Abstract:

In this work, the study of corona domination in graphs is carried over which was initially proposed by G. Mahadevan et al. Let \( H \) be a simple graph. A dominating set \( S \) of a graph is said to be a corona-dominating set if every vertex in \( S \) is either a pendant vertex or a support vertices. The minimum cardinality among all corona-dominating sets is called the corona-domination number and is denoted by \( \gamma_{CD}(H) \). In this work, we investigate the corona domination number for some specific types of graphs are given. Also, some results on the corona domination number for some classes of graphs are obtained and the method used in this paper is a well-known number theory concept with some modification this method can also be applied to obtain the results on other domination parameters.

Keywords: Corona dominating set, Dominating set, Jahangir graph, Pendant and Support vertex, Tadpole graph.

Introduction:

Every graph \( H = (X(H), Y(H)) \) considered here are connected finite, undirected, without isolated vertex and loops. A dominating set \( S \) is a set of vertices of \( H \) with the condition that every \( x \in X - S \), \( d(x, S) = 1 \). The minimum cardinality among all the dominating sets is called the domination number of \( H \), denoted by \( \gamma(H) \). The concept of corona domination was introduced by G. Mahadevan et al. \(^2\). The corona domination number (CD number) \( \gamma_{CD} \) is a minimum cardinality of the dominating set \( S \), with the subgraph induced by \( S \) having either pendant or support vertices only. For example see Fig. 1. In recent years many authors have studied the different concept in graph theory and domination theory such as order sum graph \(^3\), tadpole domination \(^4\) etc. Let \( M'(H) \) be the middle graph of \( H \) if two vertices \( x \) and \( y \) in the vertex set of \( M'(H) \) are adjacent if \( x, y \) are in \( Y(H) \) and \( x, y \) are adjacent in \( H \) or \( x \) is in \( X(H) \), \( y \) is in \( y(H) \) and \( x \) is incident to \( y \) in \( H \). The central graph \( C'(H) \) of \( H \) is obtained by subdividing each edge in \( Y(H) \) and joining all the non-adjacent vertices in \( H \). A wheel graph \(^5\) \( W_{3,r} \), \( r \geq 3 \) is obtained by joining a single vertex to all the vertices of a cycle \( C_r \). A graph obtained by attaching a pendant edge at each vertex of \( C_r \) is called helm graph \( H_r \). Joining the pendant vertices of the helm graph to form a cycle will give a closed helm graph \( CH_r \). The friendship graph \( F_r \) is obtained by attaching the \( r \)-copies of \( C_3 \) at a common vertex. A graph constructed by joining a \( C_r \) to an end vertex of \( P_r \) and a bridge is called the tadpole graph \( T_{r,s} \). The Cartesian product of two paths \( P_2 \) and \( P_r \) gives the ladder graph \( P_2 \boxtimes P_r \). A single vertex is adjacent to \( s \) vertices of \( C_r \) at a distance \( r \) to one another on \( C_r \) is called a Jahangir graph \( J_{r,s} \). Consider a sequence of cycle \( C_4 \) say \( C_4^1, C_4^2, C_4^3, \ldots, C_4^r \). a diamond snake graph \( D_r \) is obtained by pasting \( x_1 \) to \( x_{n-1} \) where \( 1 \leq i \leq r \). The \( k^{th} \) power \( H_k \) of the graph \( H \) is graph with same set of vertices and two vertices \( x, y \) in \( H \) are adjacent whenever \( d(x, y) \leq k \). The shadow graph \( S'(H) \) of a graph \( H \) is that a graph obtained by adding a new vertex \( x' \) for each vertex \( x \) of \( H \) and joining \( x' \) to the neighbors of \( x \) in \( H \).
The CD-number for the fan and generalized fan graph $F_{r,s}$, the complement of the ladder graph $P_2 \square P_r$, the GoldnerHarary graph, the CD-number for the central graph of $K_{1,r}$, the CD-number for the Bull graph, the CD-number for the claw graph and the Königsberg bridge graph are 2, the CD-number for m-shadow and m-splitting graph of $P_r$ and $C_r$ is same as the CD-number of $P_r$ and $C_r$, the CD-number for the Moster spindle graph 5, the CD-number for Wagner graph is 3, the CD-number for King's tour graph is 12.

Examine the CD-number for some special graphs

**Theorem 1**: Let $D_r$ be a diamond snake graph. Then $\gamma(D_r) = r + 1$

**Theorem 2**: Let $D_r$ be a diamond snake graph. Then $\gamma_{CD}(D_r) = r + 1$

**Proof**: Let $X(D_r) = \{x_1,x_2,\ldots,x_r,y_1,y_2,\ldots,y_{r-1},z_1,z_2,\ldots,z_{r-1}\}$ then $Y(D_r) = \{x_1y_1,x_1z_1,x_1y_{r-1},x_1z_{r-1}: 1 \leq i \leq r \leq 1\}$. Let $S_1 = \{x_i : i \text{ is even}\} \cup \{y_i : i \text{ is odd}\}$.

Then $S = \begin{cases} S_1 & \text{if } r \text{ is odd,} \\ S_1 \cup \{y_{r-1}\} & \text{if } r \text{ is even.} \end{cases}$ is a CD-set of $D_r$. Thus $\gamma_{CD}(D_r) \leq |S| = r + 1$. Since $\gamma(D_r) = r + 1$, the result follows.

**Theorem 3**: Let $H$ be the middle graph of a friendship graph. Then $\gamma_{CD}(H) = 2r$.

**Proof**: Let $X(H) = \{x_0,x_1,\ldots,x_r,y_0,y_1,\ldots,y_{r-1},y_1,\ldots,y_1\}$, then $Y(H) = \{x_0y_0,x_0y_1,x_0y_2,\ldots,x_0y_{r-1},x_1y_1,x_1y_2,\ldots,x_1y_{r-1},y_0y_1,y_0y_2,\ldots,y_0y_{r-1},y_1y_2,\ldots,y_1y_{r-1},y_1y_2,\ldots,y_1y_{r-1} : 1 \leq i \leq r \text{ and } 1 \leq j \leq r - 1\}$. Let $S = \{y_0i, y_1i : 1 \leq i \leq 2r\}$. Suppose there exist a dominating set $D$ of cardinality at most $d = 2r - 1$, then $D > |S| = 2r$. Hence $|D| \geq d + 1 = 2r$. Therefore the proof.

**Example 1**:

![Middle graph of M(F₄)](image)

**Theorem 4**: For any sand $r \equiv 0 \text{ or } 1 \text{ or } 3 \text{ (mod 4)},$

$$\gamma_{CD}(T_{r,s}) = \begin{cases} \left\lceil \frac{r}{2} \right\rceil + \left\lceil \frac{s}{2} \right\rceil & \text{if } s \equiv 0 \text{ or } 2 \text{ or } 3 \text{(mod 4)}, \\ \left\lceil \frac{r}{2} \right\rceil + \left\lceil \frac{s}{2} \right\rceil & \text{if } s \equiv 1 \text{(mod 4)}. \end{cases}$$

**Proof**: Let $X(T_{r,s}) = \{x_1,x_2,\ldots,x_r,y_1,y_2,\ldots,y_{r-1}\}$. Then $Y(T_{r,s}) = \{x_1x_1,x_1x_2,y_1y_2,\ldots,y_{r-1}\}$, for any $i \leq r - 1, 1 \leq j \leq s - 1$. Let $S_1 = \{x_i : i \equiv 1 \text{ or } 2 \text{ (mod 4)}\} \cup \{y_i : i \equiv 0 \text{ or } 3 \text{ (mod 4)}\}$.

Then
Then $Y(T_{r,s}) = \{x_i x_{i+1}, x_i x_r, y_j y_{j+1}, x_i y_1 : 1 \leq i \leq r, 1 \leq j \leq s - 1\}$. Let $S_1 = \{x_i : i \equiv 0 \text{ or } 3 \pmod{4}\}$.

Then $\gamma_{CD}(T_{r,s}) \leq |S| = \begin{cases} \left\lceil \frac{r}{2} \right\rceil + \left\lceil \frac{s}{2} \right\rceil - 1 & \text{if } s \equiv 0 \text{ or } 2 \pmod{3}, \\ \left\lceil \frac{r}{2} \right\rceil + \left\lceil \frac{s}{2} \right\rceil + 1 & \text{if } s \equiv 1 \pmod{3}. \end{cases}$

Therefore the proof.

**Theorem 5**: For any sand $r \equiv 2 \pmod{3}$,

$$\gamma_{CD}(T_{r,s}) = \begin{cases} \left\lceil \frac{r}{2} \right\rceil + \left\lceil \frac{s}{2} \right\rceil + 1 & \text{if } s \equiv 0 \text{ or } 2 \pmod{3}, \\ \left\lceil \frac{r}{2} \right\rceil + \left\lceil \frac{s}{2} \right\rceil + 1 & \text{if } s \equiv 1 \pmod{3}. \end{cases}$$

**Proof:**

$X(T_{r,s}) = \{x_1, x_2, x_3, \ldots, x_r, y_1, y_2, y_3, \ldots, y_s\}$.

Figure 3. Tadpole $T_{10,6}$

**Theorem 6**: Let $H$ be the middle graph of $W_{1,r}$.

Then

$$\gamma_{CD}(H) = \begin{cases} r + 1 & \text{if } r \equiv 0 \pmod{3}, \\ \left\lceil \frac{r}{3} \right\rceil + \left\lceil \frac{r}{3} \right\rceil + 1 & \text{if } r \equiv 1 \text{ or } 2 \pmod{3}. \end{cases}$$

**Proof:** Let $X(H) = \{x_1, x_2, x_3, \ldots, x_r, y_1, y_2, y_3, \ldots, y_r, z_1, z_2, z_3, \ldots, z_r, x_0\}$

and
Theorem 7: For any \( r \geq 3, y_{CD}(J_{r,2}) = \left[ \frac{r}{2} \right] + 1 \).

**Proof:** Let \( X(J_{r,2}) = \{x_1, x_2, x_3, \ldots, x_{2r}, x\} \) and \( Y(J_{r,2}) = \{xx_j, xi_{i+1}, xi_{i+2}, xi_{i+2r}, : 1 \leq i \leq 2r - 1, j = 1 (mod 2)\} \) is a dominating set for \( J_{r,2} \). Then \( |S| = |x| = \left[ \frac{r}{2} \right] + 1 \). Suppose there exist a dominating set \( D \) of cardinality at most \( |D| = \left[ \frac{r}{2} \right] + 1 \), then \( D \) has an isolated vertex. Thus \(|D| \geq d + 1 = \left[ \frac{r}{2} \right] + 1 \). Therefore the proof.

Theorem 8: For any \( r \geq 3, y_{CD}(J_{r,3}) = r + 1 \).

**Proof:** Let \( X(J_{r,3}) = \{x_1, x_2, x_3, \ldots, x_{2r}, x\} \) and \( Y(J_{r,3}) = \{xx_j, xi_{i+1}, xi_{i+2}, xi_{i+2r}, : 1 \leq i \leq 3r - 1, j = 1 (mod 3)\} \) is a dominating set for \( J_{r,3} \). Then \( |S| = |x| = r + 1 \). Suppose there exist a dominating set \( D \) of cardinality at most \( |D| = r + 1 \), then \( D \) has an isolated vertex. Thus \(|D| \geq d + 1 = r + 1 \). Therefore the proof.

Theorem 9: For any \( r \geq 3, y_{CD}(J_{r,5}) = 2(r + 1) \).

**Proof:** Let \( X(J_{r,5}) = \{x_1, x_2, x_3, \ldots, x_{2r}, x\} \) and \( Y(J_{r,5}) = \{xx_j, xi_{i+1}, xi_{i+2}, xi_{i+3}, xi_{i+4}, xi_{i+2r}, : 1 \leq i \leq 5r - 1, j = 1 (mod 5)\} \). Then \( |S| = |x| = 2(r + 1) \). Suppose there exist a dominating set \( D \) of cardinality at most \( |D| = 2(r + 1) \), then \( D \) has an isolated vertex. Thus \(|D| \geq d + 1 = 2(r + 1) \). Therefore the proof.

Example 3: Consider the graph \( J_{3,5} \) and \( \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}\} \) in Fig. 4 gives the minimum corona dominating set, hence \( y_{CD}(J_{3,5}) = 8 \).

![Figure 4. Jahangir J_{3,5}](image-url)
Example 4:

For Fig.5, the white vertices give the minimum corona dominating set and hence $\gamma_{CD}(J_{3,6}) = 10$ and $\gamma_{CD}(J_{5,6}) = 16$.

**Conclusion:**
In this work, the CD-number for some special graphs and the Jahangir graph are found. Moreover, these results are characterized with other domination parameters, which will be reported in the successive papers.

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**Author’s Declaration:**
- Conflict of interest: None
- We hereby confirm that all the Figures in the manuscript are ours. Besides, the Figures and images, which are not ours, have been given permission for the re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in the Gandhigram Rural Institute.

**Author’s contribution statement:**
This work was carried out in collaboration between all authors. L P designed the idea and developed the theory through discussions with G M. Moreover, G M supervised the study and verified the analytical methods. C S designed the examples to illustrate the results. All authors read and approved the final manuscript.

References:
الخلاصة:
في هذا العمل ، نستمر في دراسة هيمنة الاكليل في الرسوم البيانية التي تم اقتراحها لأول مرة في وثيقة جي ماهاديفان وجميع الآخرين . لنكن رسم بياني بسيط يقال أن المجموعة المستمرة S من الرسم البياني هي مجموعة تهيمن على الاكليل إذا كان كل فكرة في قيمة قلادة أو قيمة دعم يسيئ الحد الأدنى من الأصالة بين جميع مجموعات الهيمنة على الاكليل برقم الهيمنة كورونا ويشار إليه بـ \( \gamma_{CD}(H) \). في هذا العمل ، قمنا بفهم الدقة لرقم هيمنة الاكليل لبعض أنواع معينة من الرسوم البيانية أن أيضًا ، حصلنا على بعض النتائج على رقم هيمنة الاكليل لبعض فئات الرسوم البيانية والطريقة المستخدمة في هذه الورقة هي مفهوم نظرية الأعداد المعروف مع بعض التعديلات يمكن أيضًا تطبيق هذه الطريقة للحصول على النتائج على معلمات الهيمنة الأخرى.

الكلمات المفتاحية: تعيين هيمنة كورونا، تعيين مهيمنة، رسم بياني جاهانجير، قلادة ورأس دعم، رسم بياني الشرغوف.