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Odd Fibonacci edge irregular labeling for some trees obtained from subdivision and vertex identification operations

*M. Uma Devi** *M.Kamaraj* *S.Arockiaraj*

Department of Mathematics, Government Arts & Science College, Sivakasi, Tamilnadu, India.
*Corresponding author: moorthy.uma17@gmail.com
E-mail addresses: kamarajm17366@gmail.com , psarockiaraj@gmail.com
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Abstract:

Let G be a graph with p vertices and q edges and $f: V(G) \rightarrow \{0,1,2,\ldots,k\}$ be an injective function, where k is a positive integer. If the induced edge labeling $f^*: E(G) \rightarrow \left\{F_2, F_4, F_5, F_7, F_8, F_{10}, \ldots, F_{q+\left\lfloor\frac{q}{2}\right\rfloor+1}\right\}$ defined by $f^*(uv) = f(u) + f(v)$, for each $uv \in E(G)$, is a bijection, then the labeling f is called an odd Fibonacci edge irregular labeling of G. A graph which admits an odd Fibonacci edge irregular labeling is called an odd Fibonacci edge irregular graph. The odd Fibonacci edge irregularity strength ofes(G) is the minimum k for which G admits an odd Fibonacci edge irregular labeling. In this paper, the odd Fibonacci edge irregularity strength for some subdivision graphs and graphs obtained from vertex identification is determined.

Keywords: Edge irregular labeling, Edge irregularity strength, Irregular labeling, Odd Fibonacci edge irregular labeling, Odd Fibonacci sequence.

Introduction:

By a graph, it means a finite undirected graph without loops or multiple edges with p vertices and q edges[1-3]. Graph labeling is an assignment of integers to the vertices or edges or both. Rosa[4] introduced the concept of graceful labeling. The Fibonacci numbers can be defined by the linear recurrence $F_n = F_{n-1}+F_{n-2}$, $n \geq 3$. This generates an infinite sequence of integers $F_1=1$, $F_2=1$, $F_3=2$, $F_4=3$, $F_5=5$, $F_6=8$, $F_7=13$, etc. The concept of Fibonacci graceful graphs, total edge Fibonacci irregular labeling, and odd Fibonacci mean labeling based on Fibonacci numbers are studied in[5-8]. Also, some results related to irregular total labeling are obtained from[9]. An odd Fibonacci edge irregular labeling has been introduced in[10] as follows. An odd Fibonacci edge irregular labeling (OFEIL) which is an injective function $f: V(G) \rightarrow \{0,1,2,\ldots,k\}$, k is a positive integer if the induced edge labeling $f^*: E(G) \rightarrow \left\{F_2, F_4, F_5, F_7, F_8, F_{10}, \ldots, F_{q+\left\lfloor\frac{q}{2}\right\rfloor+1}\right\}$ defined by $f^*(uv) = f(u) + f(v)$, for each $uv \in E(G)$, is a bijection. If such labeling exists, then G is called an odd Fibonacci edge irregular graph (OFEIG) and the minimum possible k is called the odd Fibonacci edge irregularity strength ofes(G). Fig. 1 illustrates ofes.

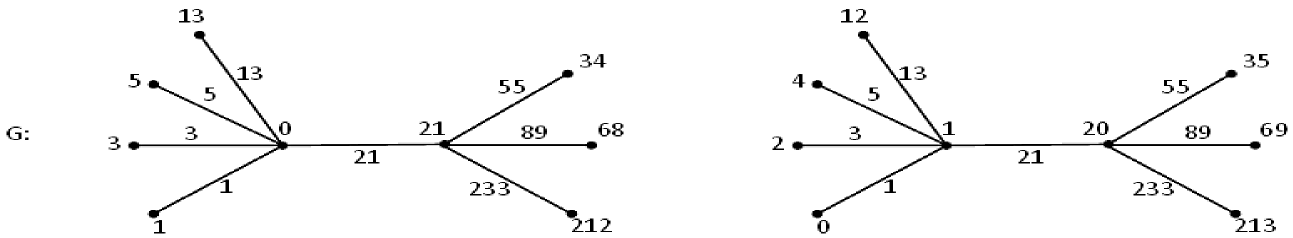

Figure 1. A graph G with two OFEIL and ofes(G) = 212

In[10], the odd Fibonacci edge irregularity strength for the graphs path, star, caterpillar, and bistar graph have been found and the nonexistence of an odd Fibonacci edge irregular labeling for the complete graph and the complete bipartite graph has been discussed. If each edge of a graph G is broken into two by exactly one vertex, then the resultant graph is S(G), the subdivision graph of G. In this paper, the odd Fibonacci edge irregularity strength for some subdivision graphs and graphs obtained from vertex identification is determined.

Theorem. 1: [10] Every path P_n, $(n \geq 2)$ is an odd Fibonacci irregular graph and

$$\mathrm{ofes}(P_n) = \begin{cases} \left\lceil \frac{1}{2} F_{\frac{3n-2}{2}} \right\rceil, & \text{if n is even} \\ F_{n+\left\lfloor \frac{n}{2} \right\rfloor - 1}, & \text{if n is odd.} \end{cases}$$

Theorem. 2:[10] Every star graph $K_{1,n}$, $(n \geq 1)$ is an odd Fibonacci irregular graph and

$$\mathrm{ofes}(K_{1,n}) = F_{q+\left\lfloor \frac{q}{2} \right\rfloor + 1} - 1$$

Main Results:

Theorem. 3: The subdivision of $P_n \odot K_1$, $(n \geq 2)$ is an OFEIG and its strength is F_{6n-3}.

Proof: Let $G = S(P_n \odot K_1)$. In G, $q = 4n - 2$. Let $V(G) = \{v_1, v_2, \ldots, v_n, x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_{n-1}, w_1, w_2, \ldots, w_n\}$ and $E(G) = \{v_i u_i \,/\, 1 \leq i \leq n-1\} \cup \{u_{i-1} v_i \,/\, 2 \leq i \leq n\} \cup \{v_i w_i \,/\, 1 \leq i \leq n\} \cup \{w_i x_i / 1 \leq i \leq n\}$.

Define $f: V(G) \to \{0,1,2,3,\ldots,F_{6n-3}\}$ as follows:
$f(v_1) = 0$, $f(u_1) = 1$, $f(w_1) = F_{6n-4}$, $f(w_2) = F_4$,
$f(x_1) = F_{6n-3}$,
$f(v_i) = F_3 + F_9 + F_{15} + \ldots + F_{6i-9}$, $2 \leq i \leq n$,
$f(u_i) = F_{6i-4} - (F_3 + F_9 + F_{15} + \ldots + F_{6i-9})$, $2 \leq i \leq n-1$,
$f(w_i) = F_{6i-8} - (F_3 + F_9 + F_{15} + \ldots + F_{6i-15})$, $3 \leq i \leq n$ and $f(x_i) = F_{6i-6} + (F_3 + F_9 + F_{15} + \ldots + F_{6i-9})$, $2 \leq i \leq n$.
Then f* is obtained as follows:
$f^*(v_1u_1) = F_2$, $f^*(v_1w_1) = F_{6n-4}$, $f^*(v_2w_2) = F_5$, $f^*(w_1x_1) = F_{6n-2}$, $f^*(w_2x_2) = F_7$,
$f^*(v_iu_i) = F_{6i-4}$, $2 \leq i \leq n-1$, $f^*(u_{i-1}v_i) = F_{6i-8}$, $2 \leq i \leq n$,
$f^*(v_iw_i) = F_{6i-7}$, $3 \leq i \leq n$ and $f^*(w_ix_i) = F_{6i-5}$, $3 \leq i \leq n$.
Since q is even, the last two odd Fibonacci numbers are not consecutive Fibonacci numbers. In order to obtain the minimum value for k, the Fibonacci numbers $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$ and $F_{q+\left\lfloor \frac{q}{2} \right\rfloor-1}$ are to be obtained in the adjacent edges. If $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$ is an edge label of a non-pendant edge, then one adjacent edge of it will be received $F_{q+\left\lfloor \frac{q}{2} \right\rfloor-1}$ as an edge label but the other adjacent edge cannot have any odd Fibonacci number. Hence $f(x_1) = F_{6n-3}$ is the required minimum value for k. □

Theorem. 4: The subdivision of $K_{1,n}$, $(n \geq 1)$ is an OFEIG and its strength is F_{3n}.

Proof: Let $G = S(K_{1,n})$. In G, $q = 2n$.
Let $\{v, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ and $E(G) = \{vv_i \,/\, 1 \leq i \leq n\} \cup \{v_iu_i \,/\, 1 \leq i \leq n\}$.
For n=1, the subdivision of $K_{1,1}$ is a path P_3 and by Theorem 1, $\mathrm{ofes}(P_3) = F_3$.
For n>1, define $f: V(G) \to \{0,1,2,3,\ldots,F_{3n}\}$ as follows:
$f(v) = 0$, $f(v_n) = F_{3n-1}$, $f(u_n) = F_{3n}$,
$f(v_i) = F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1}$, $1 \leq i \leq n-1$ and
$f(u_i) = F_{n+i+\left\lfloor \frac{n-1+i}{2} \right\rfloor} - f(v_i)$, $1 \leq i \leq n-1$.
Then f* is obtained as follows:
$f^*(vv_n) = F_{3n-1}$, $f^*(v_nu_n) = F_{3n+1}$,
$f^*(vv_i) = F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1}$, $1 \leq i \leq n-1$ and
$f^*(v_iu_i) = F_{n+i+\left\lfloor \frac{n-1+i}{2} \right\rfloor}$, $1 \leq i \leq n-1$.
To obtain F_2 as an edge label, it is necessary to assign 0 and 1 to a pair of adjacent vertices.

Case (i) Suppose 0 (or 1) is assigned to the pendant vertex then 1 (or 0) is to be assigned as a label of a vertex whose degree is 2. Therefore, it is necessary to assign 2 (or 3) to the central vertex in order to obtain the edge label F_4. Then the remaining n-1 vertices which are adjacent to central vertices can be assigned by the labels $F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1} - 2$ (or $F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1} - 3$), $3 \leq i \leq n-1$. Hence to obtain the edge label F_{3n+1} the pendant vertex can be labeled by $F_{3n+1} - F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1} + 2$ (or $F_{3n+1} - F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1} + 3$), $3 \leq i \leq n-1$. Suppose one of the adjacent vertices of the central vertex can be labeled with $F_{3n-1} - 2$ (or $F_{3n-1} - 3$) then to obtain the edge label $F_{3n+1}$the pendant vertex can be labeled by $F_{3n} + 2$ (or $F_{3n} + 3$).

Case (ii) If 0 (or 1) is assigned to the central vertex then its n number of adjacent vertices can be assigned by the labels $F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1}$ (or $F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1} - 1$), $1 \leq i \leq n$ in order to obtain the minimum label. Hence to obtain the edge label F_{3n+1} the pendant vertex can be labeled by $F_{3n+1} - F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1}$ (or $F_{3n+1} - F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1} + 1$), $1 \leq i \leq n$. Suppose one of the adjacent vertices of the central vertex can be labeled with F_{3n-1} (or $F_{3n-1} - 1$) then to obtain the edge label F_{3n+1} the pendant vertex can be labeled by F_{3n} (or $F_{3n} + 1$). Hence $f(u_n) = F_{3n}$ is the required minimum value for k. □

The Coconut Tree T(n, m) was obtained by identifying the central vertex of $K_{1,m}$ with a pendant vertex of a path P_n[11].

Theorem. 5: The graph T(n, m), $(n \geq 2, m \geq 1)$ is an OFEIG and its strength is

$$\begin{cases} F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor} - F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor-2} & \text{if } n+m \text{ is odd} \\ \sum_{i=1}^{\frac{n}{2}-1} F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3i+2} + F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3\left(\frac{n}{2}\right)+4} - 1 & \text{if } n+m \text{ is even} \\ \sum_{i=1}^{\frac{n-3}{2}} F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3i+2} + F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3\left(\frac{n-3}{2}\right)} - 1 & \text{if } n(\geq 5), m \text{ are odd} \\ F_{q+\left\lfloor\frac{q}{2}\right\rfloor} - 1 & \text{if } n=3 \text{ and } m \text{ is odd.} \end{cases}$$

Proof: Let G = T(n, m). In G, q = n+m-1.Let $V(G) = \{v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_m\}$ and $E(G) = \{v_iv_{i+1} / 1 \leq i \leq n-1\} \cup \{v_nv_{n+i} / 1 \leq i \leq m\}$.

For n=2, graph G is a star $K_{1,n}$ and its strength is given in Theorem 2. For m=1, G is a path P_n and its strength is given in Theorem 1.

Let $n \geq 3$, $m \geq 2$. To obtain F_2 as an edge label, it is necessary to assign 0 and 1 to a pair of adjacent vertices.

Case (i) n+m is odd (i.e q is even)

Let $f(v_3) = 0$ and $f(v_4) = 1$.

Define $f: V(G) \to \{0,1,2,\dots, F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor} - F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor-2}\}$ as follows:

$f(v_1) = F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor} - F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor-2}$,

$f(v_2) = F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor-2}$, $f(v_i) = \begin{cases} \left\lfloor\frac{1}{2}F_{\frac{3i-5}{2}}\right\rfloor, 3 \leq i \leq n & \text{and } i \text{ is odd} \\ \left\lceil\frac{1}{2}F_{\frac{3i-8}{2}}\right\rceil, 4 \leq i \leq n & \text{and } i \text{ is even} \end{cases}$ and

$f(v_{n+i}) = \begin{cases} F_{n+\left\lfloor\frac{n-3+i}{2}\right\rfloor-2+i} - \left\lfloor\frac{1}{2}F_{\frac{3n-5}{2}}\right\rfloor, 1 \leq i \leq m & \text{and } n \text{ is odd} \\ F_{n+\left\lfloor\frac{n-3+i}{2}\right\rfloor-2+i} - \left\lceil\frac{1}{2}F_{\frac{3n-8}{2}}\right\rceil, 1 \leq i \leq m & \text{and } n \text{ is even} \end{cases}$

Then f* is obtained as follows:

$f^*(v_1v_2) = F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor}$,

$f^*(v_2v_3) = F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor-2}$,

$$f^*(v_iv_{i+1}) = \begin{cases} F_{\frac{3i-5}{2}}, 3 \leq i \leq n-1 & \text{and } i \text{ is odd} \\ F_{\frac{3i-4}{2}}, 4 \leq i \leq n-1 & \text{and } i \text{ is even} \end{cases}$$

and $f^*(v_nv_{n+i}) = F_{n+\left\lfloor\frac{n-3+i}{2}\right\rfloor-2+i}, 1 \leq i \leq m$.

If 0 is assigned to any one of the pendant vertex or its adjacent vertex it leads to an OFEIL with k is more than $F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor} - F_{n+m+\left\lfloor\frac{n+m-1}{2}\right\rfloor-2}$.

Case (ii) n+m is even (i.e q is odd).

Sub Case (i) n, m is even.

Define $f: V(G) \to \{0,1,2,\dots,\ell\}$,

where $\ell = \sum_{i=1}^{\frac{n}{2}-1} F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3i+2} + F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3\left(\frac{n}{2}\right)+4} - 1$ as follows:

$f(v_1) = \sum_{i=1}^{\frac{n}{2}-1} F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3i+2} + F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3\left(\frac{n}{2}\right)+4} - 1$,

$f(v_n) = 1$,

$f(v_i) = F_{m+n-i+\left\lfloor\frac{m+n-i}{2}\right\rfloor+1} - f(v_{i+1}),\ 2 \leq i \leq n-1$

and $f(v_{n+i}) = F_{i+\left\lfloor\frac{i}{2}\right\rfloor+1} - 1, 1 \leq i \leq m$.

Then f* is defined as follows:

$f^*(v_iv_{i+1}) = F_{m+n-i+\left\lfloor\frac{m+n-i}{2}\right\rfloor+1}, 1 \leq i \leq n-1$ and

$f^*(v_nv_{n+i}) = F_{i+\left\lfloor\frac{i}{2}\right\rfloor+1}, 1 \leq i \leq m$.

Sub Case (ii) n, m is odd & $n \geq 5$.

Define $f: V(G) \to \{0,1,2,\dots,\ell\}$,

where $\ell = \sum_{i=1}^{\frac{n-3}{2}} F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3i+2} + F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3\left(\frac{n-3}{2}\right)} - 1$ as follows:

$f(v_1) = \sum_{i=1}^{\frac{n-1}{2}} F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3i+2} + 1$,

$f(v_2) = \sum_{i=1}^{\frac{n-3}{2}} F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3i+2} + F_{q+\left\lfloor\frac{q}{2}\right\rfloor-3\left(\frac{n-3}{2}\right)} - 1$,

$f(v_i) = F_{m+n-i+\left\lfloor\frac{m+n-i}{2}\right\rfloor+1} - f(v_{i+1}), 3 \leq i \leq n-1$,

$f(v_n) = 1$ and

$f(v_{n+i}) = F_{i+\left\lfloor\frac{i}{2}\right\rfloor+1} - 1, 1 \leq i \leq m$.

Then f* is defined as follows:

$f^*(v_iv_{i+1}) = F_{m+n-i+\left\lfloor\frac{m+n-i}{2}\right\rfloor+1}, 1 \leq i \leq n-1$ and

$f^*(v_nv_{n+i}) = F_{i+\left\lfloor\frac{i}{2}\right\rfloor+1}, 1 \leq i \leq m$.

Sub Case (iii) n = 3 & m is odd.

Define $f: V(G) \to \left\{0,1,2,\dots, F_{q+\left\lfloor\frac{q}{2}\right\rfloor} - 1\right\}$ as follows:

$f(v_1) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor-1} + 1$, $f(v_2) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor} - 1$,

$f(v_3) = 1$ and $f(v_{3+i}) = F_{i+\left\lfloor\frac{i}{2}\right\rfloor+1} - 1, 1 \leq i \leq m$.

Then f* is defined as follows:
$f^*(v_1v_2) = F_{q+\lfloor \frac{q}{2} \rfloor+1}$, $f^*(v_2v_3) = F_{q+\lfloor \frac{q}{2} \rfloor}$ and $f^*(v_3v_{3+i}) = F_{i+\lfloor \frac{i}{2} \rfloor+1}$, $1 \le i \le m$.

Since q is odd, the last two odd Fibonacci numbers are consecutive Fibonacci numbers. So case (i) labeling does not give the minimum value for k. Hence by case (ii), if both n and m are even, $f(v_1)$ is the required minimum value for k. If both n ($n \ge 5$) and m are odd, $f(v_2)$ is the required minimum value for k. If n = 3 and m is odd, $f(v_2)$ is the required minimum value for k.□

Let $K_{1,m_1}, K_{1,m_2}, \ldots, K_{1,m_n}$ be the n number of star graphs. Then graph $T(K_{1,m_1}, K_{1,m_2}, \ldots, K_{1,m_n})$ is obtained by identifying an end vertex of each of the K_{1,m_i} stars, $1 \le i \le n$. Let the identified vertex be u, say.

Theorem. 6: $T(K_{1,m_1}, K_{1,m_2}, \ldots, K_{1,m_n})$, ($n \ge 2$, $m_i \ge 2$, $1 \le i \le n$) is an OFEIG and its strength is $F_{\sum_{t=1}^{n} m_t+\left\lfloor \frac{\sum_{t=1}^{n} m_t}{2} \right\rfloor+1} - F_{\sum_{t=1}^{n-1} m_t+\left\lfloor \frac{\sum_{t=1}^{n-1} m_t+1}{2} \right\rfloor+2}$.

Proof: Let G = $T(K_{1,m_1}, K_{1,m_2}, \ldots, K_{1,m_n})$. In G, $q = m_1+m_2+\ldots+m_n$. Then $V(G) = \{v_{i,0}/1 \le i \le n\} \cup \{v_{i,j}/1 \le i \le n, 1 \le j \le m_i - 1\} \cup \{u\}$ and
$E(G) = \{uv_{i,0}/1 \le i \le n\} \cup \{v_{i,0}v_{i,j}/1 \le i \le n, 1 \le j \le m_i - 1\}$.

Define $f: V(G) \to \left\{0,1,2,\ldots, F_{\sum_{t=1}^{n} m_t+\left\lfloor \frac{\sum_{t=1}^{n} m_t}{2} \right\rfloor+1} - F_{\sum_{t=1}^{n-1} m_t+\left\lfloor \frac{\sum_{t=1}^{n-1} m_t+1}{2} \right\rfloor+2}\right\}$ as follows:

$f(V_{1,0}) = 1, f(u) = 0,$ $f(v_{I,j}) = F_{\sum_{t=1}^{i-1} m_t+j+\left\lfloor \frac{\sum_{t=1}^{i-1} m_t+j+1}{2} \right\rfloor+2} - f(v_{I,o}), 2 \le i \le n, 1 \le j \le m_i - 1,$

$f(v_{1,j}) = F_{j+\lfloor \frac{j+1}{2} \rfloor+2} - 1, 1 \le j \le m_1 - 1$ and

$f(v_{i,0}) = F_{\sum_{t=1}^{i-1} m_t+\left\lfloor \frac{\sum_{t=1}^{i-1} m_t+1}{2} \right\rfloor+2}, 2 \le i \le n.$

Then f* is obtained as follows:

$f^*(uv_{1,0}) = 1,$

$f^*(uv_{i,0}) = F_{\sum_{t=1}^{i-1} m_t+\left\lfloor \frac{\sum_{t=1}^{i-1} m_t+1}{2} \right\rfloor+2}, 2 \le i \le n,$

$f^*(v_{1,0}v_{1,j}) = F_{j+\lfloor \frac{j+1}{2} \rfloor+2}, 1 \le j \le m_1 - 1$ and

$f^*(v_{i,0}v_{i,j}) = F_{\sum_{t=1}^{i-1} m_t+j+\left\lfloor \frac{\sum_{t=1}^{i-1} m_t+j+1}{2} \right\rfloor+2}, 2 \le i \le n, 1 \le j \le m_i - 1.$

In order to obtain the minimum value of k, the last m_n odd Fibonacci numbers are to be given for the edge labels of the pendant edge attached at $v_{n,0}$ and $q + \left\lfloor \frac{q}{2} \right\rfloor - m_n$, the greatest odd Fibonacci number. Suppose 0 is assigned to the central vertex of any one of the stars. Then 1 is assigned either to the pendant vertex or the identified vertex u. If 1 is assigned to the pendant vertex then in order to obtain the minimum value of k, the successive odd Fibonacci labeling $F_4, F_5, F_7, \ldots, F_{m_1+\lfloor \frac{m_1}{2} \rfloor+1}$ are obtained in the first m_1 pendant edges of the star K_{1,m_n}. Hence the identified vertex u can get the label $F_{m_1+\lfloor \frac{m_1}{2} \rfloor+1}$. Depending upon this u, the central vertex of the star K_{1,m_n} can get the label. The last m_n odd Fibonacci numbers are to be given for the edge labels incident at $v_{n,0}$ and out of them $F_{q+\lfloor \frac{q}{2} \rfloor-m_n}$ the odd Fibonacci number is the edge label of $uv_{n,0}$. If f(u) is non-zero say x then $f(v_{n,0})$ is less than $F_{q+\lfloor \frac{q}{2} \rfloor-m_n}$. This induces the value of k as $F_{q+\lfloor \frac{q}{2} \rfloor-m_n} - F_{q+\lfloor \frac{q}{2} \rfloor-m_n} + x$. This will be the minimum for x = 0. So ofes(G)= $F_{\sum_{t=1}^{n} m_t+\left\lfloor \frac{\sum_{t=1}^{n} m_t}{2} \right\rfloor+1} - F_{\sum_{t=1}^{n-1} m_t+\left\lfloor \frac{\sum_{t=1}^{n-1} m_t+1}{2} \right\rfloor+2}$. □

Theorem 7: $P_n \Theta mK_1$, ($n \ge 2$, $m \ge 2$) is an OFEIG and its strength is
$$\begin{cases} F_{q+\lfloor \frac{q}{2} \rfloor+1} - F_{q+\lfloor \frac{q}{2} \rfloor-\frac{3m-2}{2}}, & \text{if m is even} \\ F_{q+\lfloor \frac{q}{2} \rfloor+1} - F_{q+\lfloor \frac{q}{2} \rfloor-\frac{3m-3}{2}}, & \text{if m is odd} \end{cases}.$$

Proof: Let G = $P_n \Theta mK_1$. In G, q = n(m+1)-1.
Let V(G) = $\{v_1, v_2, \ldots, v_n\} \bigcup \{u_{i,1}, u_{i,2}, \ldots, u_{i,m} / 1 \le i \le n\}$ and E(G) = $\{v_iv_{i+1} / 1 \le i \le n-1\} \bigcup \{v_iu_{i,t} / 1 \le i \le n, 1 \le t \le m\}$.
Define. $f: V(G) \to \{0,1,2,\ldots,\ell\}$
where $\ell = \begin{cases} F_{q+\lfloor \frac{q}{2} \rfloor+1} - F_{q+\lfloor \frac{q}{2} \rfloor-\frac{3m-2}{2}}, & \text{if m is even} \\ F_{q+\lfloor \frac{q}{2} \rfloor+1} - F_{q+\lfloor \frac{q}{2} \rfloor-\frac{3m-3}{2}}, & \text{if m is odd} \end{cases}$
as follows:
$$f(v_1) = \begin{cases} F_{q+\lfloor \frac{q}{2} \rfloor-\frac{3m-2}{2}}, & \text{if m is even} \\ F_{q+\lfloor \frac{q}{2} \rfloor-\frac{3m-3}{2}}, & \text{if m is odd} \end{cases},$$
$f(v_i) = \sum_{j=1}^{i-2} (-1)^{i+j} F_{j(m+1)+\lfloor \frac{j(m+1)}{2} \rfloor+1}, 3 \le i \le n,$
$f(v_2) = 0, f(u_{2,t}) = F_{t+\lfloor \frac{t}{2} \rfloor+1}, 1 \le t \le m,$ $f(u_{1,t}) =$
$$\begin{cases} F_{q-m+t+\lfloor \frac{q-m+t}{2} \rfloor+1} - F_{q+\lfloor \frac{q}{2} \rfloor-\frac{3m-2}{2}}, & \text{if m is even and } 1 \le t \le m \\ F_{q-m+t+\lfloor \frac{q-m+t}{2} \rfloor+1} - F_{q\lfloor \frac{q}{2} \rfloor-\frac{3m-3}{2}}, & \text{if m is odd and} 1 \le t \le m \end{cases}$$
and

$f(u_{i,t}) = F_{(i-2)(m+1)+t+\left\lfloor\frac{(i-2)(m+1)+t}{2}\right\rfloor+1} - \sum_{j=1}^{i-2}(-1)^{i+j}F_{j(m+1)+\left\lfloor\frac{j(m+1)}{2}\right\rfloor+1}, 3 \le i \le n, 1 \le t \le m.$

Then f* is defined as follows:

$$f^*(v_1v_2) = \begin{cases} F_{q+\left\lfloor\frac{q}{2}\right\rfloor-\frac{3m-2}{2}}, & \text{if m is even} \\ F_{q+\left\lfloor\frac{q}{2}\right\rfloor-\frac{3m-3}{2}}, & \text{if m is odd} \end{cases},$$

$f^*(v_iv_{i+1}) = F_{(i-1)(m+1)+\left\lfloor\frac{(i-1)(m+1)}{2}\right\rfloor+1}, 2 \le i \le n,$

$f^*(v_1u_{1,t}) = F_{q-m+t+\left\lfloor\frac{q-m+t}{2}\right\rfloor+1}, 1 \le t \le m,$

$f^*(v_2u_{2,t}) = F_{t+\left\lfloor\frac{t}{2}\right\rfloor+1}, 1 \le t \le m$ and

$f^*(v_iu_{i,t}) = F_{(i-2)(m+1)+t+\left\lfloor\frac{(i-2)(m+1)+t}{2}\right\rfloor+1}, 3 \le i \le n, 1 \le t \le m.$

If 0 is a vertex label of the path, the value of k is more than $f(u_{1,m})$. Hence $f(u_{1,m}) = \begin{cases} F_{q+\left\lfloor\frac{q}{2}\right\rfloor+1} - F_{q+\left\lfloor\frac{q}{2}\right\rfloor-\frac{3m-2}{2}}, & \text{if m is even} \\ F_{q+\left\lfloor\frac{q}{2}\right\rfloor+1} - F_{q+\left\lfloor\frac{q}{2}\right\rfloor-\frac{3m-3}{2}}, & \text{if m is odd} \end{cases}$ is the required minimum value for k. □

The graph P(1,2, …, n) is obtained by joining i pendant vertices at each of i^th vertex of the path P_n.

Theorem. 8: P(1,2, …, n) , (n ≥ 2) is an OFEIG and its strength is $\begin{cases} F_{q+\left\lfloor\frac{q}{2}\right\rfloor}, & \text{if } n \equiv 2,3 (\text{mod} 4) \\ F_{q+\left\lfloor\frac{q}{2}\right\rfloor} - F_{q+\left\lfloor\frac{q}{2}\right\rfloor-5}, & \text{if } n \equiv 0,1 (\text{mod} 4) \end{cases}.$

Proof: Let G = P(1,2, …, n) .

In G, $q = n - 1 + \frac{n(n+1)}{2}$.

Let V(G) = $\{v_1, v_2, \dots, v_n\} \cup \{u_{i,t} / 1 \le i \le n, 1 \le t \le i\}$ and E(G) = $\{v_iv_{i+1} / 1 \le i \le n-1\} \cup \{v_iu_{i,t} / 1 \le i \le n, 1 \le t \le i\}$.

Case (i) $n \equiv 2,3 (\text{mod } 4)$ (i.e q is even).

Define $f: V(G) \to \{0,1,2,\dots, F_{q+\left\lfloor\frac{q}{2}\right\rfloor}\}$ as follows:

$f(v_1) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor-1}$, $f(v_2) = 0$, $f(u_{1,1}) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor}$,

$f(v_i) = \sum_{j=1}^{i-2}(-1)^{j+1}F_{\sum_{t=1}^{i-j}t+(i-j-2)+\left\lfloor\frac{\sum_{t=1}^{i-j}t+(i-j-2)}{2}\right\rfloor+1}, 3 \le i \le n$, $f(u_{2,t}) = F_{t+\left\lfloor\frac{t}{2}\right\rfloor+1}$ and

$f(u_{i,t}) = F_{\sum_{j=1}^{i-1}j+(t+i-3)+\left\lfloor\frac{\sum_{j=1}^{i-1}j+(t+i-3)}{2}\right\rfloor+1} - f(v_i), 3 \le i \le n, 1 \le t \le i.$

Then f* is defined as follows:

$f^*(v_1v_2) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor-1}$, $f^*(v_1u_{1,1}) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor+1}$,

$f^*(v_2u_{2,t}) = F_{t+\left\lfloor\frac{t}{2}\right\rfloor+1}, 1 \le t \le 2$

$f^*(v_iv_{i+1}) = F_{\sum_{t=1}^{i}t+(i-2)+\left\lfloor\frac{\sum_{t=1}^{i}t+(i-2)}{2}\right\rfloor+1}, 2 \le i \le n-1$ and

$f^*(v_iu_{i,t}) = F_{\sum_{j=1}^{i-1}j+(t+i-3)+\left\lfloor\frac{\sum_{j=1}^{i-1}j+(t+i-3)}{2}\right\rfloor+1}, 3 \le i \le n, 1 \le t \le i.$

As in the proof of Theorem 3, $f(u_{1,1})$ is the required minimum value for k.

Case (ii) $n \equiv 0,1 (\text{mod } 4)$ (i.e q is odd).

Define $f: V(G) \to \{0,1,2,\dots, F_{q+\left\lfloor\frac{q}{2}\right\rfloor} - F_{q+\left\lfloor\frac{q}{2}\right\rfloor-5}\}$ as follows:

$f(v_1) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor} - F_{q+\left\lfloor\frac{q}{2}\right\rfloor-5}$, $f(v_2) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor-5}$,

$f(v_3) = 0$, $f(u_{1,1}) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor-1} + F_{q+\left\lfloor\frac{q}{2}\right\rfloor-5}$,

$f(v_i) = \sum_{j=1}^{i-3}(-1)^{j+1}F_{\sum_{t=3}^{i-j}t+(i-j-2)+\left\lfloor\frac{\sum_{t=3}^{i-j}t+(i-j-2)}{2}\right\rfloor+1}, 4 \le i \le n,$

$f(u_{2,t}) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor-4+t} - F_{q+\left\lfloor\frac{q}{2}\right\rfloor-5}, 1 \le t \le 2,$

$f(u_{3,t}) = F_{t+\left\lfloor\frac{t}{2}\right\rfloor+1}, 1 \le t \le 3$ and

$f(u_{i,t}) = F_{\sum_{j=3}^{i-1}j+(t+i-3)+\left\lfloor\frac{\sum_{j=3}^{i-1}j+(t+i-3)}{2}\right\rfloor+1} - f(v_i), 4 \le i \le n, 1 \le t \le i.$

Then f* is defined as follows:

$f^*(v_1v_2) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor}$, $f^*(v_2v_3) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor-5}$,

$f^*(v_iv_{i+1}) = F_{\sum_{t=3}^{i}t+(i-2)+\left\lfloor\frac{\sum_{t=3}^{i}t+(i-2)}{2}\right\rfloor+1}, 3 \le i \le n-1$, $f^*(v_1u_{1,1}) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor+1}$,

$f^*(v_2u_{2,t}) = F_{q+\left\lfloor\frac{q}{2}\right\rfloor-4+t}, 1 \le t \le 2$, $f^*(v_3u_{3,t}) = F_{t+\left\lfloor\frac{t}{2}\right\rfloor+1}, 1 \le t \le 3$ and

$f^*(v_iu_{i,t}) = F_{\sum_{j=3}^{i-1}j+(t+i-3)+\left\lfloor\frac{\sum_{j=3}^{i-1}j+(t+i-3)}{2}\right\rfloor+1}, 4 \le i \le n, 1 \le t \le i.$

Since q is odd, the last two odd Fibonacci numbers are consecutive Fibonacci numbers. So case (i) labeling does not give the minimum value for k. If 0 is a vertex label of the path, then the value of k is more than $F_{q+\lfloor\frac{q}{2}\rfloor} - F_{q+\lfloor\frac{q}{2}\rfloor-5}$ in order to obtain the OFEIL. Hence ofes(G) = $F_{q+\lfloor\frac{q}{2}\rfloor} - F_{q+\lfloor\frac{q}{2}\rfloor-5}$. □

Conclusion:

In this paper, the odd Fibonacci edge irregularity strength for some subdivision graphs and graphs obtained from vertex identification is determined.

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Author's contributions statement:

M.UD analyzes the existence of OFEIL by collecting the papers, proposing the conjecture, and trying to prove it. M.K edited the manuscript. S.A read and approved the final manuscript by verifying it.

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تبويب حافات فيبوناتشي الفردية غير المنتظمة لبعض الاشجار المستحصلة من عمليات التقسيم الفرعي وتحديد قمة الرأس

م. أوما ديفي * **م. كاماراج** **س. أروكياراج**

قسم الرياضيات ، كلية الفنون والعلوم الحكومية ، سيفاكاسي ، تاميل نادو ، الهند.

الخلاصة:

ليكن **G** رسما بيانيا برؤوس **p** وحواف **q** و $f: V(G) \to \{0,1,2,\dots,k\}$ دالة متباينة وشاملة , حيث **k** عدد صحيح موجب. إذا كانت تسمية الحافة المستحثة $f^*: E(G) \to \left\{F_2, F_4, F_5, F_7, F_8, F_{10}, \dots, F_{q+\lfloor \frac{q}{2} \rfloor+1}\right\}$ معرفة ب $f^*(uv) = f(u) + f(v)$, لكل $uv \in E(G)$, المتباينة, فان علامة التبويب f تدعى وضع علامات غير منتظمة على حافة فيبوناتشي الفردية ل **G**. الرسم البياني الذي يعترف بوضع علامات غير منتظمة لحافة فيبوناتشي الفردية يسمى الرسم البياني غير المنتظم لحافة فيبوناتشي الفردية. قوة عدم انتظام حافة فيبوناتشي الفردية هي الحد الأدنى **k** الذي يعترف **G** بوضع علامات غير منتظمة لحافة فيبوناتشي الفردية. في هذا البحث ، تم تحديد قوة عدم انتظام حافة فيبوناتشي الفردية لبعض الرسوم البيانية للتقسيمات الفرعية والرسوم البيانية التي تم الحصول عليها من تحديد الرأس.

الكلمات المفتاحية: تبويب حافات غير منتظمة, قوة عدم انتظام الحافة, وضع العلامات غير النظامية, تبويب حافات فيبوناتشي الفردية غير المنتظمة, متتابعة فيبوناتشي الفردية.