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Existence of Fixed Points for Expansive Mappings in Complete Strong Altering JS-metric space

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Abstract:
The paper aims at initiating and exploring the concept of extended metric known as the Strong Altering JS-metric, a stronger version of the Altering JS-metric. The interrelation of Strong Altering JS-metric with the b-metric and dislocated metric has been analyzed and some examples have been provided. Certain theorems on fixed points for expansive self-mappings in the setting of complete Strong Altering JS-metric space have also been discussed.

Keywords: Altering distance, b-metric space, Dislocated metric space, Expansive mapping, Fixed points, Strong Altering JS-metric.

Introduction:
Theory of fixed points is a significant branch of analysis which plays a key role in the non-linear analysis. Fixed point theorems has immense applications in various areas of mathematics like differential equations, integral equations, numerical analysis, mathematical economics, game theory etc. A fixed point of a self-map T is a point that remains unaltered by T. The contraction mapping emerged when the lipschitz constant is restricted to atmost 1. These maps possess a unique fixed point under complete metric space. Contractive and expansive maps have been studied over the years and fixed point theorems for such functions are determined. Several concepts of metric spaces have been playing significant roles in various disciplines of science and social science. Most importantly it plays a key role in fixed point theory. Wang et al.[1] examined the fixed-point results using Expansive type mappings in their paper in 1984 which motivated several researchers to study the expansive mappings[2-5] in various extensions of metric spaces. Dislocated metric space explored by Hitzler and Seda[6] in 2000 and the b-metric space initiated by Bhaktin[7] and Czerwik[8] independently are some notable generalized metric spaces available in the literature. Many more results on fixed points in some generalized metric spaces have been studied in recent years[9-12]. Khan[13] utilized the altering distance function in proving theorems of fixed points in complete metric spaces. These functions are initiated in the aim of altering the distances between any two points. Many researchers[14-16] have worked using these functions in various contractive and expansive conditions.

Definition. 1: Let η be a non-negative function from $\mathbb{R}^+$ to itself where, $\mathbb{R}^+$ is the set of non-negative reals then ϕ is an altering distance function if

(1) η is continuous and monotonically non-decreasing,
(2) $\eta(p) = 0 \Leftrightarrow p = 0$,
(3) $h.p^r \leq \eta(p)$ for all $p > 0$ and $h,r > 0$ are constants.

The collection of altering distance functions is denoted by Σ.

Example. 1:
(1) The trivial example, $\eta: \mathbb{R}^+ \to \mathbb{R}^+$ defined by η(p)=p, the identity function is an altering distance function.
(2) Let, $\eta: \mathbb{R}^+ \to \mathbb{R}^+$ given by $\eta(p) = 2p$, is an altering distance function.

The following is the definition of Altering JS-metric space which is an extended metric utilizing the altering distance function.

Consider a set S and a mapping $J_\phi: S \times S \to [0, \infty]$. For every $j \in S$, let the set $C(J_\phi, S, j)$ be defined as follows.

$$C(J_\phi, S, j) = \{\{j_n\} \subset S: \lim_{n\to\infty} J_\phi(j_n, j) = 0\}$$

Definition. 2: Let S be a non-void set and the map $J_\phi: S \times S \to [0, \infty]$ be given. Then J_ϕ is an Altering JS-metric on S if it satisfies the axioms for all $j, p \in S$:

(A1) $J_\phi\ (j, p) = 0 \Rightarrow j = p$,

(A2) $J_\phi(j, p) = J_\phi(p, j)$,

(A3) there exists $\phi \in \Sigma$ and $k > 0$ such that

$$j, p \in S, \{j_n\} \in C(J_\phi, S, j) \Rightarrow \phi(J_\phi(j, p)) \leq \limsup_{n\to\infty} [\phi(k.J_\phi(j_n, p))]$$

The pair (S, J_ϕ) is an Altering JS-metric space.

Suppose the set $C(J_\phi, S, j)$ is empty for every element of S, then (S, J_ϕ) it is enough to check the axioms (A1) and (A2).

Definition. 3:[7,8] A b-metric on a non-void set S is a fuction $J_\phi: S \times S \to [0, \infty)$ satisfying the conditions, for all $j, p, r \in S$:

(i) $J_\phi(j, p) = 0 \Rightarrow j = p$,

(ii) $J_\phi(j, p) = J_\phi(p, j)$,

(iii) There exists $b \geq 1$, such that, $J_\phi(j, p) \leq b[J_\phi(j, r) + J_\phi(r, p)]$.

Then (S, J_ϕ) is the b-metric space.

Example. 2: For the set $S = \mathbb{R}$, the function $J_\phi: S \times S \to [0, \infty)$ defined by $J_\phi(j, p) = (j - p)^2$ forms a b-metric space for $b = 2$.

Definition. 4: [6] Consider a non-void set S. The function $J_\phi: S \times S \to [0, \infty)$ is a dislocated metric if it satisfies the conditions, for $j, p, r \in S$:

(i) $J_\phi(j, p) = 0 \Rightarrow j = p$,

(ii) $J_\phi(j, p) = J_\phi(p, j)$,

(iii) $J_\phi(j, p) \leq J_\phi(j, r) + J_\phi(r, p)$.

The ordered pair (S, J_ϕ) is known as dislocated metric space.

Example. 3: The function $J_\phi: S \times S \to [0, \infty)$ on a non-void ordered set given by $J_\phi(j, p) = \max\{j, p\}$ is a dislocated metric.

Now, what happens when the Altering JS-metric is compressed? This question motivated to develop the idea of stronger version of this generalized metric. In this work, the Strong Altering JS-metric space has been elucidated which is a compressed version of Altering JS-metric space. Some properties of this generalized metric space have been given and have also showed some Fixed point theorems for Expansive mappings in the setting of complete Strong Altering JS-metric space.

Strong Altering JS-metric space:

This section deals with the Strong Altering JS-metric space and some fundamental results on this generalized space.

Definition. 5: Let S be a non-void set and $J_\phi: S \times S \to [0, \infty]$ be a given mapping. The map J_ϕ is a Strong Altering JS-metric on S if it satisfies the following axioms, for all $j, p \in S$:

(S1) $J_\phi(j, p) = 0 \Rightarrow j = p$,

(S2) $J_\phi(j, p) = J_\phi(p, j)$,

(S3) there exists $0 < k \leq 1$ and $\phi \in \Sigma$ such that

$$j, p \in S, \{j_n\} \in C(J_\phi, S, j)$$
$$\Rightarrow \phi(J_\phi(j, p)) \leq \limsup_{n\to\infty} [\phi(k.J_\phi(j_n, p))]$$

The pair (S, J_ϕ) is a Strong Altering JS-metric space.

Suppose the set $C(J_\phi, S, j)$ is empty for every element of S and if the the axioms (S1) and (S2) are satisfied, then both Altering JS-metric and Strong Altering JS-metric are one and the same.

Example. 4: Let $S = \mathbb{R}^+$ and $J_\phi: S \times S \to [0, \infty]$ be defined by $J_\phi(j, p) = |j^2 - p^2|$.

Let $\phi(t) = t$. $|j^2 - p^2| = 0 \Rightarrow j^2 - p^2 = 0 \Rightarrow j = p$.

Also J_ϕ is symmetric. Hence axioms (S1) and (S2) are satisfied. If $\{j_n\} \in C(J_\phi, S, j)$ then $\lim_{n\to\infty} J_\phi(j_n, j) = 0$.

Now,

$$|j^2 - p^2| \leq |j^2 - j_n^2 + j_n^2 - p^2| \leq |j^2 - j_n^2| + |j_n^2 - p^2|$$
$$2|j^2 - p^2| \leq 2|j^2 - j_n^2| + 2|j_n^2 - p^2|$$

$$\phi(|j^2 - p^2|) \leq \phi(|s^2 - j_n^2|) + \phi(|j_n^2 - t^2|)$$

$$\phi(J_\phi(j, p)) \leq \phi(J_\phi(j, j_n)) + \phi(J_\phi(j_n, p))$$

By letting $n \to \infty$ the inequality becomes,

$$\phi(J_\phi(j, p)) \leq \limsup_{n\to\infty} \phi((J_\phi(j_n, p))$$

Axiom (S3) is satisfied for $k = 1$.

Thus, the pair (S, J_ϕ) is a Strong Altering JS-metric space.

The example with the function J_ϕ given by $J_\phi(j, p) = \max\{j, p\}$, satisfies the axioms (S1) – (S3) and hence is a Strong Altering JS-metric space. In general, it can be noted that a dislocated metric is a Strong Altering JS-metric and is proved via the following proposition.

Proposition. 1: A dislocated metric space is always a Strong Altering JS-metric space.

Proof: Suppose that (S, J_ϕ) is a dislocated metric space. The axioms (S1) and (S2) are trivially true. Now, let there be a sequence $\{j_n\} \in C(J_\phi, S, j)$ where $s \in S$. Since (S, J_ϕ) is a dislocated metric space, so by the triangle inequality,

$$J_\phi(j,p) \le J_\phi(j,j_n) + J_\phi(j_n,p)$$

Letting $n \to \infty$, $J_\phi(j,p) \le \limsup_{n\to\infty}[J_\phi(j,j_n) + J_\phi(j_n,p)]$

$$\le \limsup_{n\to\infty}[J_\phi(j,j_n)] + \limsup_{n\to\infty}[J_\phi(j_n,p)]$$

$$= \limsup_{n\to\infty}[J_\phi(j_n,p)]$$

For any $\phi \in \Sigma$, the inequality becomes,

$$\phi\left(J_\phi(j,p)\right) \le \phi\left(\limsup_{n\to\infty} J_\phi(j_n,p)\right)$$

Since ϕ is continuous and monotonically non-decreasing, thus

$$\phi\left(J_\phi(j,p)\right) \le \limsup_{n\to\infty} \phi[J_\phi(j_n,p)].$$

Hence, (S, J_ϕ) is a Strong Altering JS-metric space with $k = 1$.

Proposition. 2: Any metric space is a Strong Altering JS-metric space.

Proof: The proof is identical to the above proposition.

Remark. 1: It is obvious that a Strong Altering JS-metric is an Altering JS-metric where the constant $k \le 1$, but the converse is not necessarily true. It has been proved[12] that every b-metric is an Altering JS-metric. But a b-metric is not necessarily a Strong Altering JS-metric. The function defined by $J_\phi(j,p) = (j-p)^2$, is a b-metric and also an Altering JS-metric but not a Strong Altering JS-metric. It is observed that the Strong Altering JS-metric compresses the distance whereas a b-metric magnifies it. Thus a b-metric becomes a Strong Altering JS-metric in the case where the constant $b = 1$.

Proposition. 3: If J_ϕ is a Strong Altering JS-metric space then so is $n.J_\phi$ for positive finite n.

Proof: Let $n.J_\phi(s,t) = 0$ which implies $J_\phi(s,t) = 0 \Rightarrow s = t$. Hence, the axiom (S1) is true.

Also, $J_\phi(s,t) = J_\phi(t,s) \Rightarrow n.J_\phi(s,t) = n.J_\phi(t,s)$. Thus, (S2) is true.

From axiom (S3),

$$\phi\left(J_\phi(s,t)\right) \le \limsup_{n\to\infty}\left[\phi\left(k.J_\phi(s_n,t)\right)\right]$$

Multiplying both sides by n where $n \in (0,\infty)$,

$$n.\phi\left(J_\phi(s,t)\right) \le n.\limsup_{n\to\infty}\left[\phi\left(k.J_\phi(s_n,t)\right)\right]$$

By continuity and monotonicity of ϕ,

$$\phi\left(n.J_\phi(s,t)\right) \le \limsup_{n\to\infty}\left[\phi\left(k.n.J_\phi(s_n,t)\right)\right]$$

Hence, $n.J_\phi$ is an Altering JS-metric.

Similarly, the following proposition can be proved.

Proposition. 4: If J_ϕ is a Strong Altering JS-metric space then so is $\sqrt[n]{J_\phi}$ for positive finite n.

The following definition gives the fundamental topological concepts of the convergence of a sequence and Cauchy sequence in Strong Altering JS-metric space.

Definition. 6: A sequence $\{j_n\}$ in a Strong Altering JS-metric space (S, J_ϕ) is:

(i) J_ϕ-convergent to $j \in S$ if $\lim_{n\to\infty} J_\phi(j_n,j) = 0$ i.e. $\{j_n\} \in C(J_\phi, S, j)$,

(ii)(ii) J_ϕ-Cauchy if for given $\varepsilon > 0, \exists\ s \in N$ such that $J_\phi(j_n,j_m) < \varepsilon$ for all $m,n \ge s$.

i.e. $\lim_{n,m\to\infty} J_\phi(j_n,j_m) = 0$

The Strong Altering JS-metric space (S, J_ϕ) is J_ϕ-complete if every J_ϕ-Cauchy sequence in S is J_ϕ-convergent.

For the space $(\mathrm{S}, \mathrm{J}_\phi)$, with $S = \mathbb{R}$ and $J_\phi(j,p) = \max\{j,p\}$, the set $C(J_\phi, S, j)$ is non-void only for the point 0 which is the only point that is not dislocated in the space. Thus all the sequences converging to zero are the J_ϕ-convergent sequences.

Expansive mappings in Strong Altering JS-metric space:

This section presents the Exapansive mappings and fixed point results for such mappings in complete Strong Altering JS-metric space. These results are not always true in an Altering JS-metric space but they hold for the Strong Altering JS-metric space. The compression in the space aids the fixed point of the exapansive mapping.

Definition. 7: Let (S, J_ϕ) be a Strong Altering JS-metric space and a self-map on S be T. Then T is a α-Kannan expansion if for every $j,p \in S$, there exists $\alpha \ge 1/2$ such that

$$J_\phi(Tj,Tp) \ge \alpha[J_\phi(j,Tj) + J_\phi(p,Tp)]$$

Theorem. 1: Suppose T be a α-Kannan expansion on a Complete Strong Altering JS-metric space $(S,J_\phi)$with α ≥ 1 then it has a fixed point in S.

Proof: Let $j_n = Tj_{n+1}$ be a sequence in S. If $j_n = j_{n+1}$, then the result becomes obvious. Let $j_n \neq j_{n+1}$. Now consider,

$$J_\phi(j_{n-1},j_n) = J_\phi\,(Tj_n,Tj_{n+1})$$
$$\geq \alpha[J_\phi\,(j_n,Tj_n) + J_\phi(j_{n+1},Tj_{n+1})]$$
$$= \alpha[J_\phi\,(j_n,j_{n-1}) + J_\phi\,(j_{n+1},j_n)]$$
$$\Rightarrow \left(\frac{1-\alpha}{\alpha}\right)J_\phi\,(j_n,j_{n-1}) \geq J_\phi(j_{n+1},j_n)$$
$$\Rightarrow J_\phi(j_{n+1},j_n) \leq \left(\frac{1-\alpha}{\alpha}\right)J_\phi(j_n,j_{n-1}) \qquad 1$$

Now,

$$J_\phi(j_{n-2},j_{n-1}) = J_\phi(Tj_{n-1},Tj_n) \geq \alpha[J_\phi\,(j_{n-1},Tj_{n-1}) + J_\phi(j_n,Tj_n)]$$

$$= \alpha[J_\phi\,(j_{n-1},j_{n-2}) + J_\phi(j_n,j_{n-1})]$$

$$\Rightarrow \left(\frac{1-\alpha}{\alpha}\right)J_\phi\,(j_{n-1},j_{n-2}) \geq J_\phi(j_n,j_{n-1})$$
$$\Rightarrow J_\phi(j_n,j_{n-1}) \leq \left(\frac{1-\alpha}{\alpha}\right)J_\phi(j_{n-1},j_{n-2}) \qquad 2$$

From Eq.1 and Eq.2,

$$J_\phi\,(j_{n+1},j_n) \leq \left(\frac{1-\alpha}{\alpha}\right)^2 J_\phi(j_{n-1},j_{n-2}) \qquad 3$$

Consider,

$$J_\phi(j_{n-3},j_{n-2}) = J_\phi(Tj_{n-2},Tj_{n-1}) \geq \alpha[J_\phi(j_{n-2},Tj_{n-2}) + J_\phi(j_{n-1},Tj_{n-1})] \geq \alpha[J_\phi(j_{n-2},j_{n-3}) + J_\phi(j_{n-1},j_{n-2})]$$
$$\left(\frac{1-\alpha}{\alpha}\right)J_\phi(j_{n-3},j_{n-2}) \geq J_\phi(j_{n-1},j_{n-2})$$

$$J_\phi(j_{n-1},j_{n-2}) \leq \left(\frac{1-\alpha}{\alpha}\right)J_\phi(j_{n-3},j_{n-2})$$

Thus Eq.3 becomes,

$$J_\phi(j_{n+1},j_n) \leq \left(\frac{1-\alpha}{\alpha}\right)^3 J_\phi(j_{n-3},j_{n-2})$$

Continuing this way,

$$J_\phi(j_{n+1},j_n) \leq \left(\frac{1-\alpha}{\alpha}\right)^n J_\phi(j_0,j_1)$$

As $n \to \infty, J_\phi(j_{n+1},j_n) \to 0$. Now to prove $\{j_n\}$ is a Cauchy sequence, consider,

$$J_\phi(j_n,j_m) = J_\phi(Tj_{n+1},Tj_{m+1})$$
$$\geq \alpha[J_\phi(j_{n+1},Tj_{n+1}) + J_\phi(j_{m+1},Tj_{m+1})]$$
$$= \alpha[J_\phi(j_{n+1},j_n) + J_\phi(j_{m+1},j_m)]$$
$$-J_\phi(j_n,j_m) \leq -\alpha[J_\phi(j_{n+1},j_n) + J_\phi(j_{m+1},j_m)]$$

As $n,m \to \infty, J_\phi(j_n,j_m) \to 0$. Hence $\{j_n\}$ is a Cauchy sequence. By the completeness of (S,J_ϕ), $j_n \xrightarrow{J_\phi} j$, where $j \in S$.

By Axiom (S3) of Strong Altering JS-metric, there exists $\phi \in \Sigma$ and $0 < k \leq 1$ such that

$$\phi\left(J_\phi(j,Tj)\right) \leq \limsup_{n\to\infty}\left[\phi\left(k.J_\phi(j_n,Ts)\right)\right]$$

$$= \phi\left[\limsup_{n\to\infty}\left(k.J_\phi(j_n,Tj)\right)\right]$$

Since ϕ is monotonically non decreasing,

$$J_\phi(j,Tj) \leq \limsup_{n\to\infty}[k.J_\phi(j_n,Tj)]$$
$$= \limsup_{n\to\infty}[k.J_\phi(Tj_{n+1},Tj)]$$
$$-(J_\phi(j,Tj)) \geq -\limsup_{n\to\infty}[k.J_\phi(Tj_{n+1},Tj)]$$
$$\geq -k.\limsup_{n\to\infty}\left[\alpha\left(J_\phi(j_{n+1},Tj_{n+1}) + J_\phi(j,Tj)\right)\right]$$
$$= -k.\,\alpha\limsup_{n\to\infty}[J_\phi(j_{n+1},j_n) + J_\phi(j,Tj)]$$
$$= k.\,\alpha\liminf_{n\to\infty}[(-J_\phi(j_{n+1},j_n)) + (-J_\phi(j,Tj))]$$

Since $\alpha \geq 1$,

$$-(J_\phi(j,Tj)) \geq k.\liminf_{n\to\infty}\left[\left(-J_\phi(j_{n+1},j_n)\right) + \left(-J_\phi(j,Tj)\right)\right]$$
$$-\left(J_\phi(j,Tj)\right) \geq k.\left[\liminf_{n\to\infty}\left(-J_\phi(j_{n+1},j_n)\right) + \liminf_{n\to\infty}\left(-J_\phi(j,Tj)\right)\right]$$

Since $J_\phi(j_{n+1},j_n) \to 0$,

$$\liminf_{n\to\infty}\left(J_\phi(j_{n+1},j_n)\right) = \limsup_{n\to\infty}\left(J_\phi(j_{n+1},j_n)\right)$$
$$= \lim_{n\to\infty}\left(J_\phi(j_{n+1},j_n)\right) = 0$$
$$\Rightarrow -\left(J_\phi(j,Tj)\right) \geq -k.\left(J_\phi(j,Tj)\right)$$
$$\Rightarrow \left(J_\phi(j,Tj)\right) \leq k.\left(J_\phi(j,Tj)\right)$$

Since $k < 1$, this inequality is possible only if $J_\phi(j,Tj) = 0$

$\Rightarrow J_\phi(j,Tj) = 0 \Rightarrow j = Tj$

The expansive mapping T has fixed point in the Strong Altering JS-metric space S.

Theorem. 2: Let T be an expansive map on a complete Strong Altering JS-metric space (S,J_ϕ) satisfying

$$J_\phi(Tj,Tp) \geq \delta\left[J_\phi(j,Tj) + J_\phi(p,Tp) + \phi[J_\phi(j,p)]\right]$$

for all $j,p \in S$ where $\phi \in \Sigma$ and $\delta \geq 1$. Then it has a fixed point in S.

Proof: Let $\{j_n\} \in S$ be a sequence defined by $j_n = Tj_{n+1}$.

Now,

$$J_\phi(Tj_1,Tj_2) \geq \delta\left[J_\phi(j_1,Tj_1) + J_\phi(j_2,Tj_2) + \phi[J_\phi(j_1,j_2)]\right]$$

$\Rightarrow J_\phi(j_0,j_1) \geq \delta\left[J_\phi(j_1,j_0)+J_\phi(j_2,j_1)+\phi[J_\phi(j_1,j_2)]\right]$

$\Rightarrow (1-\delta)J_\phi(j_0,j_1) \geq \delta\{J_\phi(j_2,j_1)+\phi[J_\phi(j_1,j_2)]\}$

$\geq \delta\{J_\phi(j_2,j_1)+J_\phi(j_2,j_1)\}$

$= 2\delta\{J_\phi(j_2,j_1)\}$

$\Rightarrow \left(\frac{1-\delta}{2\delta}\right)J_\phi(j_0,j_1) \geq J_\phi(j_1,j_2)$

$\Rightarrow J_\phi(j_1,j_2) \leq \left(\frac{1-\delta}{2\delta}\right)J_\phi(j_0,j_1)$ 4

$J_\phi(Tj_2,Tj_3) \geq \delta\left[J_\phi(j_2,Tj_2)+J_\phi(j_3,Tj_3)+\phi[J_\phi(j_2,j_3)]\right]$

$\Rightarrow J_\phi(j_1,j_2) \geq \delta\left[J_\phi(j_2,j_1)+J_\phi(j_3,j_2)+\phi[J_\phi(j_2,j_3)]\right]$

$\Rightarrow (1-\delta)J_\phi(j_1,j_2) \geq \delta\{J_\phi(j_3,j_2)+\phi[J_\phi(j_2,j_3)]\}$

$\geq \delta\{J_\phi(j_3,j_2)+J_\phi(j_3,j_2)\}$

$= 2\delta\{J_\phi(j_3,j_2)\}$

$\Rightarrow \left(\frac{1-\delta}{2\delta}\right)J_\phi(j_1,j_2) \geq J_\phi(j_2,j_3)$

$\Rightarrow J_\phi(j_2,j_3) \leq \left(\frac{1-\delta}{2\delta}\right)J_\phi(j_1,j_2)$ 5

From Eq.4 and Eq.5,

$$J_\phi(j_2,j_3) \leq \left(\frac{1-\delta}{2\delta}\right)^2 J_\phi(j_0,j_1)$$

So in general,

$$J_\phi(j_n,j_{n+1}) \leq \left(\frac{1-\delta}{2\delta}\right)^n J_\phi(j_0,j_1)$$

Letting $n\to\infty, J_\phi(j_n,j_{n+1})\to 0$ To prove $\{j_n\}$ is a Cauchy sequence, consider,

$J_\phi(j_n,j_m) = J_\phi(Tj_{n+1},Tj_{m+1})$

$\geq \delta\left[J_\phi(j_{n+1},Tj_{n+1})+J_\phi(j_{m+1},Tj_{m+1})+\phi[J_\phi(j_{n+1},j_{m+1})]\right]$

$= \delta\left[J_\phi(j_{n+1},j_n)+J_\phi(j_{m+1},j_m)+\phi[J_\phi(j_{n+1},j_{m+1})]\right]$

$\geq \delta[J_\phi(j_{n+1},j_n)+J_\phi(j_{m+1},j_m)]$

$\Rightarrow -J_\phi(j_n,j_m) \leq -\delta[J_\phi(j_{n+1},j_n)+J_\phi(j_{m+1},j_m)]$

As $n,m\to\infty, J_\phi(j_n,j_m)\to 0$. Since the Strong Altering JS-metric space (S,J_ϕ) is a complete, there exists $j\in S$ such that $j_n \overset{J_\phi}{\rightarrow} j$. Axiom (A3) is used to ensure that fixed point exists.

$$\phi(J_\phi(j,Tj)) \leq \limsup_{n\to\infty}[\phi(k.J_\phi(j_n,Tj))]$$

Since ϕ is a continuous monotonically non-decreasing map,

$$J_\phi(j,Tj) \leq \limsup_{n\to\infty}[k.J_\phi(j_n,Tj)]$$

$$= \limsup_{n\to\infty}[k.J_\phi(Tj_{n+1},Tj)]$$

$$-\left(J_\phi(j,Tj)\right) \geq -\limsup_{n\to\infty}[k.J_\phi(Tj_{n+1},Tj)]$$

$$\geq -k.\limsup_{n\to\infty}[\delta\left[J_\phi(j_{n+1},Tj_{n+1})+J_\phi(j,Tj)+\phi[J_\phi(j_{n+1},j)]\right]$$

$$= -k.\delta\limsup_{n\to\infty}[J_\phi(j_{n+1},Tj_{n+1})+J_\phi(j,Tj)+\phi[J_\phi(j_{n+1},j)]]$$

$$= k.\delta\liminf_{n\to\infty}[\left(-J_\phi(j_{n+1},Tj_{n+1})\right)+\left(-J_\phi(j,Tj)\right)+(-\phi[J_\phi(j_{n+1},j)])]$$

$$\geq k.\liminf_{n\to\infty}[\left(-J_\phi(j_{n+1},Tj_{n+1})\right)+\left(-J_\phi(j,Tj)\right)+(-\phi[J_\phi(j_{n+1},j)])]$$

$$-J_\phi(j,Tj) \geq k.\left[\liminf_{n\to\infty}\left(-J_\phi(j_{n+1},Tj_{n+1})\right)+\liminf_{n\to\infty}\left(-J_\phi(j,Tj)\right)+\liminf_{n\to\infty}(-\phi[J_\phi(j_{n+1},j)])\right]$$

Since $j_n \overset{J_\phi}{\rightarrow} j$ and $\phi(0)=0$,

$$\Rightarrow -J_\phi(j,Tj) \geq -k.J_\phi(j,Tj)$$

$$\Rightarrow J_\phi(j,Tj) \leq k.J_\phi(j,Tj) \Rightarrow J_\phi(j,Tj)=0 \Rightarrow j = Tj.$$

Thus a fixed point j is obtained.

Conclusion:

In this work, the study of Strong Altering JS-metric has been initiated and its relation with b-metric, dislocated metric has been discussed. Theorems on fixed points for expansive maps in the complete Strong Altering JS-metric space have been analyzed. It was observed that the compression of the space helped to obtain the fixed point in an expansive map. Theorems on fixed points for contractive type maps in this space will be presented in future works.

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Author's contributions statement:

This work was carried out in collaboration between all authors. X. M. J V developed the theory, studied the idea and wrote the manuscript. P. G initiated the idea and supervised the work. B. A revised the final output. All authors read and approved the final manuscript.

References:

1. Wang SZ, Li BY, Gao ZM, Iseki K. Some fixed point theorems on expansion mappings. Math Japon. 1984; 29: 631-636.
2. Parkhey B, Daheriya RD, Ughade M. Fixed Point Theorems for Expansive Mapping in A-Metric Space. Int J Comput Sci. 2018; 9(9): 1142-1150. http://dx.doi.org/10.29055/jcms/853
3. Pandey B, Ughade M, Pandey AK. Fixed-Points Theorems For Expansive Type Mapping On F-Cone Metric Spaces. Adv Appl Math Sci. 2022; 21(10): 6129-6146. https://www.mililink.com/upload/article/198997632aams_vol_2110_august_2022_a43_p6129-6146_babita_pandey,_manoj_ughade_and_amit_kumar_pandey.pdf
4. Chaipornjareansri S. Fixed Point and Coincidence Point Theorems for Expansive Mappings in Partial b-Metric Spaces. Thai J Math. 2018; Special Issue: 169-185. http://thaijmath.in.cmu.ac.th/index.php/thaijmath/article/viewFile/3184/354354368
5. Yeşilkaya SS, Aydın C. Fixed Point Results of Expansive Mappings in Metric Spaces. Mathematics. 2020; 8(10): 1 –10. https://doi.org/10.3390/math8101800
6. Hitzler P, Seda AK. Dislocated topologies. J Electr Eng. 2000; 51(12): 3–7. https://people.cs.ksu.edu/~hitzler/pub/pdf/scam00tr.pdf
7. Bakhtin IA. The contraction principle in quasimetric spaces. Funct Anal Unianowsk Gos Ped Inst. 1989; 30: 26–37.
8. Czerwik S. Contraction mappings in b-metric spaces. Acta. Math. Inf Univ Ostrav. 1993; 1(1): 5–11. https://dml.cz/bitstream/handle/10338.dmlcz/120469/ActaOstrav_01-1993-1_2.pdf
9. Hashim AT, Hashim AM. Some New Fixed Point Theorems in Weak Partial Metric Spaces. Baghdad Sci J. 2022; 1: 175 – 180. http://dx.doi.org/10.21123/bsj.2022.6724
10. Hussein LH, Abed SS. Fixed Point Theorems in General Metric Space with an Application. Baghdad Sci J. 2021; 18(1): 0812. https://doi.org/10.21123/bsj.2021.18.1(Suppl.).0812
11. Sanatammappa NP, Krishnakumar R, Dinesh K. Some results in b-metric spaces. Malaya J Mat. 2021; 9(1): 539-541
12. Karapınar E. Revisiting the Kannan Type Contractions via Interpolation. Adv Theory Nonlinear Anal Appl. 2018; 2(2): 85 – 87. https://doi.org/10.31197/atnaa.431135
13. Khan MS, Swaleh M, Sessa S. Fixed point theorems by altering distances between the points. Bull Austral Math Soc. 1984; 30(1): 1 - 9. https://doi.org/10.1017/S0004972700001659
14. Rao CS, Murthy KS, Murthy MPR. Common Fixed Point Theorems on Complete Metric Space for Two self Maps Using Generalized Altering Distance Functions in Five Variables and Deficit functions. Glob J Pure Appl Math. 2019; 15(3): 241 -250. http://ripublication.com/gjpam19/gjpamv15n3_02.pdf
15. Hamaizia T. Fixed point theorems for generalized (ψ, ϕ, F)-contraction type mappings in b-metric spaces with applications. Open J Math Anal. 2021; 5(1): 35-41. https://doi.org/10.30538/psrp-oma2021.0080
16. Kumar M, Devi S, Singh P. Fixed Point Theorems by Using Altering Distance Function in S-Metric Spaces. Commun Math Appl. 2022; 13(2): 553 – 573. http://doi.org/10.26713/cma.v13i2.1787

وجود النقاط الثابتة للخرائط التوسعية في فضاء جي اس المتري البديل القوي التام

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الخلاصة:

تهدف الورقة ألى بدء واستكشاف مفهوم مقياس موسع معروف بمقياس جي اس البديل القوي، نسخة اقوى من مقياس جي اس البديل.
لقد ناقشنا العلاقة مع مقياس "ب" والمقياس المخلوع وقدمت بعض الامثلة. وتمت أيضا مناقشة بعض النظريات حول النقاط ثابتة للتحويلات الذاتية في انشاء مقياس جي اس البديل.

الكلمات المفتاحية: المسافة البديلة، فضاء بي المتري، الفضاء المتري المخلوع، التحويلات الموسعة، النقاط الثابتة، جي اس المتري البديل القوي.