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A Study on Nψβ and Nβψ - Closed sets in Neutrosophic Topological spaces

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Abstract:

The aim of this paper is to introduce the concept of Nψβ and Nβψ-closed sets in terms of neutrosophic topological spaces. Some of its properties are also discussed.

Keywords: Neutrosophic set, Neutrosophic Topology, Nψβ-closed set, Nβψ-closed set, Nβ^*-closed set.

Introduction:

In general topological space, several authors introduced various open sets such as Pre-stable set by Anmar Hashim Jasim[1]and G-open set by Jalal Hatem Hussein[2]. Florentin Smarandache[3] defined the neutrosophicset on three components (t,f,i) = (truth, falsehood, indeterminacy).This opened up a wide range of investigation in terms of neutosophic topology and its application in decision-making algorithms. Dhavaseelan.R and Jafari[4] introduced generalized Neutrosophic closed sets in 2018.Mani Parimala et al.[5]introduced neutrosophic αψ-closed sets in 2018. Pushpalatha A et al.[6] introduced generalized closed sets via Neutrosophic topological spaces in 2019.Renu Thomas et al.[7] introduced and studied semi pre-open(or β-open) sets in neutrosophic topological spaces. Recently, Subasree R and Basari Kodi K[8], introduced and studied Nβ*-closed sets in Neutrosophic Topological spaces in 2022. In this article, a new class of sets namely N$\boldsymbol{\psi}$**β**-closed sets and Nβ$\boldsymbol{\psi}$-closed sets are introduced in neutrosophic topological space. Moreover, some of its properties are investigated.

Preliminaries

Definition 1: Let X be a non-empty fixed set. A neutrosophic set (NS) A is aobject having the form A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element x ∈ X to the set A.

A Neutrosophic set A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ can be identified as an ordered triple $\langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$in $]-0, 1+[$ on X.

Definition 2: Let A =$\langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ be a NS on X, then the complement C(A) may be defined as

1. $C(A) = \{\langle x, 1-\mu_A(x), 1-\nu_A(x)\rangle : x \in X\}$
2. $C(A) = \{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$
3. $C(A) = \{\langle x, \nu_A(x), 1-\sigma_A(x), \mu_A(x)\rangle : x \in X\}$

Note that for any two neutrosophic sets A and B,

4. $C(A \cup B) = C(A) \cap C(B)$
5. $C(A \cap B) = C(A) \cup C(B)$.

Definition 3: For any two neutrosophic sets A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ and B = $\{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$. The following definitions hold:

1. $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x)$, $\sigma_A(x) \le \sigma_B(x)$ and $\nu_A(x) \ge \nu_B(x) \forall\, x \in X$

2. $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x)$, $\sigma_A(x) \ge \sigma_B(x)$ and $\nu_A(x) \ge \nu_B(x) \forall\, x \in X$

3. $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)\rangle$

4. $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)\rangle$

5. $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)\rangle$
6. $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)\rangle$

For Simplicity, throughout this paper, NS denote Neutrosophic set and NT denote Neutrosophic Topology.

Definition 4: A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:
(NT1) 0_N, $1_N \in \tau$
(NT2) $G_1 \cap G_2 \in \tau$ for any G_1 , $G_2 \in \tau$
(NT3) $\cup G_i \in \tau$ for all $\{G_i : i \in J\} \subseteq \tau$

Definition 5: Let A be a NS in NTS X. Then
Nint(A) = ∪{G : G is an NOS in X and G ⊆ A} is called a neutrosophic interior of A.
Ncl(A) = ∩ {K : K is an NCS in X and A ⊆ K} is called a neutrosophic closure of A.

Definition 6: A NS A of a NTS X is said to be
(i) A neutrosophic semi-open set (NSOS) if A ⊆NCl(NInt(A)) and a neutrosophic semi-closed set (NSCS) if NInt(NCl(A)) ⊆ A.
(ii) Aneutrosophic α-open set (NαOS) if A ⊆NInt(NCl(NInt(A))) and a neutrosophic α-closed set (NαCS) if NCl(NInt(NCl(A))) ⊆ A.
(iii) A neutrosophic semi-preopen set or β-open(NβOS) if A ⊆NCl(NInt(NCl(A))) and a neutrosophic semi-pre closed set or β-closed(NβCS) if NInt(NCl(NInt(A))) ⊆ A.

Definition 7: A subset A of a neutrosophic topological space (X, τ) is called a neutrosophic generalized closed (neutrosophic g-closed) set if Ncl(A) ⊆ U whenever A ⊆ U and U is neutrosophic open set in (X, τ).

Definition 8: A subset A of a neutrosophic topological space (X, τ) is called a neutrosophic semi generalized − closed (Nsg-closed) set if Nscl(A)⊆ U whenever A ⊆ U and U is Nsemi-open set in (X, τ).

Definition 9: A subset A of a neutrosophic topological space (X, τ) is called a neutrosophicψ−closed (Nψ-closed) set if Nscl(A)⊆ U whenever A ⊆ U and U is Nsg-open set in (X, τ).

Definition 10: Consider a NS A in a NTS (X, τ). Then the neutrosophic ψ interior and the neutrosophic ψ closure are defined as
Nψint(A) = ∪ {G: G is a Nψ-open set in X and G ⊆ A}
Nψcl(A) = ∩{K: K is a Nψ-closed set in X and A ⊆ K }

Definition 11: Consider a NS A in a NTS (X, τ). Then the neutrosophic β interior and the neutrosophic β closure are defined as
Nβint(A) = ∪ {G: G is a Nβ-open set in X and G ⊆ A}
Nβcl(A) = ∩{K: K is a Nβ-closed set in X and A ⊆ K }

Neutrosophic*ψ*β-closed set

In this section, the new concept of neutrosophic*ψ*β-closed sets in neutrosophic topological spaces was defined and studied some of its properties.

Definition 12: A subset A of a neutrosophic topological space (X, τ) is called a neutrosophic ψβ-closed (Nψβ-closed) set if Nψcl(A) ⊆ U whenever A ⊆ U and U is neutrosophic β-open set in (X, τ).

Example 1: Let X = {a,b,c} with $\tau_N = \{0_N, G, 1_N\}$ where

G = ⟨(a,0.5,0.6,0.4),(b,0.4,0.5,0.2), (c,0.7,0.6,0.9)⟩.

Here
A = ⟨(a,0.2,0.2,0.1),(b,0.6,0.6,0.6),(c,0.8,0.9,0.9)⟩
B = ⟨(a,0.2,0.2,0.7),(b,0.1,0.3,0.7),(c,0.8,0.2,0.9)⟩
C = ⟨(a,0.4,0.4,0.5),(b,0.2,0.5,0.4),(c,0.9,0.4,0.7)⟩
D = ⟨(a,0.5,0.5,0.4),(b,0.3,0.8,0.3),(c,0.1,0.6,0.5)⟩
are some examples of Nψβ-closed sets.

Theorem 1: Each Neutrosophic closed Set is Nψβ-closed set in X.
Proof: Let A ⊆ U where U is a neutrosophic β-open set in X. Since A is a neutrosophic closed set, Ncl(A) ⊆ A. Then Nψcl(A) ⊆Ncl(A) ⊆ U. Therefore A is a Nψβ-closed set in X.

The converse of theorem 1 need not be true as shown in the following example.
Example 2: Let X = {a,b,c} with $\tau_N = \{0_N, G, 1_N\}$ where
G = ⟨(a,0.5,0.6,0.4),(b,0.4,0.5,0.2), (c,0.7,0.6,0.9).
Here
E = ⟨(a,0.3,0.2,0.8),(b,0.1,0.3,0.6),(c,0.7,0.2,0.8)⟩ is an Nψβ-closed set, however E is not a Neutrosophic closed Set in X.

Theorem 2: Each Nψβ − closed set is Nψ-closed set in X and the converse is also true.
Proof: Let A be an Nψβ−closed set in X, then Nψcl(A) ⊆ A. Also A⊆Nψcl(A). Therefore A is Nψ-closed set in X. Conversely, Let A be a Nψ−closed set in X, then A = Nψcl(A) ⊆ A. Also Nψcl(A) ⊆U, where U is a neutrosophic β-open set in X. Therefore A is a Nψβ-closed set in X.

Theorem 3: Each Nsemi-closed set is Nψβ-closed set in X.
Proof: Let A ⊆ U where U is a neutrosophic β-open set in X. Since A is aNsemi-closed set in X, thenNψcl(A) ⊆Nscl(A) ⊆ A ⊆ U. Therefore A is aNψβ-closed set in X.

The converse of theorem 2 need not be true as shown in the following example.

Example 3: Let X = {a,b,c} with τ_N = {0_N,A,1_N} where
A = ⟨(a,0.5,0.6,0.4),(b,0.4,0.5,0.2),(c,0.7,0.6,0.9)⟩
Here
B = ⟨(a,0.2,0.2,0.1),(b,0.6,0.6,0.6),(c,0.8,0.9,0.9)⟩ is an Nψβ-closed set, but B is not an N semi-closed Set.

Remark 1: The following example shows that the Neutrosophic generalized-closed set and an Nψβ-closed set are independent of each other.

Example 4: Let X = {a,b} with τ_N = {0_N,A,B,1_N} where A = ⟨(a,0.4,0.3,0.5),(b,0.1,0.2,0.5)⟩ and
B = ⟨(a,0.4,0.4,0.5),(b,0.4,0.3,0.4)⟩.
Here B = ⟨(a,0.4,0.4,0.5),(b,0.4,0.3,0.4)⟩is an Nψβ-closed set, but B is not a Neutrosophic g-Closed Set and also C = ⟨(a,0.4,0.6,0.5),(b,0.3,0.6,0.9)⟩ is a Neutrosophic g−Closed Set but not an Nψβ-closed set.

Theorem 4: If A and B are Nψβ-closed sets, then A∪B is Nψβ-closed set.
Proof: If A∪B ⊆ U and U is Neutrosophic β-open set, then A ⊆ U and B ⊆ U. Since A and B are Nψβ-closed sets, Nψcl(A) ⊆ U and Nψcl(B) ⊆ U and hence Nψcl(A) ∪ Nψcl(B) ⊆ U. This implies Nψcl(A ∪ B) ⊆ U. Hence A∪B is an Nψβ-closed set in X.

Theorem 5:A neutrosophic set A is Nψβ-closed set then Nψcl(A) – A does not contain any nonempty Nψβ-closed sets.
Proof: Suppose that A is aNψβ-closed set. Let F be an Nψβ-closed set such that F ⊆Nψcl(A) – A which implies F ⊆Nψcl(A)∩A^c. Then A ⊆ F^c. Since A is Nψβ-closed set, thenNψcl(A) ⊆ F^c. Consequently F ⊆ $(N\psi cl(A))^c$.ThenF ⊆Nψcl(A). Thus F ⊆Nψcl(A)∩$(N\psi cl(A))^c$ = ϕ. Hence F is empty.

Theorem 6: If A is a Nψβ-closed set in (X,τ_N) and A ⊆ B ⊆Nψcl(A), then B is Nψβ-closed.
Proof: Let B ⊆ U where U is a Neutrosophic β-open set in (X,τ_N). Then A ⊆ B implies A ⊆ U. Since A is aNψβ-closed set, thenNψcl(A)⊆U. Also, A ⊆Nψcl(B) implies Nψcl(B) ⊆Nψcl(A). Thus Nψcl(B)⊆ U and so B is an Nψβ-closed set in (X,τ_N).

Theorem 7: If A is Neutrosophic ψ-open and Nψβ-closed, then A is Nψ-closed set.
Proof: Since A is Neutrosophic ψ-open and Nψβ-closed, then Nψcl(A) ⊆ A. Therefore Nψcl(A)=A. Hence A is Nψ-closed.

Neutrosophicβψ-closed set
In this section, the new concept of neutrosophic**βψ**-closed sets in neutrosophic topological spaces was defined and studied some of its properties.

Definition 13:A subset A of a neutrosophic topological space (X, τ) is called a neutrosophic βψ- closed (Nβψ-closed) set if Nβcl(A) ⊆ U whenever A ⊆ U and U is neutrosophic ψ-open set in (X, τ).

Example 5: Let X = {a,b} with τ_N = {0_N,A,B,1_N} where A = ⟨(a,0.4,0.3,0.5),(b,0.1,0.2,0.5)⟩ and
B = ⟨(a,0.4,0.4,0.5),(b,0.4,0.3,0.4)⟩.
Here C = ⟨(a,0.3,0.3,0.6),(b,0.3,0.2,0.5)⟩
D = ⟨(a,0.6,0.7,0.3),(b,0.5,0.8,0.3)⟩
E = ⟨(a,0.4,0.6,0.5),(b,0.3,0.6,0.9)⟩
F=⟨(a,0.5,0.4,0.4),(b,0.9,0.4,0.3)⟩ are some examples of Nβψ-closed sets in X.

Theorem 8: Each Neutrosophic closed Set is an Nβψ-closed set in X.
Proof: Let A ⊆ U where U is a neutrosophic ψ-open set in X. Since A is a neutrosophic closed set Ncl(A) = A. Then Nβcl(A) ⊆ Ncl(A) ⊆ U. ThereforeA is aNβψ-closed set in X.

The converse of theorem 8 need not be true as shown in the following example.
Example 6: Let X = {a,b,c} with τ_N = {0_N,G,1_N} where
G = ⟨(a,0.5,0.6,0.4),(b,0.4,0.5,0.2), (c,0.7,0.6,0.9)⟩.
Here
F = ⟨(a,0.2,0.2,0.1),(b,0.6,0.6,0.6),(c,0.8,0.9,0.9)⟩ is an Nβψ-closed set, however F is not a Neutrosophic Closed Set in X.

Theorem 9: Each Nβψ − closed set is an Nβ-closed set in X and the converse is also true.
Proof: Let A be an Nβψ−closed set in X, then Nβcl(A) ⊆ A. Also, A⊆ Nβcl(A). Therefore, A is a Nβ-closed set in X.Conversely, Let A be an Nβ−closed set in X, then A = Nβcl(A) ⊆ A. Also, Nβcl(A) ⊆U, where U is a neutrosophic ψ-open set in X. Therefore, A is a Nβψ-closed set in X.

Theorem 10: Each Nsemi-closed set is an Nβψ-closed set in X.

Proof: Let A ⊆ U where U is a neutrosophic ψ-open set in X. Since A is a Nsemi-closed set in X, then Nβcl(A) ⊆Nscl(A) ⊆ A ⊆ U. Therefore A is a Nβψ-closed set in X.

The converse of theorem 10 need not be true as shown in the following example.

Example 7: Let X = {a,b,c} with τ_N = {0_N,A,1_N} where
A = ⟨(a,0.5,0.6,0.4),(b,0.4,0.5,0.2),(c,0.7,0.6,0.9)⟩
Here B = ⟨(a,0.1,0.7,0.3),(b,0.2,0,0),(c,0.8,0.3,0.9)⟩is an Nβψ-closed set, but B is not an N semi − Closed Set.

Remark 2: The following example shows that the Neutrosophic generalized-closed set and an Nβψ-closed set are independent of each other.

Example 8: Let X = {a,b,c} with τ_N = {0_N,A,B,1_N} where
A = ⟨(a,0.5,0.5,0.4),(b,0.7,0.5,0.5),(c,0.4,0.5,0.5)⟩,
B = ⟨(a,0.3,0.4,0.4),(b,0.4,0.5,0.5),(c,0.3,0.4,0.6)⟩
Here
D = ⟨(a,0.7,0.6,0.3),(b,0.9,0.7,0.2),(c,0.5,0.7,0.3)⟩ is a Neutrosophic g−Closed Set but D is not an Nβψ-closed set and also
E = ⟨(a,0.2,0.3,0.5),(b,0.3,0.2,0.6),(c,0.1,0.2,0.9)⟩ is an Nβψ-closed set but E is not a Neutrosophic g−Closed Set.

Theorem 11: If A and B are Nβψ-closed sets, then A ∪ B is Nβψ-closed set.
Proof: If A ∪ B ⊆ U and U is Neutrosophic ψ-open set, then A ⊆ U and B ⊆ U. Since A and B are Nβψ-closed sets, Nβcl(A) ⊆ U and Nβcl(B) ⊆ U and hence Nβcl(A)∪Nβcl(B) ⊆ U. This implies Nβcl(A ∪ B) ⊆ U. Hence A∪B is an Nβψ-closed set in X.

Theorem 12:A neutrosophic set A is Nβψ-closed set then Nβcl(A) – A does not contain any nonempty Nβψ-closed sets.
Proof: Suppose that A is aNβψ-closed set. Let F be an Nβψ-closed set such that F ⊆ Nβcl(A) – A which implies F ⊆ Nβcl(A)∩A^c. Then A ⊆ F^c. Since A is Nβψ-closed set, thenNβcl(A) ⊆F^c .Consequently F ⊆ $(N\psi cl(A))^c$. Then F ⊆ Nβcl(A). Thus F ⊆ Nβcl(A)∩$(N\beta cl(A))^c$ = ϕ. Hence F is empty.

Theorem 13: If A is a Nβψ-closed set in (X,τ_N) and A ⊆ B ⊆ Nβcl(A), then B is Nβψ-closed.
Proof: Let B ⊆ U where U is a Neutrosophic ψ-open set in (X,τ_N). Then A ⊆ B implies A ⊆ U. Since A is aNβψ-closed set, thenNβcl(A)⊆U. Also, A ⊆ Nβcl(B) implies Nβcl(B) ⊆ Nβcl(A). Thus Nβcl(B)⊆ U and so B is an Nβψ-closed set in (X,τ_N).

Theorem 14: If A is Neutrosophic β-open and Nβψ-closed, then A is Nβ-closed set.
Proof: Since A is Neutrosophic β-open and Nβψ-closed, then Nβcl(A) ⊆ A. Therefore Nβcl(A) =A. Hence A is Nβ-closed.

Remark 3: The following diagram Fig. 1. Relationship between various Neutrosophic sets shows the relationship between Nψβ-closed set,Nβψ-closed set with the known existing neutrosophic sets.A → B represents A implies B but not conversely.

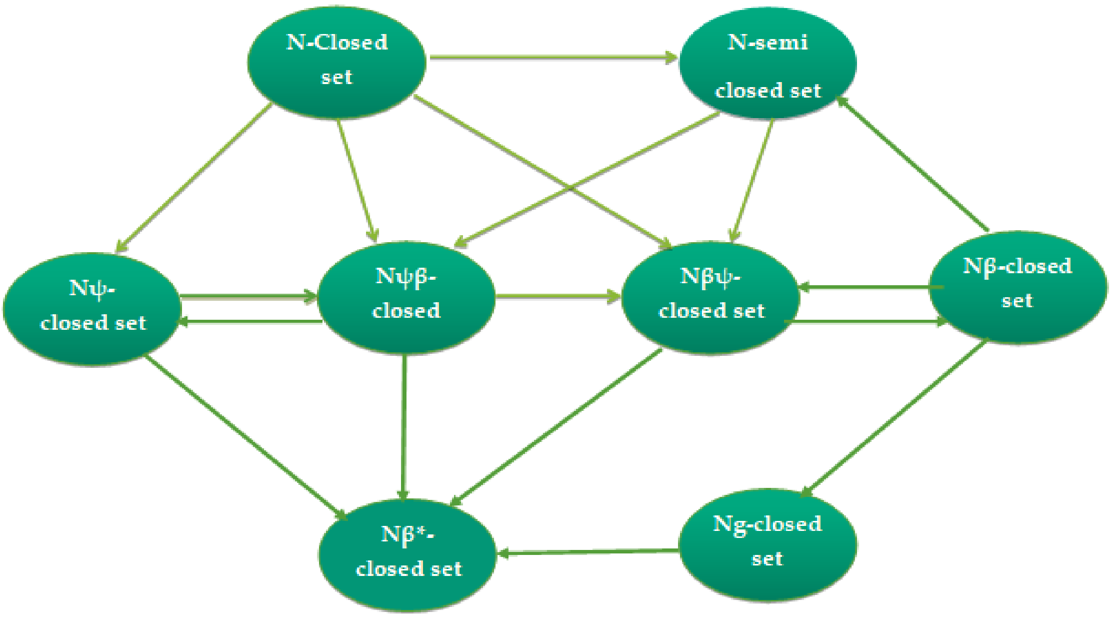

Figure 1. Relationship between various Neutrosophic sets.

Conclusion and Future work:

In this paper, Nβψ-closed and Nψβ-closed sets are introduced and some of its properties were discussed. Some contradicting examples are also given. This idea can be developed and extended in the area of continuous functions, contra continuous functions, homeomorphisms, compactness, connectedness and so on.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Besides, the Figures and images, which are not ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Ramco Institute of Technology.

Author's Contribution Statement:

R.Subasree contributed to the definition and investigation of the research, and K. Basari Kodi contributed to the analysis of results and to the writing of the manuscript.

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دراسة على Nψβ و Nβψ- مجموعات مغلقة في المساحات التوبولوجية العصبية

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الخلاصة:

الهدف من هذه الورقة هو تقديم مفهوم المجموعات N و Nβ المغلقة من حيث المساحات الطوبولوجية للمجموعات. كما تمت مناقشة بعض خصائصه.

الكلمات المفتاحية: المجموعة العصبية، توبولوجيا العصبية، مجموعة N المغلقة، مجموعة Nβ المغلقة.