Open Access | Baghdad Science Journal | P-ISSN: 2078-8665
2023, 20(1 Special Issue) ICAAM: 371-379 | E-ISSN: 2411-7986

DOI: https://dx.doi.org/10.21123/bsj.2023.8430

Nordhaus-Gaddum Type Relations on Open Support Independence Number of Some Path Related Graphs Under Addition and Multiplication

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ICAAM= International Conference on Analysis and Applied Mathematics 2022.

Received 20/1/2023, Revised 13/2/2023, Accepted 14/2/2023, Published 4/3/2023

Abstract:

In this paper, Nordhaus-Gaddum type relations on open support independence number of some derived graphs of path related graphs under addition and multiplication are studied.

Keywords: Derived graphs, Nordhaus-Gaddum type relations, Open support independence number, Open support independence number under addition, Open support independence number under multiplication.

Introduction:

Graphs considered in this paper are finite, undirected, and without loops or multiple edges[1]. Let $G = (V, E)$ be a graph with p vertices and q edges. For each vertex $v \in V$, the open neighborhood[2] of v is the set $N(v)$ containing all the vertices u adjacent to v. In a graph G, an independent set is a subset S of $V(G)$ such that no two vertices in S are adjacent. A maximum independent set is an independent set[3] of maximum size.

Recently the concept of open support of a graph under addition was introduced by Balamurugan *et al.* [4] and further studied in [5]. Open support of a graph under multiplication was introduced in [6]. Open support independence number of a graph under addition and multiplication was introduced in [7]. In this paper, Nordhaus-Gaddum type relations on open support independence number of some derived graphs of path related graphs under addition and multiplication are studied. The following definitions are necessary for the present study.

Definition. 1: Let $G = (V, E)$ be a graph. A subset S of V is called an independent set of G if no two vertices in S are adjacent in G.

Definition. 2: An independent set 'S' is maximum in G if G has no independent set S' with $|S'| > |S|$.

Definition. 3: The number of vertices in a maximum independent set of G is called the independence number of G and is denoted by $\alpha(G)$.

Definition. 4: Let $G = (V, E)$ be a graph. An open support of a vertex v under addition is defined by $\sum_{u\in N(v)} deg\, u$ and is denoted by $supp\ (v)$.

Definition. 5: Let $G = (V, E)$ be a graph. Open support of the graph G under addition is defined by $\sum_{v\in V(G)} supp\ (v)$ and is denoted by $supp\ (G)$.

Definition. 6: Let $G = (V, E)$ be a graph. An open support of a vertex v under multiplication is defined by $\prod_{u\in N(v)} deg\, u$ and is denoted by $mult\ (v)$.

Definition. 7: Let $G = (V, E)$ be a graph. Open support of the graph G under multiplication is defined by $\prod_{v\in V(G)} mult\ (v)$ and is denoted by $mult\ (G)$.

Definition. 8: The line graph[8] $L(G)$ of G is the graph whose vertex set is $E(G)$ in which two vertices are adjacent if and only if they are adjacent in G.

Definition. 9: The jump graph $J(G)$ of G is the graph whose vertex set is $E(G)$ in which two vertices are adjacent if and only if they are non-adjacent in G.

Definition. 10: The paraline graph $P\,L(G)$ is a line graph of the subdivision graph of G.

Definition. 11: The complement of a graph G has V as its vertex set and two vertices are adjacent in if and only if they are not adjacent in G.

Definition. 12: The subdivision graph[9] $S(G)$ of a graph G is obtained from G by inserting a new vertex into every edge of G.

Definition. 13: The semi-total point graph $T_2(G)$ is the graph whose vertex set is $V(G) \cup E(G)$ where two vertices are adjacent if and only if (i) they are adjacent vertices of G or
(ii) one is a vertex of G and the other is an edge of G incident with it.

Definition. 14: The semi-total line graph $T_1(G)$ is the graph whose vertex set is $V(G) \cup E(G)$ where two vertices are adjacent if and only if (i) they are adjacent edges of G or
(ii) one is a vertex of G and the other is an edge of G incident with it.

Definition. 15: The quasi-total graph $P(G)$ is the graph with vertex set $V(G) \cup E(G)$ where two vertices are adjacent if and only if (i) they are non-adjacent vertices of G or
(ii) they are adjacent edges of G or (iii) one is a vertex of G and the other is an edge of G incident with it.

Definition. 16: The quasi vertex-total graph $Q(G)$ is the graph with vertex set $V(G) \cup E(G)$ where two vertices are adjacent if and only if
(i) they are adjacent vertices of G or (ii) they are non-adjacent vertices of G or
(iii) they are adjacent edges of G or (iv) one is a vertex of G and the other is an edge of G incident with it.

Definition. 17: The complementary prism[10] of a graph G, denoted as $G\bar{G}$, is obtained from the graph $G \cup \bar{G}$ by adding a perfect matching between the corresponding vertices of G and $\bar{G}$.

Definition. 18: Let $G = (V, E)$ be a graph. Let S denote the maximum independent set of G. Open support independence number of the set S under addition, denoted by $supp\, S^{+}(G)$, is defined by $supp\, S^{+}(G) = \sum_{v \in S} supp\,(v)$. Open support independence number of G under addition, denoted by $supp\,\alpha^{+}(G)$, is defined by $p\,\alpha^{+}(G) = max\,\{supp\, {S_i}^{+}(G); i \geq 1\}$.

Definition. 19: Let $G = (V, E)$ be a graph. Let S denote the maximum independent set of G.
G Open support independence number of the set S under multiplication, denoted by $supp\, S^{\times}(G)$, is defined by $supp\, S^{\times}(G) = \prod_{v \in S} mult\,(v)$. Open support independence number of G under multiplication, denoted by $supp\,\alpha^{\times}(G)$ is defined by $supp\,\alpha^{\times}(G) = max\,\{mult\, {S_i}^{\times}(G); i \geq 1\}$.

Result. 1: Let $G = P_n$ where $n > 2$ is a path on n vertices. Then

$$supp\,\alpha^{+}(G) = \begin{cases} 2n-2 \; if\; n\; is\; odd \\ 2n-3 \; if\; n\; is\; even \end{cases}$$ and

$$supp\,\alpha^{\times}(G) = \begin{cases} 2^{n-1} if\; n\; is\; odd \\ 2^{n-2} \; if\; n\; is\; even \end{cases}$$

Main results:

Theorem. 1: Let $L(P_n)$ be the line graph of P_n where $n \geq 2$. Then $supp\,\alpha^{+}(P_n) + supp\,\alpha^{+}[L(P_n)] = 4n - 7$, $supp\,\alpha^{\times}(P_2) + supp\,\alpha^{\times}[L(P_2)] = 1$ and $supp\,\alpha^{\times}(P_n) + supp\,\alpha^{\times}[L(P_n)] =$
$$\begin{cases} 5 \times 2^{n-3} if\; n\; is\; odd \\ 2^{n-1} \; if\; n\; is\; even, n \neq 2 \end{cases}$$

Proof: Let $L(P_n)$ be the line graph of P_n where $n \geq 2$. $L(P_n) = P_{n-1}$.

Case (i): Suppose n is odd.

Then $supp\,\alpha^{+}[L(P_n)] = 2(n-1) - 3$
$= 2n - 5$

Hence $supp\,\alpha^{+}(P_n) + supp\,\alpha^{+}[L(P_n)] = (2n-2) + (2n-5) = 4n - 7$

Now $supp\,\alpha^{\times}[L(P_n)] = 2^{(n-1)-2} = 2^{n-3}$

Hence $supp\,\alpha^{\times}(P_n) + supp\,\alpha^{\times}[L(P_n)] = 2^{n-1} + 2^{n-3} = 5 \times 2^{n-3}$

Case (ii): Suppose n is even.

Then $supp\,\alpha^{+}[L(P_n)] = 2(n-1) - 2$
$= 2n - 4$

Hence $supp\,\alpha^{+}(P_n) + supp\,\alpha^{+}[L(P_n)] = (2n-3) + (2n-4) = 4n - 7$

Now $supp\,\alpha^{\times}[L(P_n)] = 2^{(n-1)-1}$
$= 2^{n-2}$

Hence $supp\,\alpha^{\times}(P_n) + supp\,\alpha^{\times}[L(P_n)] = 2^{n-2} + 2^{n-2} = 2^{n-1}$.

Theorem. 2: Let $[J(P_n)]$ be the jump graph of P_n where ≥ 2. Then

$supp\,\alpha^{+}(P_2) + supp\alpha^{+}[J(P_2)] = 1$, $supp\,\alpha^{+}(P_3) + supp\alpha^{+}[J(P_3)] = 4$, $supp\,\alpha^{+}(P_n) + supp\alpha^{+}[J(P_n)] =$
$$\begin{cases} 2n^2 - 13n + 28 \;\; if\; n\; is\; odd, n \neq 3 \\ 2n^2 - 13n + 27 \;\; if\; n\; is\; even, n \neq 2 \end{cases},$$
$supp\,\alpha^{\times}(P_2) + supp\alpha^{\times}[J(P_2)] = 1$, $supp\,\alpha^{\times}(P_3) + supp\alpha^{\times}[J(P_3)] = 4$ and

$supp\,\alpha^{\times}(P_n) + supp\alpha^{\times}[J(P_n)] =$
$$\begin{cases} (n-4)^{2n-9} \times (n-3)^2 + 2^{n-1} \;\; if\; n\; is\; odd, n \neq 3 \\ (n-4)^{2n-9} \times (n-3)^2 + 2^{n-2} \;\; if\; n\; is\; even, n \neq 2 \end{cases}.$$

Proof: Let $J(P_n)$ be the jump graph of P_n where $n \geq 2$. Let $e_1, e_2, \dots, e_{n-1}$ be the vertices of $J(P_n)$. Then $deg\, e_i = n - 4, 2 \leq i \leq n - 2$ and $deg\, e_1 = deg\, e_{n-1} = n - 3$. Obviously the independence number of $J(P_n)$ is 2. To get the maximum support, the degree of those two vertices should be maximum, but there are only two maximum-degree vertices that are adjacent. Hence $S_1 = \{e_1, e_2\}$ and $S_2 = \{e_{n-2}, e_{n-1}\}$ are such maximum independent sets. Consider the set S_1. The proof is similar to the set S_2.

$$supp\ (e_1) = \sum_{v \in N(e_1)} deg\ v = deg\ e_3 + deg\ e_4 + \dots + deg\ e_{n-1} = (n-4) + (n-4) + \cdots + (n-4) + (n-3) = (n-4)^2 + (n-3) = n^2 - 7n + 13$$

$$supp\ (e_2) = \sum_{v \in N(e_2)} deg\ v = deg\ e_4 + deg\ e_5 + \dots + deg\ e_{n-1} = (n-5)(n-4) + (n-3) = n^2 - 8n + 17$$

Therefore $supp\alpha^+[J(P_n)] = 2n^2 - 15n + 30$

Hence $supp\ \alpha^+(P_n) + supp\alpha^+[J(P_n)] = \begin{cases} 2n^2 - 13n + 28 & if\ n\ is\ odd, n \neq 3 \\ 2n^2 - 13n + 27 & if\ n\ is\ even, n \neq 2 \end{cases}$.

Now $supp\ \alpha^+(P_2) + supp\alpha^+[J(P_2)] = 1$ and $supp\ \alpha^+(P_3) + supp\alpha^+[J(P_3)] = 4$

Now $supp\ \alpha^\times[J(P_n)] = \prod_{u \in s_1} mult(u)$

$$= mult(e_1) \times mult(e_2)$$
$$= (n-4)^{(n-4)} \times (n-3) \times (n-4)^{(n-5)} \times (n-3)$$
$$= (n-4)^{2n-9} \times (n-3)^2$$

Hence

$$supp\ \alpha^\times(P_n) + supp\alpha^\times[J(P_n)] = \begin{cases} (n-4)^{2n-9} \times (n-3)^2 + 2^{n-1} & if\ n\ is\ odd, n \neq 3 \\ (n-4)^{2n-9} \times (n-3)^2 + 2^{n-2} & if\ n\ is\ even, n \neq 2 \end{cases}.$$

Now $supp\ \alpha^\times(P_2) + supp\alpha^\times[J(P_2)] = 1, supp\ \alpha^\times(P_3) + supp\alpha^\times[J(P_3)] = 4.$

Theorem. 3: Let $S(P_n)$ be the subdivision graph of P_n, where $n \geq 2$. Then

$supp\ \alpha^+(P_n) + supp\ \alpha^+[S(P_n)] = \begin{cases} 6n - 6 & if\ n\ is\ odd \\ 6n - 7 & if\ n\ is\ even \end{cases}$ and

$supp\ \alpha^\times(P_n) + supp\ \alpha^\times[S(P_n)] = \begin{cases} 2^{n-1} + 2^{2n-2} & if\ n\ is\ odd \\ 2^{n-2} + 2^{2n-2} & if\ n\ is\ even \end{cases}$

Proof: Let $S(P_n)$ be the subdivision graph of P_n where $n \geq 2$. $S(P_n) = P_{2n-1}$. The subdivision graph of a path is a path of odd length. Let $v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_{n-1}$ be the vertices of $S(P_n)$.

Hence $supp\ \alpha^+[S(P_n)] = 4n - 4$.

Therefore $supp\ \alpha^+(P_n) + supp\ \alpha^+[S(P_n)] = \begin{cases} 6n - 6 & if\ n\ is\ odd \\ 6n - 7 & if\ n\ is\ even \end{cases}$

Now $supp\ \alpha^\times[S(P_n)] = 2^{2n-2}$

Hence $supp\ \alpha^\times(P_n) + supp\alpha^\times[S(P_n)] = \begin{cases} 2^{n-1} + 2^{2n-2} & if\ n\ is\ odd \\ 2^{n-2} + 2^{2n-2} & if\ n\ is\ even \end{cases}$

Theorem. 4: Let $PL(P_n)$ be the paraline graph of P_n where ≥ 2. Then

$supp\ \alpha^+(P_n) + supp\ \alpha^+[PL(P_n)] = \begin{cases} 6n - 9 & if\ n\ is\ odd \\ 6n - 10 & if\ n\ is\ even \end{cases}$ and

$supp\ \alpha^\times(P_n) + supp\alpha^\times[PL(P_n)] = \begin{cases} 2^{n-1} + 2^{2n-4} & if\ n\ is\ odd \\ 5 \times 2^{n-2} & if\ n\ is\ even \end{cases}$

Proof: Let $PL(P_n)$ be the paraline graph of P_n, where $n \geq 2$.

$PL(P_n) = P_{2n-2}$ A paraline graph of a path is a path of even length. Let $e_1, e_2, \dots, e_{2n-2}$ be the vertices of $PL(P_n)$.

Hence $supp\ \alpha^+[PL(P_n)] = 4n - 7$.

Therefore $supp\ \alpha^+(P_n) + supp\ \alpha^+[PL(P_n)] = \begin{cases} 6n - 9 & if\ n\ is\ odd \\ 6n - 10 & if\ n\ is\ even \end{cases}$

Now $supp\ \alpha^\times[PL(P_n)] = 2^{(2n-2)-2} = 2^{2n-4}$

Therefore $supp\ \alpha^\times(P_n) + supp\ \alpha^\times[PL(P_n)] = \begin{cases} 2^{n-1} + 2^{2n-4} & if\ n\ is\ odd \\ 5 \times 2^{n-2} & if\ n\ is\ even \end{cases}$

Theorem. 5: Let $\overline{P_n}$ be the complement graph of P_n where $n \geq 2$. Then

$supp\ \alpha^+(P_2) + supp\ \alpha^+(\overline{P_2}) = 1$, $supp\ \alpha^\times(P_2) + supp\ \alpha^\times(\overline{P_2}) = 1$,

$supp\ \alpha^+(P_n) + supp\ \alpha^+(\overline{P_n}) = \begin{cases} 2n^2 - 9n + 15 & if\ n\ is\ odd \\ 2n^2 - 9n + 14 & if\ n\ is\ even, n \neq 2 \end{cases}$ and

$supp\ \alpha^\times(P_n) + supp\ \alpha^\times(\overline{P_n}) = \begin{cases} (n-3)^{2n-7}(n-2)^2 + 2^{n-1} & if\ n\ is\ odd \\ (n-3)^{2n-7}(n-2)^2 + 2^{n-2} & if\ n\ is\ even, n \neq 2 \end{cases}$.

Proof: Let $\overline{P_n}$ be the complement graph of P_n where $n \geq 2$.

$\overline{P_n} = J(P_{n+1})$.

Let $v_1, v_2, \dots, v_n$ be the vertices of $\overline{P_n}$.

Hence $supp\ \alpha^+(\overline{P_n}) = 2(n+1)^2 - 15(n+1) + 30$

$= 2n^2 - 11n + 17$

Therefore $supp\ \alpha^+(P_n) + supp\ \alpha^+(\overline{P_n}) = \begin{cases} 2n^2 - 9n + 15 & if\ n\ is\ odd \\ 2n^2 - 9n + 14 & if\ n\ is\ even \end{cases}$.

Also $supp\ \alpha^+(P_2) + supp\ \alpha^+(\overline{P_2}) = 1$

Now $supp\ \alpha^\times[\overline{P_n}] = (n+1-4)^{2(n+1)-9} \times (n+1-3)^2$

$= (n-3)^{2n-7}(n-2)^2$

Therefore $supp\ \alpha^\times(P_n) + supp\ \alpha^\times(\overline{P_n}) = \begin{cases} (n-3)^{2n-7}(n-2)^2 + 2^{n-1} & if\ n\ is\ odd \\ (n-3)^{2n-7}(n-2)^2 + 2^{n-2} & if\ n\ is\ even \end{cases}$

Also $supp\ \alpha^{\times}(P_2) + supp\ \alpha^{\times}(\overline{P_2}) = 1$.

Theorem. 6: Let $T_2(P_n)$ be the semi-total point graph of P_n where $n \geq 3$. Then $supp\ \alpha^{+}(P_n) + supp\ \alpha^{+}[T_2(P_n)] = \begin{cases} 10n-14 & if\ n\ is\ odd \\ 10n-15 & if\ n\ is\ even \end{cases}$
and
$supp\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}[T_2(P_n)] = \begin{cases} 64 \times 16^{n-3} + 2^{n-1} & if\ n\ is\ odd \\ 64 \times 16^{n-3} + 2^{n-2} & if\ n\ is\ even \end{cases}$

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices and $e_i = v_i v_{i+1}$, $1 \leq i \leq n-1$ be the edges of the path P_n. In $T_2(P_n)$, v_i is adjacent with $v_{i-1}, v_{i+1}, e_{i-1}, e_i$ for $2 \leq i \leq n-3$, v_1 is adjacent with v_2, e_1 and v_n is adjacent with v_{n-1}, e_{n-1}. e_i is adjacent with v_i, v_{i+1}, $1 \leq i \leq n-1$. Then $deg\ v_i = 4,\ 2 \leq i \leq n-1, deg\ v_1 = deg\ v_n = 2$ and $deg\ e_i = 2,\ 1 \leq i \leq n-1$. $S_1 = \{e_1, e_2, \dots, e_{n-1}\}$, $S_2 = \{v_1, e_2, e_3, \dots, e_{n-1}\}$ and $S_3 = \{v_1, e_2, e_3, \dots, e_{n-2}, v_n\}$ are the three maximum independent sets. Consider the set S_1. The proof is similar for the sets S_2 and S_3.

$$supp(e_1) = \sum_{v \in N(e_1)} deg\ v$$
$$= deg\ v_1 + deg\ v_2 = 2 + 4 = 6$$

Similarly $supp(e_{n-1}) = \sum_{v \in N(e_{n-1})} deg\ v = 6$

$$supp(e_2) = \sum_{v \in N(e_2)} deg\ v$$
$$= deg\ v_2 + deg\ v_3 = 4 + 4 = 8$$

Similarly

$$supp(e_3) = supp(e_4) = \cdots = supp(e_{n-2}) = 8$$

Hence $supp\ \alpha^{+}[T_2(P_n)] = \sum_{v \in s_1} supp\ (v)$
$$= 6 + (n-3)8 + 6.$$
$$= 8n - 12$$

Therefore $supp\ \alpha^{+}(P_n) + supp\ \alpha^{+}[T_2(P_n)] = \begin{cases} 10n-14 & if\ n\ is\ odd \\ 10n-15 & if\ n\ is\ even \end{cases}$

Now $supp\ \alpha^{\times}[T_2(P_n)] = \prod_{u \in s_1} mult(u) = mult(e_1) \times mult(e_2) \times \dots \times mult(e_{n-1})$
$$= 8 \times 16^{(n-3)} \times 8 = 64 \times 16^{n-3}$$

Therefore $p\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}[T_2(P_n)] = \begin{cases} 64 \times 16^{n-3} + 2^{n-1} & if\ n\ is\ odd \\ 64 \times 16^{n-3} + 2^{n-2} & if\ n\ is\ even \end{cases}$.

Theorem. 7: Let $T_1(P_n)$ be a semi-total line graph of P_n where $n \geq 2$. Then $p\ \alpha^{+}(P_n) + supp\ \alpha^{+}[T_1(P_n)] = \begin{cases} 10n-14 & if\ n\ is\ odd \\ 10n-15 & if\ n\ is\ even \end{cases}$,
$supp\ \alpha^{\times}(P_2) + supp\ \alpha^{\times}[T_1(P_2)] = 5$ and
$supp\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}[T_1(P_n)] = \begin{cases} 2^{n-1} + 1296 \times 16^{n-4} & if\ n\ is\ odd \\ 2^{n-2} + 1296 \times 16^{n-4} & if\ n\ is\ even\ and\ n \neq 2 \end{cases}$
.

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices and $e_i = v_i v_{i+1}$, $1 \leq i \leq n-1$ be the edges of the path P_n. In $T_1(P_n)$, v_i is adjacent with e_{i-1}, e_i for $2 \leq i \leq n-1$, v_1 is adjacent with e_1 and v_n is adjacent with e_{n-1}. e_i is adjacent with $v_i, v_{i+1}, e_{i-1}, e_{i+1}$, $2 \leq i \leq n-2$, e_1 is adjacent with v_1, v_2 and e_2 and e_{n-1} is adjacent with v_{n-1}, v_n and e_{n-2}. Then $deg\ v_i = 2,\ 2 \leq i \leq n-1, deg\ v_1 = deg\ v_n = 1, deg\ e_1 = deg\ e_{n-1} = 3$ and $deg\ e_i = 4,\ 2 \leq i \leq n-2$. $S = \{v_1, v_2, \dots, v_n\}$ is the unique maximum independent set.

$$supp(v_1) = \sum_{v \in N(v_1)} deg\ v$$
$$= deg\ e_1 = 3$$

Similarly $supp(v_n) = 3$

$$supp(v_2) = \sum_{v \in N(v_2)} deg\ v$$
$$= deg\ e_1 + deg\ e_2 = 3 + 4 = 7$$

Similarly $supp(v_{n-1}) = 7$

$$supp(v_3) = \sum_{v \in N(v_3)} deg\ v$$
$$= deg\ e_2 + deg\ e_3 = 4 + 4 = 8$$

Similarly

$$supp(v_4) = supp(v_5) = \cdots = supp(v_{n-2}) = 8$$

Hence $supp\ \alpha^{+}[T_1(P_n)] = \sum_{v \in S} supp\ (v)$
$= 3 + 7 + (n-4)8 + 7 + 3 = 8n - 12$

Therefore $supp\ \alpha^{+}(P_n) + supp\ \alpha^{+}[T_1(P_n)] = \begin{cases} 10n-14 & if\ n\ is\ odd \\ 10n-15 & if\ n\ is\ even \end{cases}$

Now when $n \geq 3$,

$$supp\ \alpha^{\times}[T_1(P_n)] = \prod_{u \in S} mult(u)$$
$$= mult(v_1) \times mult(v_2) \times \dots \times mult(v_n)$$
$$= 3 \times 12 \times 16^{n-4} \times 12 \times 3 = 1296 \times 16^{n-4}$$

Therefore $supp\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}[T_1(P_n)] = \begin{cases} 2^{n-1} + 1296 \times 16^{n-4} & if\ n\ is\ odd \\ 2^{n-2} + 1296 \times 16^{n-4} & if\ n\ is\ even \end{cases}$.

when $n = 2$, $supp\ \alpha^{\times}[T_1(P_n)] = 4$.
Hence $supp\ \alpha^{\times}(P_2) + supp\ \alpha^{\times}[T_1(P_2)] = 5$.

Theorem. 8: Let $P(P_n)$ be a quasi-total graph of P_n where $n \geq 2$. Then
$supp\ \alpha^{+}(P_2) + supp\ \alpha^{+}[P(P_2)] = 5$,
$supp\ \alpha^{+}(P_3) + supp\ \alpha^{+}[P(P_3)] = 16$,
$supp\ \alpha^{+}(P_n) + supp\ \alpha^{+}[P(P_n)]$
$= \begin{cases} 3n^2 - 5n + 3 & if\ n\ is\ odd, n \neq 3 \\ 3n^2 - 4n + 2 & if\ n\ is\ even, n \neq 2 \end{cases}$,
$supp\ \alpha^{\times}(P_2) + supp\ \alpha^{\times}[P(P_2)] = 5$,
$supp\ \alpha^{\times}(P_3) + supp\ \alpha^{\times}[P(P_3)] = 76$ and

$supp\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}[P(P_n)] = \begin{cases} 2^{n-1} + 432(n-1)^{3n-8}4^{n-5} & if\ n\ is\ odd, n \neq 3 \\ 2^{n-2} + 144(n-1)^{3n-7}4^{n-4} & if\ n\ is\ even, n \neq 2 \end{cases}$
.

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices and $e_i = v_i v_{i+1}$, $1 \leq i \leq n-1$ be the edges of the path P_n. In $P(P_n)$, v_i is non-adjacent with $v_{i-1}\ and\ v_{i+1}$ for $2 \leq i \leq n-1$, v_1 is non-adjacent with v_2 and v_n is non-adjacent with v_{n-1}. e_i is adjacent with $v_i, v_{i+1}, e_{i-1}\ and\ e_{i+1}$ for $2 \leq i \leq n-2$, e_1 is adjacent with v_1, v_2 and e_2 and e_{n-1} is adjacent with v_{n-1}, v_n and e_{n-2}. Then $deg\ v_i = n-1,\ 1 \leq i \leq n, deg\ e_1 = deg\ e_{n-1} = 3$ and $deg\ e_i = 4$ for $2 \leq i \leq n-2$.

Case (i): Let n be odd. The independence number of $P(P_n)$ is $\frac{n+1}{2}$. Let $n \geq 5$. To get the maximum support consider the end vertices v_1, v_2 or v_{n-1}, v_n with the vertices lying between e_1 and e_{n-1}. Hence $S_1 = \{v_1, v_2, e_3, e_5, \dots, e_{n-4}, e_{n-2}\}$ and $S_2 = \{v_{n-1}, v_n, e_2, e_4, \dots, e_{n-5}, e_{n-3}\}$ are the two maximum independent sets with maximum support. Consider the set S_1. The proof is similar to the other set.

$$supp(v_1) = \sum_{v \in N(v_1)} deg\ v = deg\ v_3 + deg\ v_4 + \dots + deg\ v_n + deg\ e_1 = (n-2)(n-1) + 3 = n^2 - 3n + 5$$

$$supp(v_2) = \sum_{v \in N(v_2)} deg\ v = deg\ v_4 + deg\ v_5 + \dots + deg\ v_n + deg\ e_1 + deg\ e_2 = (n-3)(n-1) + 3 + 4 = n^2 - 4n + 10$$

$$supp(e_3) = \sum_{v \in N(e_3)} deg\ v = deg\ v_3 + deg\ v_4 + deg\ e_2 + deg\ e_4 = (n-1) + (n-1) + 4 + 4 = 2n + 6$$

Similarly

$$supp(e_5) = supp(e_7) = \dots = supp(e_{n-4}) = 2n + 6$$

$$supp(e_{n-2}) = \sum_{v \in N(e_{n-2})} deg\ v$$
$$= deg\ e_{n-3} + deg\ e_{n-1} + deg\ v_{n-2} + deg\ v_{n-1}$$
$$= 4 + 3 + (n-1) + (n-1) = 2n + 5$$

Hence $supp\ \alpha^{+}[P(P_n)] = \sum_{v \in S_1} supp\ (v)$
$= 3n^2 - 7n + 5$

Therefore $supp\ \alpha^{+}(P_n) + supp\ \alpha^{+}[P(P_n)]$
$= 2n - 2 + 3n^2 - 7n + 5$
$= 3n^2 - 5n + 3$

Now $supp\ \alpha^{+}[P(P_3)] = 12$

Hence $supp\ \alpha^{+}(P_3) + supp\ \alpha^{+}[P(P_3)] = 16$

Similarly $supp\ \alpha^{\times}[P(P_n)] = \prod_{u \in S_1} mult(u)$
$= mult(v_1) \times mult(v_2) \times mult(e_3) \times mult(e_5) \times \dots \times mult(e_{n-4}) \times mult(e_{n-2})$
$= 3(n-1)^{n-2} \times 12(n-1)^{n-3}[16(n-1)^2]^{\left(\frac{n-5}{2}\right)} \times 12(n-1)^2$
$= 432(n-1)^{3n-8}4^{n-5}$

Therefore $supp\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}[P(P_n)] = 2^{n-1} + 432(n-1)^{3n-8}4^{n-5}$.

Now $supp\ \alpha^{\times}[P(P_3)] = 72$

Hence $supp\ \alpha^{\times}(P_3) + supp\ \alpha^{\times}[P(P_3)] = 76$

Case (ii): Let n be even and $n \geq 4$. Let $S_3 = \{v_1, v_2, e_3, e_5, \dots, e_{n-3}, e_{n-1}\}$ and $S_4 = \{v_{n-1}, v_n, e_1, e_3, \dots, e_{n-5}, e_{n-3}\}$ are two maximum independent sets. Consider the set S_3. The proof is similar to the other set. From case (i)

$supp(v_1) = n^2 - 3n + 5$
$supp(v_2) = n^2 - 4n + 10$
$supp(e_3) = 2n + 6$ and $supp(e_5) = supp(e_7) = \dots = supp(e_{n-3}) = 2n + 6$

$$supp(e_{n-1}) = \sum_{v \in N(e_{n-1})} deg\ v = deg\ e_{n-2} + deg\ v_{n-1} + deg\ v_n = 4 + (n-1) + (n-1) = 2n + 2$$

Hence

$$supp\ \alpha^{+}[P(P_n)] = \sum_{v \in S_3} supp\ (v) = 3n^2 - 6n + 5$$

Therefore $supp\ \alpha^{+}(P_n) + supp\ \alpha^{+}[P(P_n)] = 2n - 3 + 3n^2 - 6n + 5$
$= 3n^2 - 4n + 2$

Now $supp\ \alpha^{+}[P(P_2)] = 4$

Hence $supp\ \alpha^{+}(P_2) + supp\ \alpha^{+}[P(P_2)] = 5$

Now $supp\ \alpha^{\times}[P(P_n)] = \prod_{u \in S_3} mult(u)$
$= mult(v_1) \times mult(v_2) \times mult(e_3) \times mult(e_5) \times \dots \times mult(e_{n-3}) \times mult(e_{n-1})$
$= 3(n-1)^{n-2} \times 12(n-1)^{n-3}[16(n-1)^2]^{\left(\frac{n-4}{2}\right)} \times 4(n-1)^2$
$= 144(n-1)^{3n-7}4^{n-4}$

Therefore $supp\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}[P(P_n)] = 2^{n-2} + 144(n-1)^{3n-7}4^{n-4}$

Now $supp\ \alpha^{\times}[P(P_2)] = 4$

Hence $supp\ \alpha^{\times}(P_2) + supp\ \alpha^{\times}[P(P_2)]$ =5.

Theorem. 9: Let $Q(P_n)$ be a quasi-vertex-total graph of P_n where $n \geq 2$. Then
$supp\ \alpha^{+}(P_2) + supp\ \alpha^{+}[Q(P_2)] = 3$,
$supp\ \alpha^{+}(P_n) + supp\ \alpha^{+}[Q(P_n)] = \begin{cases} 2n^2 + 6n - 12 & if\ n\ is\ odd \\ 2n^2 + 5n - 14 & if\ n\ is\ even, n \neq 2 \end{cases}$,
$supp\ \alpha^{\times}(P_2) + supp\ \alpha^{\times}[Q(P_2)] = 2$, and

$supp\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}[Q(P_n)] =$

$$\begin{cases} 2^{n-1}+144n^2(n+1)^{2n-4}4^{n-5} if\ n\ is\ odd \\ 2^{n-2}+432n(n+1)^{2n-4}4^{n-6} \quad if\ n\ is\ even, n\neq 2 \end{cases}$$
.

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices and $e_i = v_i v_{i+1}$, $1 \le i \le n-1$ be the edges of the path P_n. In Q(P_n), any two $v_i's$ are adjacent to each other for $1 \le i \le n$. e_i is adjacent with $v_i, v_{i+1}, e_{i-1}\ and\ e_{i+1}$ for $2 \le i \le n-2$, e_1 is adjacent with v_1, v_2 and e_2 and e_{n-1} is adjacent with v_{n-1}, v_n and e_{n-2}. Then $deg\ v_i = n+1$, $2 \le i \le n-1, deg\ v_1 = deg\ v_n = n,$ $deg\ e_i = 4$ for $2 \le i \le n-2$ and $deg\ e_1 = deg\ e_{n-1} = 3$.

Case (i): Let n be odd. The independence number of $Q(P_n)$ is $\frac{n+1}{2}$. There are $S_{\frac{n+1}{2}}$ maximum independent sets with maximum support. Consider the set $S_1 = \{v_1, e_2, e_4, e_6, \dots, e_{n-3}, e_{n-1}\}$. The proof is similar to the other sets.

$$supp(v_1) = \sum_{v\in N(v_1)} deg\ v$$
$$= deg\ v_2 + deg\ v_3 + \cdots + deg\ v_{n-1} + deg\ v_n + deg\ e_1$$
$$= (n-2)(n+1) + n + 3 = n^2 + 1$$
$$supp(e_2) = \sum_{v\in N(e_2)} deg\ v$$
$$= deg\ v_2 + deg\ v_3 + deg\ e_1 + deg\ e_3$$
$$= (n+1) + (n+1) + 3 + 4 = 2n + 9$$
$$supp(e_4) = \sum_{v\in N(e_4)} deg\ v$$
$$= deg\ v_4 + deg\ v_5 + deg\ e_3 + deg\ e_5$$
$$= (n+1) + (n+1) + 4 + 4 = 2n + 10$$
Similarly
$$supp(e_6) = supp(e_8) = \cdots = supp(e_{n-3}) = 2n + 10$$
$$supp(e_{n-1}) = \sum_{v\in N(e_{n-1})} deg\ v$$
$$= deg\ e_{n-2} + deg\ v_{n-1} + deg\ v_n$$
$$= 4 + (n+1) + n = 2n + 5$$
Hence $supp\ \alpha^+[Q(P_n)] = \sum_{v\in S_1} supp\ (v)$
$$= n^2 + 1 + 2n + 9 + (2n+10)\left(\frac{n-5}{2}\right) + 2n + 5$$
$$= 2n^2 + 4n - 10$$
Therefore $supp\ \alpha^+(P_n) + supp\ \alpha^+[Q(P_n)] = 2n^2 + 6n - 12$

Now $supp\ \alpha^\times[Q(P_n)] = \prod_{u\in S_1} mult(u)$
$$= mult(v_1) \times mult(e_2) \times mult(e_4) \times \dots \times mult(e_{n-3} \times mult(e_{n-1})$$
$$= 3n(n+1)^{n-2} \times 12(n+1)^2[16(n+1)^2]^{\left(\frac{n-5}{2}\right)} \times 4n(n+1)$$
$$= 144n^2(n+1)^{2n-4}4^{n-5}$$
Therefore $supp\ \alpha^\times(P_n) + supp\ \alpha^\times[Q(P_n)] = 2^{n-1} + 144n^2(n+1)^{2n-4}4^{n-5}$.

Case (ii): Let n be even. The independence number of $Q(P_n)$ is $\frac{n}{2}$. To get the maximum support to choose the end vertices v_1 or v_n with $e_2, e_4, \dots, e_{n-2}$, otherwise, choose any $v_i's$ between v_1 and v_n with $e_i's$ which are non-adjacent between e_1 and e_n with e_1 or e_n since $v_i's$ have maximum support than $e_i's$. Consider the set $S_2 = \{v_1, e_2, e_4, \dots, e_{n-4}, e_{n-2}\}$. The proof is similar to the other sets. From case (i),
$$supp(v_1) = n^2 + 1$$
$$supp(e_2) = 2n + 9$$
$$supp(e_4) = supp(e_6) = \cdots = supp(e_{n-4}) = 2n + 10$$
And $supp(e_{n-2}) = 2n + 9$
Hence $supp\ \alpha^+[Q(P_n)] = \sum_{v\in S_2} supp\ (v)$
$$= (n^2+1) + (2n+9) + (2n+10)\left(\frac{n-6}{2}\right) + (2n+9)$$
$$= 2n^2 + 3n - 11$$
Therefore $supp\ \alpha^+(P_n) + supp\ \alpha^+[Q(P_n)] = 2n - 3 + 2n^2 + 3n - 11$
$$= 2n^2 + 5n - 14$$
Now when $= 2$, $supp\ \alpha^+[Q(P_n)] = 2$
Hence $supp\ \alpha^+(P_2) + supp\ \alpha^+[Q(P_2)] = 3$.
Also $\alpha^\times[Q(P_n)] = \prod_{u\in S_2} mult(u)$
$$= mult(v_1) \times mult(e_2) \times mult(e_4) \times \dots \times mult(e_{n-4}) \times mult(e_{n-2})$$
$$= 3n(n+1)^{n-2} \times 12(n+1)^2[16(n+1)^2]^{\left(\frac{n-6}{2}\right)} \times 12(n+1)^2$$
$$= 432n(n+1)^{2n-4}4^{n-6}$$
Therefore $supp\ \alpha^\times(P_n) + supp\ \alpha^\times[Q(P_n)] = 2^{n-2} + 432n(n+1)^{2n-4}4^{n-6}$.
Now when $= 2$, $supp\ \alpha^\times[Q(P_n)] = 2$
Hence $supp\ \alpha^\times(P_2) + supp\ \alpha^\times[Q(P_2)] = 2$.

Theorem. 10: Let $P_n \bar{P}_n$ be a complementary prism of P_n, where $n \ge 2$. Then
$supp\ \alpha^+(P_2) + supp\ \alpha^+(\ P_2\,\bar{P}_2\ ) = 6, supp\ \alpha^+(P_3) + supp\ \alpha^+(\ P_3\,\bar{P}_3\ ) = 16, supp\ \alpha^+(P_4) + supp\ \alpha^+(\ P_4\,\bar{P}_4\ ) = 29,$
$supp\ \alpha^+(P_n) + supp\ \alpha^+(\ P_n\,\bar{P}_n\ ) =$
$$\begin{cases} \frac{5}{2}n^2 - \frac{11}{2}n + 11 \quad if\ n\ is\ odd, n\neq 3 \\ \frac{5}{2}n^2 - 6n + 14 \quad if\ n\ is\ even, n\neq 2\ and\ \ n\neq 4 \end{cases}$$
$supp\ \alpha^\times(P_2) + supp\ \alpha^\times(\ P_2\,\bar{P}_2\ ) = 5,$
$supp\ \alpha^\times(P_3) + supp\ \alpha^\times(\ P_3\,\bar{P}_3\ ) = 76,$
$supp\ \alpha^\times(P_4) + supp\ \alpha^\times(\ P_4\,\bar{P}_4\ ) = 6565$ and
$supp\ \alpha^\times(P_n) + supp\ \alpha^\times(\ P_n\,\bar{P}_n\ ) =$
$$\begin{cases} 2^{n-1} + (n-2)^{\frac{5n-17}{2}} \times 2 \times 3^{n-1} \times (n-1)^3\ if\ n\ is\ odd, n\neq 3 \\ 2^{n-2} + (n-2)^{\frac{5n}{2}-11} \times 3^n \times (n-1)^5\ \ if\ n\ is\ even, n\neq 2\ and\ \ n\neq 4 \end{cases}$$
.

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices of the path P_n and $\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n$ be the vertices in the copy of $\bar{P}_n$. In $P_n\,\bar{P}_n$, $\bar{v}_i$ is non-adjacent with $\bar{v}_{i-1}$ and $\bar{v}_{i+1}$ for $2 \le i \le n-1$, $\bar{v}_1$ is non-adjacent with $\bar{v}_2$ and

$\bar{v}_n$ is non-adjacent with $\bar{v}_{n-1}$. v_i is adjacent with v_{i-1}, v_{i+1} and $\bar{v}_i$ for $2 \le i \le n-1$, v_1 is adjacent with $\bar{v}_1$ and v_2 similarly v_n is adjacent with $\bar{v}_n$ and v_{n-1}. Then $deg\ \bar{v}_i = n-2,\ 2 \le i \le n-1, deg\ \bar{v}_1 = deg\ \bar{v}_n = n-1,\quad deg\ v_i = 3,$ $2 \le i \le n-1$ and $deg\ v_1 = deg\ v_n = 2$.

Case (i): Suppose n is odd and $n \ge 5$. In this case the end vertices namely $\bar{v}_1$ and $\bar{v}_n$ have the maximum support when compared with other $v_i's$ for $2 \le i \le n-1$. So choose either $\bar{v}_1, \bar{v}_2$ or $\bar{v}_{n-1}, \bar{v}_n$ vertices with $v_3, v_5, \dots, v_n$ or $v_1, v_3, \dots, v_{n-2}$ to get the maximum support. Hence $S_1 = \{\bar{v}_1, \bar{v}_2, v_3, v_5, \dots, v_n\}$ and $S_2 = \{\bar{v}_{n-1}, \bar{v}_n, v_1, v_3, \dots, v_{n-2}\}$ are the only two maximum independent sets of $P_n \bar{P}_n$. Consider the set S_1. The proof is similar to the other set.

$$\begin{aligned} supp(\bar{v}_1) &= \sum_{v \in N(\bar{v}_1)} deg\ v \\ &= deg\ \bar{v}_3 + deg\ \bar{v}_4 + \cdots + deg\ \bar{v}_n + deg\ v_1 \\ &= (n-2) + (n-2) + \cdots + (n-2) + (n-1) + 2 \\ &= (n-2)(n-3) + (n-1) + 2 \\ &= n^2 - 4n + 7 \end{aligned}$$

$$\begin{aligned} supp(\bar{v}_2) &= \sum_{v \in N(\bar{v}_2)} deg\ v \\ &= deg\ \bar{v}_4 + deg\ \bar{v}_5 + deg\ \bar{v}_6 + \cdots + deg\ \bar{v}_{n-1} + deg\ \bar{v}_n + deg\ v_2 \\ &= (n-2) + (n-2) + \cdots + (n-2) + (n-1) + 3 \\ &= (n-4)(n-2) + (n-1) + 3 \\ &= n^2 - 5n + 10 \end{aligned}$$

$$\begin{aligned} supp(v_3) &= \sum_{v \in N(v_3)} deg\ v \\ &= deg\ v_2 + deg\ v_4 + deg\ \bar{v}_3 \\ &= 3 + 3 + (n-2) = n+4 \end{aligned}$$

Similarly

$$supp(v_5) = supp(v_7) = \cdots = supp(v_{n-2}) = n+4$$

$$\begin{aligned} supp(v_n) &= \sum_{v \in N(v_n)} deg\ v \\ &= deg\ v_{n-1} + deg\ \bar{v}_n \\ &= 3 + (n-1) = n+2 \end{aligned}$$

Hence $supp\ \alpha^{+}(P_n \bar{P}_n) = \sum_{v \in S_1} supp\ (v) = \frac{5}{2}n^2 - \frac{15}{2}n + 13$

Therefore $supp\ \alpha^{+}(P_n) + supp\ \alpha^{+}(P_n \bar{P}_n) = \frac{5}{2}n^2 - \frac{11}{2}n + 11$

Now when $n = 3$ $supp\ \alpha^{+}(P_n \bar{P}_n) = 12$

Hence $supp\ \alpha^{+}(P_3) + supp\ \alpha^{+}(P_3 \bar{P}_3) = 16$

Now $supp\ \alpha^{\times}(P_n \bar{P}_n) = \prod_{u \in S_1} mult(u)$

$$= mult(\bar{v}_1) \times mult(\bar{v}_2) \times mult(v_3) \times mult(v_5) \times \dots \times mult(v_n)$$

$$= (n-2)^{n-3} \times 2(n-1)(n-2)^{n-4} \times 3(n-1)[9(n-2)]^{\left(\frac{n-3}{2}\right)} \times 3(n-1)$$

$$= (n-2)^{\frac{5n-17}{2}} \times 2 \times 3^{n-1} \times (n-1)^3$$

Therefore $supp\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}(P_n \bar{P}_n) = 2^{n-1} + (n-2)^{\frac{5n-17}{2}} \times 2 \times 3^{n-1} \times (n-1)^3$

Now when $n = 3$ $supp\ \alpha^{\times}(P_n \bar{P}_n) = 72$

Hence $supp\ \alpha^{\times}(P_3) + supp\ \alpha^{\times}(P_3 \bar{P}_3) = 76$.

Case (ii): Let n be even and $n \ge 6$. $S_3 = \{\bar{v}_2, \bar{v}_3, v_1, v_4, v_6, \dots, v_n\}$ and $S_4 = \{\bar{v}_{n-2}, \bar{v}_{n-1}, v_n, v_1, v_3, \dots, v_{n-3}\}$ are the only two maximum independent sets of $P_n \bar{P}_n$. Consider the set S_3. The proof is similar to the other set.

$$\begin{aligned} supp(\bar{v}_2) &= \sum_{v \in N(\bar{v}_2)} deg\ v \\ &= deg\ \bar{v}_4 + deg\ \bar{v}_5 + \cdots + deg\ \bar{v}_n + deg\ v_2 \\ &= (n-2) + (n-2) + \cdots + (n-2) + (n-1) + 3 \\ &= (n-4)(n-2) + (n-1) + 3 \\ &= n^2 - 5n + 10 \end{aligned}$$

$$\begin{aligned} supp(\bar{v}_3) &= \sum_{v \in N(\bar{v}_3)} deg\ v \\ &= deg\ \bar{v}_1 + deg\ \bar{v}_5 + deg\ \bar{v}_6 + \cdots + deg\ \bar{v}_n + deg\ v_3 \\ &= (n-1) + (n-2) + (n-2) + \cdots + (n-2) + (n-1) + 3 \\ &= n^2 - 5n + 11 \end{aligned}$$

$$\begin{aligned} supp(v_1) &= \sum_{v \in N(v_1)} deg\ v \\ &= deg\ v_2 + deg\ \bar{v}_1 \\ &= 3 + (n-1) = n+2 \end{aligned}$$

$$\begin{aligned} supp(v_4) &= \sum_{v \in N(v_4)} deg\ v \\ &= deg\ v_3 + deg\ v_5 + deg\ \bar{v}_4 \\ &= 3 + 3 + n - 2 = n+4 \end{aligned}$$

Similarly $supp(v_6) = supp(v_8) = \cdots = supp(v_{n-2}) = n+4$

$$\begin{aligned} supp(v_n) &= \sum_{v \in N(v_n)} deg\ v \\ &= deg\ v_{n-1} + deg\ \bar{v}_n \\ &= 3 + (n-1) = n+2 \end{aligned}$$

Hence $supp\ \alpha^{+}(P_n \bar{P}_n) = \sum_{v \in S_3} supp\ (v)$

$$= \frac{5}{2}n^2 - 8n + 17$$

Therefore $supp\ \alpha^{+}(P_n) + supp\ \alpha^{+}(P_n \bar{P}_n) = \frac{5}{2}n^2 - 6n + 14$

Now when $= 2$ $supp\ \alpha^{+}(P_n \bar{P}_n) = 5$. Hence $supp\ \alpha^{+}(P_2) + supp\ \alpha^{+}(P_2 \bar{P}_2) = 6$

when $= 4$ $supp\ \alpha^{+}(P_n \bar{P}_n) = 24$. Hence $supp\ \alpha^{+}(P_4) + supp\ \alpha^{+}(P_4 \bar{P}_4) = 29$

Also $supp\ \alpha^{\times}(P_n\,\bar{P}_n) = \prod_{u\in S_3} mult(u)$

$$= mult(\bar{v}_2) \times mult(\bar{v}_3) \times mult(v_1) \times mult(v_4) \times mult(v_6) \times \ldots \times mult(v_n)$$

$$= (n-2)^{n-4} \times 3(n-1)(n-2)^{n-5} \times 3(n-1)^2 \times 3(n-1)[9(n-2)]^{\left(\frac{n-4}{2}\right)} \times 3(n-1)$$

$$= (n-2)^{\frac{5n}{2}-11} \times 3^n \times (n-1)^5$$

Therefore $supp\ \alpha^{\times}(P_n) + supp\ \alpha^{\times}(P_n\,\bar{P}_n) = 2^{n-2} + (n-2)^{\frac{5n}{2}-11} \times 3^n \times (n-1)^5$

Now when $= 2$ $supp\ \alpha^{\times}(P_n\,\bar{P}_n) = 4$. Hence $supp\ \alpha^{\times}(P_2) + supp\ \alpha^{\times}(P_2\,\bar{P}_2) = 5$

when $= 4$ $supp\ \alpha^{\times}(P_n\,\bar{P}_n) = 6561$. Hence $supp\ \alpha^{\times}(P_4) + supp\ \alpha^{\times}(P_4\,\bar{P}_4) = 6565$.

Conclusion:

In this paper, Nordhaus-Gaddum type relations on the open support independence number of some path related graphs under addition and multiplication are studied.

Authors' declaration:

- Conflicts of Interest: None.
- Ethical Clearance: The project was approved by the local ethical committee at The Madurai Diraviyam Thayumanavar Hindu College, India.

Authors' contributions:

This work was carried out in collaboration between the authors. Mary Vithya T conceived of the presented idea and developed the theory through discussions with Murugan K. All authors read and approved the final manuscript.

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Open Access | Baghdad Science Journal | P-ISSN: 2078-8665
2023, 20(1 Special Issue) ICAAM: 371-379 | E-ISSN: 2411-7986

علاقات من نوع نوردهاوس-جادوم في عدد استقلالية الدعم المفتوح لبعض الرسوم البيانية ذات الصلة بالمسار تحت الجمع والضرب

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قسم الرياضيات ، كلية مادوراي ديرافيام تايومانافار الهندوسية ، تيرونلفيلي ، تاميل نادو ، الهند.

الخلاصة:
في هذا البحث ، تمت دراسة العلاقات من نوع نوردهاوس - جادوم في عدد استقلالية الدعم المفتوح لبعض الرسوم البيانية المشتقة من الرسوم البيانية المتعلقة بالمسار تحت الجمع والضرب.

الكلمات الرئيسية: الرسوم البيانية المشتقة، علاقات نوع نوردهاوس - جادوم، عدد استقلال الدعم المفتوح،عدد استقلال الدعم المفتوح تحت الجمع، عدد استقلال الدعم المفتوح تحت الضرب.