيطر على الحي ، الذي يُشار إليه بالرمز $\gamma_{mN}(\mathcal{G})$، هو الحد الأدنى لعدد العناصر المسيطرة على الجوار. بمعنى آخر ، هو أصغر عدد من الرؤوس اللازمة لتشكيل مجموعة مسيطرة على الجوار الأدنى. يرتبط هذا المفهوم بدراسة بنية وخصائص الرسوم البيانية ويستخدم في مختلف المجالات مثل علوم الكمبيوتر وبحوث العمليات وتصميم الشبكات. تعتبر مجموعة السيطرة على الجوار الأدنى مفيدة أيضًا في دراسة نظرية الرسم البياني ولها تطبيقات في مجالات مثل تصميم الشبكة ونظرية التحكم. هذا المفهوم هو تباين في مشكلة المجموعة المهيمنة التقليدية ويضيف قيدًا إضافيًا على تقاطع الجوارات للرؤوس في المجموعة. كما إنها تعتبر مشكلة NP - صعبة. الهدف الرئيسي من هذه الورقة هو دراسة الحد الأدنى لعدد هيمنة الجوارات للرسم البياني المقسم لبعض الرسوم البيانية.

الكلمات المفتاحية: المجموعة المسيطرة ، الرقم المهيمن ، الحد الأدنى من المجموعة المسيطرة على الجوار ، الحد الأدنى من العدد المسيطر على الجوار ، الرسم البياني المقسم.