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A Study on Co – odd (even) Sum Degree Edge Domination Number in Graphs

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Abstract:
An edge dominating set $T \subseteq E(G)$ of a graph $G = (V, E)$ is said to be an odd (even) sum degree edge dominating set (osded (esded) - set) of G if the sum of degree of all the edges in T is an odd (even) number. The odd (even) sum degree edge domination number $\gamma'_{osd}(G)$ ($\gamma'_{esd}(G)$) is the minimum cardinality taken over all odd (even) sum degree edge dominating sets of G and is defined as zero if no such odd (even) sum degree edge dominating set exists in G. In this paper, the odd (even) sum degree domination concept is extended on the co-dominating set E-T of a graph G, where T is an edge dominating set of G. The corresponding parameters co-odd (even) sum degree edge dominating set, co-odd (even) sum degree edge domination number and co-odd (even) sum degree edge domination value are defined. Further, the exact values of the above said parameters are found for some standard classes of graphs. The bounds of the co-odd (even) sum degree edge domination number are obtained in terms of basic graph terminologies. The co-odd (even) sum degree edge dominating sets are characterized. The relationships with other edge domination parameters are also studied.

Keywords: Co-odd (even) sum degree edge dominating set, Co-odd (even) sum degree edge domination number, Co-odd (even) sum degree edge domination value, Odd (even) sum degree edge domination number, Odd (even) sum degree edge domination value.

Introduction:
Let G = (V, E) be a simple, connected, finite, and undirected graph. The maximum and minimum degree of a graph G is respectively denoted by $\Delta(G)$ and $\delta(G)$. The cardinality of the vertex (edge) set of a graph G is called the order (size) of G and is denoted by p(q). A graph with p vertices and q edges is called a (p, q) graph. A regular graph is a graph where each vertex has the same number of neighbors. If 't' is a vertex of G, then its degree is denoted by deg(t). A set $T \subseteq E(G)$ is an edge dominating set if every edge in E(G) – T is adjacent to at least one edge in T. The edge domination number $\gamma'(G)$ is the minimum cardinality of an edge dominating set in G. The edge dominating set with cardinality $\gamma'(G)$ is denoted as γ' – set of G. The edge set E (G) - T is said to be a co-edge dominating set[1-3] of G. All the basic graph terminologies are used in the sense of Harary[4]. In the year 2003, the odd domination number of graph G is introduced by Yair Caro and William F. Klostermeyer[5]. The odd geo-domination number of a graph is introduced by Anto Kinsley A and Karthika K John in the year 2020[6]. Motivated by the notion of the above parameters and their applicability, the odd (even) sum degree edge dominating set (oded (eded) - set) is introduced by posting odd (even) condition on the sum of degree of edges of edge dominating set of a graph G. In this paper, by extending the above concept on co-edge dominating set, the co-odd (even) sum degree edge dominating set is defined for a graph G[7-9]. Then the corresponding co-odd (even) sum degree edge domination number and value are defined and studied. All the graphs considered in this article are referred from Joshep A. Gallian[10]. In this paper, co-odd (even) sum degree edge domination numbers are found for some standard classes of graphs such as Path, Cycle, Wheel, Comb, Star, Crown, Friendship, Helm, Triangular Snack, Fan, Book, Dumbbell, Flag, Todpole[11]and Caterpillar graphs. Further, the bounds of the above parameters are obtained and the relationships between some of the existing edge domination parameters are studied.

Also, the co-odd (even) sum degree edge dominating sets are characterized.

Co-odd (even) Sum Degree Edge Domination

Definition. 1:

An edge dominating set T(G) of a graph G = (V, E) is said to be a co-odd (even) sum degree edge dominating set (cosded – set (cesded - set)) of G if the sum of the degree of all edges in E-T is an odd (even) number. The co-odd (even) sum degree edge domination number γ'_{cosd} (G) (γ'_{cesd} (G)) is the minimum cardinality taken over all cosded (cesded) - set of G and it is defined as zero if no such cosded (cesded) - set exists in G. The co-odd (even) sum degree edge dominating set with cardinality γ'_{cosd} (G) (γ'_{cesd} (G)) is denoted by γ'_{cosd} – set (γ'_{cesd} - set) of G.

Definition. 2:

Let T be a γ'_{cosd} – set of G. Then the minimum sum of the degree of the edges of the set E-T is said to be co-odd (even) sum degree edge domination value and it is denoted by $S'_{cosd}(G)$ ($S'_{cesd}(G)$) of G.

Definition. 3:

A graph G is said to be a co-odd (even) sum degree edge dominating graph (cosded- graph (cesded- graph)) if it has at least one co-odd (even) sum degree edge dominating set of G.

Example. 1:

For the graph G in Fig. 1, $T_o = \{t_3, t_7, t_{12}\}$ is a γ'_{cosd} – set, and hence, γ'_{cosd} (G) = 3 and S'_{cosd} (G) = $\sum_{t\in T-T_o} \deg(t)$ = 43. $T_e = \{t_3, t_7, t_{13}\}$ is a γ'_{cesd} – set and hence γ'_{cesd} = 3 and S'_{cesd} (G) = $\sum_{t\in T-T_e} \deg(t)$ = 44.

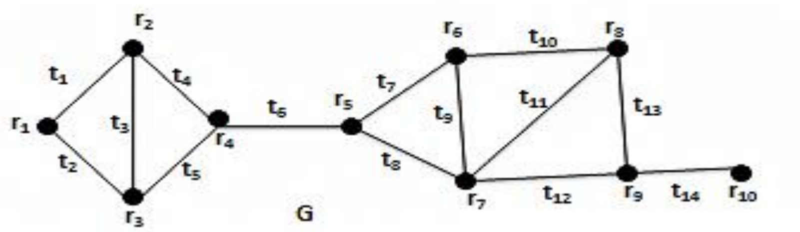

Figure 1. Co-odd (even) sum degree edge domination number and value

Observations:

1. Every graph has at least one cesded – set since every graph has an even number of odd degree edges.
2. Cosded- set need not exist in all graphs, for example, Cycle C_n, n ≥ 3 does not have a cosded – set.

Theorem. 1: For path P_n, n ≥ 2

$$(i)\ \gamma'_{cosd}(P_n)=\left\lfloor\frac{n-1}{3}\right\rfloor+1\ ,\ n\geq 3.;\quad S'_{cosd}(P_n)=\begin{cases}4\left\lfloor\frac{n}{3}\right\rfloor-1 & ,\ n\equiv 2(mod 3)\\ 4\left\lfloor\frac{n}{3}\right\rfloor-3 & ,\ n\equiv 0,1\ (mod 3)\end{cases}$$

$$(ii)\ \gamma'_{cesd}(P_n)=\begin{cases}\left\lfloor\frac{n-2}{3}\right\rfloor+1 & ,n\geq 5\\ 1 & ,\ n=4\\ 2 & ,n=3\end{cases};\ S'_{cesd}(P_n)=\begin{cases}4\left\lfloor\frac{n}{3}\right\rfloor-2, & n\equiv 2(mod 3)\\ 4\left\lfloor\frac{n-2}{3}\right\rfloor, & n\equiv 0(mod 3)\\ 4\left\lfloor\frac{n-4}{3}\right\rfloor+2, & n\equiv 1(mod 3)\end{cases}$$

Proof:

Let G be a path graph with at least three vertices. Let E(G) = $\{t_1, t_2, t_3, \dots, t_{n-1}\}$be the edge set of G with $\deg(t_1) = \deg(t_{n-1}) = 1$ and the remaining edges have degree 2.

Claim (i): Let $X_1 = \{t_{3i-1} / i = 1, 2, \dots, \left\lfloor\frac{n-1}{3}\right\rfloor\}$ and $X_2 = \{t_{n-1}\}$ be two edge sets of G. Note that X_1 is an edge dominating set of G and $X_1 U X_2$ is an odd degree edge dominating set of G.

Hence, γ'_{cosd} (G) ≤ $|X_1 U X_2|$ = $\left\lfloor\frac{n-1}{3}\right\rfloor$+1 1

On the other hand, suppose Y is a γ'_{cosd} set of G. Then for edge domination Y must have at least $\{t_{3i-1}/i=1,2,..,\left\lfloor\frac{n-1}{3}\right\rfloor\}$ edges and for the odd sum degree edge domination Y must contain $\{t_{n-1}\}$.

Hence, γ'_{cosd} (G) = |Y| ≥ $\left\lfloor\frac{n-1}{3}\right\rfloor$+1 2

From Eq.1 and 2, gives, γ'_{cosd} (G) = $\left\lfloor\frac{n-1}{3}\right\rfloor$+1.

Further, if T is γ'_{cosd} – set of G then the co-odd sum degree edge domination value

$$S'_{cosd}(G)=\sum_{t\in E-T}\deg(t)=\begin{cases}4\left\lfloor\frac{n}{3}\right\rfloor-1 & ,\ n\equiv 2(mod 3)\\ 4\left\lfloor\frac{n}{3}\right\rfloor-3 & ,\ n\equiv 0,1\ (mod 3)\end{cases}$$

It completes Claim (i).

Claim (ii): When n = 3, the edge set X = {t_1, t_2} is a γ'_{cesd} – set of G and hence, γ'_{cesd} (G) = 2.

When n = 4, the edge set X = {t_2} itself a γ'_{cesd} – set of G and hence, γ'_{cesd} (G) =1.

Let $n \geq 5$. Suppose X = {t_{3i-1}/ i=1,2,.., $\left\lfloor\frac{n-2}{3}\right\rfloor$} U {$t_{n-2}$}. Then X is an even degree edge dominating set of G. Hence, γ'_{cesd} (G) ≤ |X| = $\left\lfloor\frac{n-2}{3}\right\rfloor$+1 3

On the other hand, suppose Y is a γ'_{cesd} set of G. Then Y must contain at least {t_{3i-1}/ i=1,2,.., $\left\lfloor\frac{n-2}{3}\right\rfloor$}U {$t_{n-2}$} edges. Hence γ'_{cesd}(G)=|Y| ≥ $\left\lfloor\frac{n-2}{3}\right\rfloor$+1 4

Then the result in Claim (ii) is proved from Eq. 3 and 4.

Further, if T is γ'_{cesd} – set of G then the co-even sum degree edge domination value

$$S'_{cesd}(G)=\sum_{t\in E-T}\deg(t)=\begin{cases}4\left\lfloor\frac{n}{3}\right\rfloor-2, n\equiv 2(mod3)\\ 4\left\lfloor\frac{n-2}{3}\right\rfloor, n\equiv 0(mod3)\\ 4\left\lfloor\frac{n-4}{3}\right\rfloor+2, n\equiv 1(mod3)\end{cases}$$

It completes Claim (ii).

□

Theorem. 2: For the cycle graph C_n, $n \geq 3$

(i) $\gamma'_{cosd}(C_n)=0$; $S'_{cosd}(C_n)=0$

(ii) $\gamma'_{cesd}(C_n)=\left\lfloor\frac{n+2}{3}\right\rfloor$;

$$S'_{cesd}(C_n)=\begin{cases}4\left\lfloor\frac{n}{3}\right\rfloor, & n\equiv 0,1(mod3)\\ 4\left\lfloor\frac{n}{3}\right\rfloor+2, & n\equiv 2(mod3)\end{cases}$$

Proof:

Let G be a cycle graph C_n with at least three vertices. Let E(G) = {$t_1,t_2,\ldots,t_n$}be the edge set of G. Since all the edges in E(G) are in even degrees gives $\gamma'_{cosd}(C_n)=0$.

Let T = {t_{3i-2}/ i=1,2,..., $\left\lfloor\frac{n+2}{3}\right\rfloor$} be an edge set of G. Note that T is an edge dominating set of G and also T is an even degree edge dominating set of G. Hence, γ'_{cesd} (G) ≤ $|X|$=$\left\lfloor\frac{n+2}{3}\right\rfloor$.

Therefore, γ'_{cesd} (G) ≤ $\left\lfloor\frac{n+2}{3}\right\rfloor$ 1

On the other hand, suppose Y is a γ'_{cesd} – set of G. Then for edge domination Y must have at least {t_{3i-2}/ i=1,2,..., $\left\lfloor\frac{n+2}{3}\right\rfloor$} edges and also Y is an even degree dominating set.

Hence,

γ'_{cesd}(G) = $|Y|$ ≥ $\left\lfloor\frac{n+2}{3}\right\rfloor$ 2

From Eq.1 and 2, γ'_{cesd}(G) = $\left\lfloor\frac{n+2}{3}\right\rfloor$.

Further, if T is γ'_{cesd} – set of G then the co-even sum degree edge domination value

$$S'_{cesd}(G)=\sum_{t\in E-T}\deg(t)=\begin{cases}4\left\lfloor\frac{n}{3}\right\rfloor, & n\equiv 0,1(mod3)\\ 4\left\lfloor\frac{n}{3}\right\rfloor+2, & n\equiv 2(mod3)\end{cases}$$

It completes Claim (ii).

Theorem. 3: For Pan graph $T_{m,1}$, $m \geq 3$

(i) $\gamma'_{cosd}(T_{m,1})$= $\left\lfloor\frac{m-1}{3}\right\rfloor$+1 ;

$$S'_{cosd}(T_{m,1})=\begin{cases}4\left\lfloor\frac{m}{3}\right\rfloor+3, & m\equiv 0,1(mod3)\\ 4\left\lfloor\frac{m}{3}\right\rfloor+5, & m\equiv 2(mod3)\end{cases}$$

(ii) $\gamma'_{cesd}(T_{m,1})$=$\left\lceil\frac{m+1}{3}\right\rceil$;

$$S'_{cesd}(T_{m,1})=\begin{cases}4\left\lfloor\frac{m}{3}\right\rfloor+4, & m\equiv 1(mod3)\\ 4\left\lfloor\frac{m}{3}\right\rfloor+6, & m\equiv 2(mod3)\\ 4\left\lfloor\frac{m-3}{3}\right\rfloor+6, & m\equiv 0(mod3)\end{cases}$$

Proof:

Let G be a Pan graph with at least four vertices. Let E(G) = {$t_1,t_2,\ldots,t_m$} U {s} where deg(t_1) = deg(t_m) = 3 , deg(t_i) = deg(s) = 2, i = 2,…,m-1 and s is adjacent to t_1 and t_m.

Claim (i): Let T_1= {t_{3i-2}/ i=1,2,..., $\left\lfloor\frac{m-1}{3}\right\rfloor$} and T_2 ={s} be two edge sets of G. Note that T= T_1UT_2 is an edge dominating set of G. Since deg (t_1) = 3 and the remaining edges in T are in even degree gives T is a cosded – set of G.

Hence

$\gamma'_{cosd}(G) \leq |T_1 U T_2|$ =$\left\lfloor\frac{m-1}{3}\right\rfloor$ +1 1

On the other hand, let Y be a γ'_{cosd} – set of G. Then for edge domination Y must have at least $\left\lfloor\frac{m-1}{3}\right\rfloor$+1 edge and hence

$\gamma'_{cosd}(G) = |Y|$ ≥ $\left\lfloor\frac{m-1}{3}\right\rfloor$+1 2

Then the result in Claim (i) is followed by Eq.1 and 2.

Further, if T is γ'_{cosd} – set of G then the co-odd sum degree domination value

$$S'_{cosd}(G)=\sum_{t\in E-T}\deg(t)=\begin{cases}4\left\lfloor\frac{m}{3}\right\rfloor+3, & n\equiv 0,1(mod3)\\ 4\left\lfloor\frac{m}{3}\right\rfloor+5, & n\equiv 2(mod3)\end{cases}$$

It completes Claim (i).

Claim (ii): Let T = {t_{3i-1}/ i=1,2,..., $\left\lceil\frac{m+1}{3}\right\rceil$}. Then T is an even degree edge dominating set of G.

$$\gamma'_{cesd}(G) \le |T| = \left\lceil \frac{m+1}{3} \right\rceil \qquad 3$$

On the other hand, suppose S is a γ'_{cesd} - set of G, then for edge domination S must have at least $\{t_{3i-1}/i=1,2,..., \left\lceil \frac{m+1}{3} \right\rceil\}$ edges and also S is an even degree edge dominating set of G. It gives,

$$\gamma'_{cesd}(G) \ge \left\lceil \frac{m+1}{3} \right\rceil \qquad 4$$

Then the result in Claim (ii) is followed by Eq.3 and 4.

Further, if T is γ'_{cesd} – set of G then the co-even sum degree edge domination value

$S'_{cesd}(G) = \sum_{t\in E-T} \deg(t) =$

$$\begin{cases} 4\left\lfloor \frac{m}{3} \right\rfloor + 4, & n \equiv 1(mod 3) \\ 4\left\lfloor \frac{m}{3} \right\rfloor + 6, & n \equiv 2(mod 3) \\ 4\left\lfloor \frac{m-3}{3} \right\rfloor + 6, & n \equiv 0(mod 3) \end{cases}$$

It completes Claim (ii).

□

By similar arguments, the exact value of $\gamma'_{cosd}(G)$ and $\gamma'_{cesd}(G)$ for some standard classes of graphs are obtained and presented below.

Proposition. 1:

(i) For star graph S_n, $n \ge 2$

$\gamma'_{cosd}(S_n) = \begin{cases} 0, & n \equiv 1(mod 2) \\ 1, & n \equiv 0\ (mod 2) \end{cases}$; $S'_{cosd}(S_n) = (n-2)^2, n \equiv 1(mod 2)$

$\gamma'_{cesd}(S_n) = \begin{cases} 1, & n \equiv 1(mod 2) \\ 2, & n \equiv 0\ (mod 2) \end{cases}$; $S'_{cesd}(S_n) = \begin{cases} (n-2)^2, & n \equiv 0(mod 2) \\ (n-2)(n-3), & n \equiv 1\ (mod 2) \end{cases}$

(ii) For friendship graph F_n, $n \ge 2$

$\gamma'_{cosd}(F_n) = 0$; $S'_{cosd}(F_n) = 0$

$\gamma'_{cesd}(F_n) = n$; $S'_{cesd}(F_n) = (2n)^2$

(iii) For crown graph C_n^+, $n \ge 2$

$\gamma'_{cosd}(C_n^+) = 0$; $S'_{cosd}(C_n^+) = 0$

$\gamma'_{cesd}(C_n^+) = \left\lceil \frac{n}{2} \right\rceil$;

$$S'_{cesd}(C_n^+) = \begin{cases} 8\left\lfloor \frac{n}{2} \right\rfloor, & n \equiv 0\ (mod 2) \\ 8\left\lfloor \frac{n}{2} \right\rfloor + 2, & n \equiv 1\ (mod 2) \end{cases}$$

(iv) For dumbell graph D_n, $n \ge 3$

$\gamma'_{cosd}(D_n) = 0$; $S'_{cosd}(D_n) = 0$

$\gamma'_{cesd}(D_n) = 2$;

$$S'_{cesd}(D_n) = \begin{cases} 8\left\lfloor \frac{n}{3} \right\rfloor + 4, & n \equiv 0,2\ (mod 3) \\ 8\left\lfloor \frac{n}{3} \right\rfloor, & n \equiv 1\ (mod 3) \end{cases}$$

(v) For flag graph Fl_n, $n \ge 3$

$\gamma'_{cosd}(Fl_n) = \left\lceil \frac{n}{3} \right\rceil$;

$$S'_{cosd}(Fl_n) = \begin{cases} 4\left\lfloor \frac{n}{3} \right\rfloor + 3, & n \equiv 0,1\ (mod 3) \\ 4\left\lceil \frac{n}{3} \right\rceil + 1, & n \equiv 2\ (mod 3) \end{cases}$$

$\gamma'_{cesd}(Fl_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1$;

$$S'_{cesd}(Fl_n) = \begin{cases} 4\left\lceil \frac{n}{3} \right\rceil, & n \equiv 1\ (mod 3) \\ 4\left\lfloor \frac{n}{3} \right\rfloor + 2, & n \equiv 0,2\ (mod 3) \end{cases}$$

(vi) For bistar graph $B_{n,n}$, $n \ge 2$

$\gamma'_{cosd}(B_{n,n}) = 0$; $S'_{cosd}(B_{n,n}) = 0$

$\gamma'_{cesd}(B_{n,n}) = 1$; $S'_{cesd}(B_{n,n}) = 2(n)$

(vii) For caterpillar graph Sn_m, $m \ge 3$

$\gamma'_{cosd}(Sn_m) = 0$; $S'_{cosd}(Sn_m) = 0$

$\gamma'_{cesd}(Sn_m) = 1$; $S'_{cesd}(Sn_m) = (m-2)^2$

Bounds and Characterization of Odd (Even) Sum Degree Edge Dominating Sets of G

The following result gives the relation between the edge domination number and co-odd (even) degree edge domination number.

Theorem. 4: For any graph G, (a) $\gamma'(G) \le \gamma'_{cosd}(G)$; (b) $\gamma'(G) \le \gamma'_{cesd}(G)$

Proof:

Since every cosded-set and cesded-set is an edge dominating set of G, proves the results immediately.

For (a), the bound is sharp for star S_3.

For (b), the bound is sharp for cycle C_3 □

Theorem. 5: Let G be a r – regular graph with $r > 0$. Then

(a) $\gamma'_{cosd}(G) = 0$

(b) $\gamma'_{cesd}(G) = \gamma'(G)$.

Proof:

Since G has only even degree edges gives there is no cosded – set in G. Hence, γ'_{cosd} (G) = 0. At the same time, every γ' – set of G is an γ'_{cesd} –set, and hence $\gamma'_{cesd}(G) = \gamma(G)$. □

Theorem. 6: Let G be a graph with a *cosded – set* T_o and a *cesded – set* T_e. Then (i) T-T_o have odd number of odd degree edges. (ii) T-T_e has either even numbers of odd degree edges or any numbers of even degree edges or both.

Proof:

Claim (i): Suppose T_o have no odd degree edges or it has an even number of odd degree edges than the sum of degrees of edges of T_o become even. Which contradicts T-T_o be a cosded – set. Hence T-T_o has an odd number of odd degree edges.

Claim (ii): Suppose T_e has an odd number of odd degree edges then T-T_e is a cosded – set of G but not cesded – set. Therefore, T_e has an even number of odd degree edges. Since the sum of even numbers is even given T_e may contain any number of even degree edges. Further, the above two combinations also give T-T_e a cesded – set of G. It provestheclaim. □

Theorem. 7: Let G be a graph with distinct minimum and maximum edge degrees $\delta'(G)$ and $\Delta'(G)$. If T_o and T_e are γ'_{cosd} – set and γ'_{cesd} – set of G then

(i) $\frac{1}{\Delta'(G)} S'_{cosd}(t) \leq q - \gamma'_{cosd}(G) \leq \frac{1}{\delta'(G)} S'_{cosd}(t)$

(ii) $\frac{1}{\Delta'(G)} S'_{cesd}(t) \leq q - \gamma'_{cesd}(G) \leq \frac{1}{\delta'(G)} S'_{cesd}(t)$

Proof:

Since $\delta'(G)$ and $\Delta'(G)$ are the minimum and maximum edge degrees of graph G, gives

$\delta'(G) \leq \deg(t) \leq \Delta'(G)$, if $\delta' \neq \Delta'$ and e ∈G.

Therefore,

$$|T - T_o|\delta'(G) \leq \gamma'_{cosd}(G) \leq |T - T_o|\Delta'(G) \quad 1$$

From the left inequality, gives $|q - \gamma'_{cosd}| \leq \frac{1}{\delta'(G)} S'_{cosd}$

Therefore, $\gamma'_{cosd}(G) \leq \frac{1}{\delta'(G)} S'_{cosd} - q$

Hence, $\gamma'_{cosd}(G) \geq q - \frac{1}{\delta'(G)} S'_{cosd}$ 2

From the right inequality in (1), gives $|T - T_o| \geq \frac{1}{\Delta'(G)} S'_{cosd}$

That is $q - \gamma'_{cosd}(G)\ \Delta'(G) \geq S'_{cosd}$

$\frac{1}{\Delta'(G)} S'_{cosd} \leq q - \gamma'_{cosd}(G)$

Therefore, $\gamma'_{cosd}(G) \leq q - \frac{1}{\Delta'(G)} S'_{cosd}$ 3

Hence, from Eq.2 and 3,

$\frac{1}{\Delta'(G)} S'_{cesd}(t) \leq q - \gamma'_{cesd}(G) \leq \frac{1}{\delta'(G)} S'_{cesd}(t)$

By similar arguments, one can prove the result,

$\frac{1}{\Delta'(G)} S'_{cesd}(t) \leq q - \gamma'_{cesd}(G) \leq \frac{1}{\delta'(G)} S'_{cesd}(t)$.

For (i), the lower and upper bound is sharp for star S_3.

For (ii), the lower and upper bound is sharp for cycle C_3. □

Theorem. 8: Let *G* be a graph. Then (i) *G* is a *cosded – graph* if and only if *G* has at least one odd degree edge. (ii) *G* is a *cesded – graph* if and only if *G* has at least even numbers of odd degree edges (or) degree of all the edges are even.

Proof:

Claim (i): Let G is a cosded – graph, then there exists a cosded – set T-T_o of G. Assume that G has no odd degree edge. Since G is connected with at least two edges gives G has only edges of even degree. Then there is no cosded – set existing in G, which contradicts that G has a cosded–set T-T_o. Therefore, G has at least one odd degree edge. The converse is obvious.

Claim (ii): Let G is a cesded – graph, then there exists a cesded – set T-T_e of G. Suppose G has odd degree edges. Then for the existence of T-T_e, G must have an even number of odd degree edges. On the other hand, suppose G has only even degree edges then the result is immediate. The converse of the result is obvious. □

Theorem. 9: Let *G* is a graph with a*n osded – set* T_o and *esded – set* T_e. If $S'_{osd}(t)$ be an even number then

(a) T -T_o is a *cosded – set* of *G* and (b) T -T_e is a *cesded – set* of *G*

Proof:

Let $S'_{cosd}(t) = 2m$, m∈N. Then $S'_{osd}(t) + S'_{cosd}(t) = 2m$. Then $S'_{osd}(t) = 2m - S'_{cosd}(t)$. Since T_o be an osded – set of G given $\sum_{e \in E_0} \deg(e)$ is an odd number. It gives $\sum_{e \in E - E_0} \deg(e)$ is also an odd number. This shows that T - T_o be a cosded – set of G. By a similar argument, one can prove that T – T_e is a cesded – set of G. □

Conclusion:

In this paper, the exact values of the co-odd (even) sum degree edge domination number and co-odd (even) sum degree edge domination value are found for some standard classes of graphs described below: The bounds of the co-odd (even) sum degree edge domination number are obtained. The co-odd (even) sum degree edge dominating sets are characterized. The relationships with other edge domination parameters are also determined.

Authors' declaration:

- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Besides, the Figures and images, which are not ours, have been given permission for the re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Nehru Memorial College, India.

Authors’ contribution statement:

This work was carried out in collaboration between all authors. V.M.S. introduces the parameter. P. U. found the bounds of the parameter and D. I. wrote and edited the manuscript with revisions idea. All authors read and approved the final manuscript

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دراسة حول Co – الفردي (الزوجي) مجموع درجة الحافة للرقم المهيمن في الرسوم البيانية

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الخلاصة:

يقال إن المجموعة المهيمنة على الحافة $X \subseteq E(G)$ من الرسم البياني $G = (V, E)$ هي مجموعة مهيمنة على حافة المجموع الفردي (الزوجي) (osded (esded) - set) من G إذا كان مجموع درجة جميع الحواف في X هو رقم فردي (زوجي). المجموع الفردي (الزوجي) لدرجة الحافة لرقم الهيمنة $\gamma'_{osd}(G)$ $(\gamma'_{esd}(G))$ هو الحد الأدنى من الكاردينالية المأخوذة على جميع المجموعات المهيمنة على حافة المجموع الفردي (الزوجي) من G ويتم تعريفه على أنه صفر إذا لم يكن هناك مثل هذا المجموع المهيمن على حافة المجموع الفردي (الزوجي) في G. في هذا البحث ، تم توسيع مفهوم هيمنة درجة المجموع الفردي (الزوجي) على المجموعة المهيمنة المشتركة E-T للرسم البياني G ، حيث T هي مجموعة مهيمنة على الحافة من G. . تم تعريف المعلمات المقابلة لـCo-الفردي (الزوجي) مجموع درجة الحافة المهيمنة على مجموعة ، و Co - الفردي (الزوجي) مجموع درجة الهيمنة على الحافة و Co- الفردي (الزوجي) مجموع درجة قيمة هيمنة الحافة. علاوة على ذلك ، تم العثور على القيم الدقيقة للمعلمات المذكورة أعلاه لبعض الفئات القياسية من الرسوم البيانية. يتم الحصول على حدود رقم هيمنة حافة المجموع Co - الفردي (الزوجي) من حيث مصطلحات الرسم البياني الأساسية. تم تميز المجموعات المهيمنة على حافة المجموع Co - الفردي (الزوجي). كما تتم دراسة العلاقات مع معلمات هيمنة الحافة الأخرى.

الكلمات المفتاحية: مجموع الهيمنة على حافة المجموع الفردي (الزوجي)، رقم هيمنة حافة المجموع Co - الفردي (الزوجي)، قيمة هيمنة حافة المجموع Co - الفردي (الزوجي)، رقم هيمنة حافة المجموع الفردي (الزوجي) ، قيمة هيمنة حافة المجموع الفردي (الزوجي).