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Some New Results on Lucky Labeling

J. Ashwini [1,3]* *S.Pethanachi Selvam* [2] *R.B.Gnanajothi* [3]

[1]School of Mathematics, Madurai Kamaraj University, Madurai, Tamilnadu, India.
[2]Department of Mathematics, The Standard Fireworks Rajaratnam College for Women, Sivakasi, Tamilnadu, India.
[3]Department of Mathematics, V.V. Vanniaperumal College for Women, Virudhunagar, Tamilnadu, India.
*Corresponding author: aswiniprakash777@gmail.com
E-mail addresses: pethanachi-mat@sfrcollege.edu.in , gnanajothi_pcs@rediff.com
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Abstract:

Czerwi'nski et al. introduced Lucky labeling in 2009 and Akbari et al and A.Nellai Murugan et al studied it further. Czerwi'nski defined Lucky Number of graph as follows: A labeling of vertices of a graph G is called a Lucky labeling if $S(u) \neq S(v)$ for every pair of adjacent vertices u and v in G where $S(v) = \sum_{u \in N(v)} l(u)$. A graph G may admit any number of lucky labelings. The least integer k for which a graph G has a lucky labeling from the set 1, 2, k is the lucky number of G denoted by $\eta(G)$. This paper aims to determine the lucky number of Complete graph K_n, Complete bipartite graph $K_{m,n}$ and Complete tripartite graph $K_{l,m,n}$. It has also been studied how the lucky number changes while adding a graph G with K_n and deleting an edge e from K_n.

Keywords: Complete graph, Complete Bipartite graph, Complete Tripartite graph, Lucky Labeling, Lucky Number.

Introduction:

Graph Labeling is one of the most interesting areas in Graph Theory. Lucky Labeling is one among them yet to be studied in detail. As in other labelings, trial and error method is used to determine a lucky labeling of given graph. But it is a hectic job to determine the lucky number of a given graph since it is to be justified that there is no lucky labeling with fewer numbers. Taking little diversions and defining new labelings happen to be the second step. Edge lucky labeling has been introduced by us in the paper "An Exploration into Lucky Labeling". R. Sridevi and S. Ragavi have studied the Lucky Edge Labeling[1,2], of K_n and Special types of graphs. Numerous studies have been conducted on various families of graphs using various fortunate labelling techniques, including Lucky Edge Lableing[3,4], Proper Lucky Labeling[5], e Lucky Labeling[6], d-Lucky Labeling[7], Proper d-Lucky Labeling[8] etc. This paper aims to find the lucky number of prominent families[9-11] namely Complete graphs and Complete Bipartite graphs.

Preliminaries

The book[12], "Graph Theory with Applications" by J.A.Bondy and U.S.R.Murthy is followed for definitions of Complete graph, Complete Bipartite graph and Complete Tripartite graph.

Definition. 1: [13] Let $G_1=(V_1,X_1)$ and $G_2=(V_2,X_2)$ be two graphs with $V_1 \cap V_2 = \phi$. Then the sum G_1+G_2 is defined as $G_1 \cup G_2$ together with all the lines joining points of V_1 to points of V_2. Similarly the cartesian product $G_1 \times G_2$ is defined as having $V = V_1 \times V_2$ and $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent if $u_1 = v_1$ and u_2 is adjacent to v_2 in G_2 or u_1 is adjacent to v_1 in G_1 and $u_2 = v_2$.

Definition. 2: [14] Suppose that G is a graph and $f: V(G) \rightarrow N$ is a labeling of the vertices of G. Let S(v) denote the sum of labels of overall neighbors of the vertex in G. A labeling f of G is called lucky if $S(u) \neq S(v)$ for every pair of adjacent vertices u and v in G. The least integer k for which a graph G has a lucky labeling from the set {1,2,…,k} is the lucky number of G, denoted by $\eta(G)$.

Example 1: The following graph (Fig.1) G admits Lucky Labeling and its lucky number is 2.

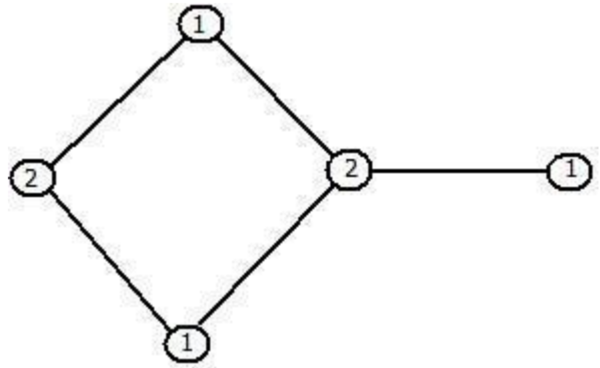

Figure 1. Lucky Labelling of a graph

Main Results:

Theorem. 1: $\eta(K_n) = n$

Proof: Let $V(K_n) = \{v_1, v_2, \dots, v_n\}$. Label v_i with i for i varies from 1 to n.

$S(v_i) = \sum_{j=1 \neq i} j = \frac{n(n+1)}{2} - i$; For $i \neq j, S(v_i) \neq S(v_j)$.

Hence it is a lucky labeling of K_n and lucky number of K_n is less than or equal to n.

Suppose there exists a lucky labeling with maximum label strictly less than n.

In that case, at least one label must be repeated.

Let $l(v_1) = r = l(v_2)$

Then $S(v_1) = \sum_{i=3}^{n} l(v_i) + r = S(v_2)$, a contradiction.

Therefore $\eta(K_n) = n$.

Theorem 2: $\eta(K_{m,n}) = \begin{cases} 1 \text{ if } m \neq n \\ 2 \text{ if } m = n \end{cases}$

Proof: Let $V(K_{m,n}) = \{u_1, u_2, \dots, u_m, v_1, v_2, \dots, v_n\}$

$E(K_{m,n}) = \{u_i v_j \text{for i lying between 1 and m and j lying between 1 and n}\}$

Case. 1: $m = n$

Define $l(u_i) = 1$ for i lying between 1 and m and $l(v_j) = 2$ for j lying between 1 and n.

Then $S(u_i) = 2n$ and $S(v_j) = n$ for $1 \leq i, j \leq n$

Clearly $S(u_i) \neq S(v_j)$ for $1 \leq i, j \leq n$. Therefore $K_{m,n}$ admits lucky labeling with lucky number 2.

Case. 2: $m \neq n$

Define $l(u_i) = 1 = l(v_j)$ for i taking integer values from 1 and m and j lying between 1 and n.

Then $S(u_i) = n$ for i lying between 1 and m and $S(v_j) = m$ for j lying between 1 and n.

Clearly $S(u_i) \neq S(v_j)$ for any i and j.

Therefore $K_{m,n}$ admits lucky labeling with lucky number 1.

Illustration: The lucky labeling of $K_{2,3}$ (Fig.2) is given below

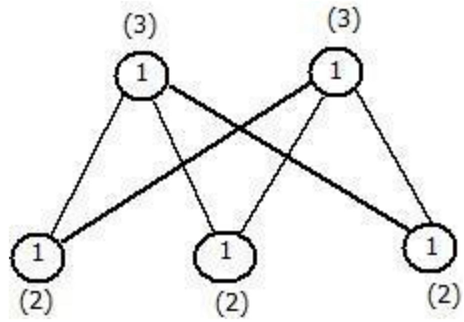

Figure 2. Lucky Labeling of $K_{2,3}$

Theorem. 3: Lucky number of $K_{l,m,n}$ is less than or equal to 3.

Proof: Let $V(K_{l,m,n}) = \{u_1, u_2, \dots, u_l, v_1, v_2, \dots, v_m, w_1, w_2, \dots, w_n\}$

$E(K_{l,m,n}) = \{u_i v_j, u_i w_k, v_j w_k / 1 \leq i \leq l, 1 \leq j \leq m, 1 \leq k \leq n\}$

Case. 1: l = m = n

Define $l(u_i) = 1$ for i lying between 1 and l; $l(v_j) = 2$ for j taking integer values from 1 and m; $l(w_k) = 3$ for k lying between 1 and n.

Then $S(u_i) = 2m + 3n$ for i taking integer values from 1 and 1.

$S(v_j) = l + 3n$ for j taking integer values from 1 and m.

$S(w_k) = n + 2m$ for k lying between 1 and n.

$S(u_i) = S(v_j)$ implies $l = 2m$, which is a contradiction.

$S(u_i) = S(w_k)$ implies $n = 0$, which is a contradiction.

$S(v_j) = S(w_k)$ implies $l = 2(m - n)$, which is a contradiction.

Therefore $K_{l,m,n}$ admits lucky labeling with lucky number 3.

Case. 2: $l = m$ and m $\neq n$

Define $l(u_i) = 1$ for i taking integer values from 1 and l. ; $l(v_j) = 2$ for j taking integer values from 1 and m; $l(w_k) = 1$ for k lying between 1 and n.

Then $S(u_i) = 2m + n$ for i taking integer values from 1 and 1 .

$S(w_k) = 2m + l$ for k lying between 1 and n.

$S(u_i) = S(v_j)$ implies $l = 2m$, which is a contradiction.

$S(u_i) = S(w_k)$ implies $n = l$, which is a contradiction.

$S(v_j) = S(w_k)$ implies $n = 2m$, which is a contradiction.

Therefore $K_{l,m,n}$ admits lucky labeling with lucky number 2.

Case. 3: $l \neq m \neq n$
Define $l(u_i) = 1 = l(v_j) = l(w_k)$ for i taking integer values from 1 and l, j taking integer values from 1 and m and k lying between 1 and n.
Then $S(u_i) = m + n$ for i taking integer values from 1 and 1.
$S(v_j) = l + n$ for j taking integer values from 1 and m.
$S(w_k) = m + l$ for k lying between 1 and n.
$S(u_i) = S(v_j)$ implies $l = m$, which is a contradiction.
$S(u_i) = S(w_k)$ implies $n = l$, which is a contradiction.
$S(v_j) = S(w_k)$ implies $n = m$, which is a contradiction.
Therefore $K_{l,m,n}$ admits lucky labeling with lucky number 1.

Theorem. 4: $\eta(K_n + G) \geq n$ for any n.
Proof: Let m be the number of vertices of the graph G. $V(K_n) = \{u_1, u_2, \dots, u_n\}$;
$V(G) = \{v_1, v_2, \dots, v_m\}$
Take $K_n + G = G'$
Let l be any labeling of the vertices of G′; Let $K = \sum_{j=1}^{m} l(v_j)$
$S_{G'}(u_i) = \sum_{j=1 \neq i}^{m} l(u_j) + K; S_{G'}(u_i) = S_{G'}(u_j)$ if and only if $S_{K_n}(u_i) = S_{K_n}(v_j)$
$S_{G'}(u_i) \neq S_{G'}(u_j)$ only when 1 is a lucky labeling of K_n.
So there should be at least n numbers to label G′.
Therefore $\eta(K_n + G) \geq n$ for any n.

Theorem. 5: $\eta(K_n + \overline{K_m}) = n$ for $n, m \geq 3$.
Proof: Let $G = K_n + \overline{K_m}$ where $n, m \geq 3$
Let $V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_m\}; E(G) = \{v_i v_j / 1 \leq i \neq j \leq n\} \cup \{v_i u_j / 1 \leq i \leq n, 1 \leq j \leq m\}$;
Label the vertices of G as follows
$l(v_i) = i$ for $1 \leq i \leq n$ and $l(u_j) = n$ for $1 \leq j \leq m$
Then $S(v_i) = \frac{n(n+1)}{2} - i + mn$ and $S(u_j) = \frac{n(n+1)}{2}$
Minimum $S(v_i)$ is attained at i = n.
i.e. $S(v_n) = \frac{n(n-1)}{2} + mn$
Claim: $S(v_i) \neq S(v_j)$ for $i \neq j$
Suppose $S(v_i) = S(v_j$; Then $\frac{n(n+1)}{2} - i + mn = \frac{n(n+1)}{2} - j + mn$
$\Longrightarrow i = j$, a contradiction.
Claim: $S(v_n) > S(u_j)$
$S(v_n) - S(u_j) = \frac{n(n-1)}{2} + mn = \frac{n(n+1)}{2} = n(m - 1) > 0$
It follows that $S(v_i) \neq S(u_j)$ for all i and j. If possible let $\eta(G) < n$
Then at least two $v_i's$ should receive same labels. Without loss of generosity assume that $l(v_1) = l(v_2)$. Then there exists a lucky labeling with lucky number number less than n. i.e there will be two vertices with same label.
Take $\sum_{i=1}^{n} l(v_i) + \sum_{i=1}^{n} l(u_j) = K($ say $). S(v_1) = K - l(v_2)$ and $S(v_2) = K - l(v_2)$
$l(v_1) = l(v_2) \Longrightarrow S(v_1) = S(v_2)$, a contradiction.
Hence $\eta(G) \geq n$.
Therefore $\eta(K_n + \overline{K_m}) = n$ for $n, m \geq 3$.

Theorem. 6: $\eta(K_2 + P_n) = 2$ for $n > 3$.
Proof: Let $V(K_2) = \{u_1, u_2\}; V(P_n) = \{v_1, v_2, \dots, v_n\}$
Take $G = K_2 + P_n$ for $n > 3$.
Let l be the labeling of the vertices of G. Define $l(u_1) = 1; l(u_2) = 2$ and

$$l(v_i) = \begin{cases} 1 & \text{for} \quad i = 1,5,9,13, \dots, 4n-3 \\ 2 & \text{for} \quad i = 2,4,6, \dots, 2n \\ 2 & \text{for} \quad i = 3,7,11, \dots, 4n-1 \end{cases}$$

Then for n even, $S(u_1) = \begin{cases} 7i + 2 & \text{for } i = 4,8, \dots, 4n \\ 7i + 5 & \text{for } i = 6,10, \dots, 4n+2 \end{cases}$

$$S(u_2) = \begin{cases} 7i + 1 & \text{for} \quad i = 4,8, \dots, 4n \\ 7i + 4 & \text{for} \quad i = 6,10, \dots, 4n+2 \end{cases}$$

For n odd, $S(u_1) = \begin{cases} 7i + 3 & \text{for } i = 5,9, \dots, 4n-1 \\ 7i + 7 & \text{for } i = 7,11, \dots, 4n+3 \end{cases}$

$$S(u_2) = \begin{cases} 7i + 2 & \text{for } i = 5,9, \dots, 4n-1 \\ 7i + 6 & \text{for} \quad i = 7,11 \dots, 4n+3 \end{cases}$$

$S(v_1) = 5$ for all $n; S(v_2) = 6$ for $i = 2,4,6,8, \dots, 2n; S(v_3) = 7$ for $i = 3,5,7,9, \dots, 2n+1$
$S(v_4) = 5$ for $n = 4; S(v_5) = 5$ for $n = 5; S(v_6) = 4$ for $n = 6; S(v_7) = 5$ for $n = 7$
$S(v_8) = 5$ for $n = 8; S(v_9) = 5$ for $n = 9; S(v_{10}) = 4$ for $n = 10$
Clearly $S(u_i) \neq S(v_j)$ and $S(u_1) \neq S(u_2)$ for i taking values 1 and 2 and j varies from 1 to n.
Therefore $\eta(K_2 + P_n) = 2$ for $n > 3$.

Illustration The lucky labeling of $K_2 + P_4$ (Fig.3) is given below

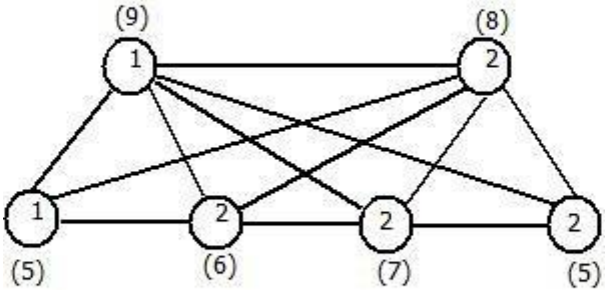

Figure 3. Lucky Labeling of $K_2 + P_4$

Theorem. 7: $\eta(P_m \times P_n) = 2$ for $m, n \geq 2$.

Proof: Consider the graph $P_m \times P_n$ where $m, n \geq 2$

Let $V(P_m \times P_n) = \{u_{ij}$ for i lying between 1 and m and j taking integer values from i to n$\}$

$E(P_m \times P_n) = \{u_{ij}u_{i,j+1}, u_{ij}u_{i+1,j}$ for i lying between 1 and $m-1$ and j taking integer values between 1 and $n-1\}$

Define $l(u_{ij}) = \begin{cases} 1 \text{ if i and j are of same parity} \\ 2 \text{ otherwise} \end{cases}$

Case. 1: Both m and n are odd Then $S(u_{11}) = S(u_{1n}) = S(u_{m1}) = S(u_{mn}) = 4$

$S(u_{1j}) = S(u_{mj}) = \begin{cases} 3 \text{ for even values of j from 2 to } m-1 \\ 6 \text{ for odd values of j from 3 to } n-2 \end{cases}$

$S(u_{i1}) = S(u_{in}) = 3$ for even values of i from 2 to $m-1$

$S(u_{ij}) = \begin{cases} 8 \text{ if } i \text{ and } j \text{ are of same even parity} \\ 4 \text{ if } i = 2,4,6 \end{cases}$

$S(u_{ij}) = \begin{cases} 8 \text{ if } i \text{ and } j \text{ are of same even parity} \\ 4 \text{ if } i = 2,4,6,\dots,m-1 \text{ and } j = 3,5,7,\dots,n-2 \end{cases}$

$S(u_{i1}) = S(u_{in}) = 6$ for odd values of i from 3 to $m-2$

$S(u_{ij}) = \begin{cases} 4 \text{ if } i \text{ and } j \text{ are of same odd parity} \\ 8 \text{ if } i = 3,5,7,\dots,m-2 \text{ and } j = 3,5,7,\dots,n-1 \end{cases}$

For $i = 1$, $S(u_{11}) = 4 \neq 3 = S(u_{12})$ and $S(u_{11}) = 4 \neq 3 = S(u_{21})$

When $i = 1$ and $j = 2,4,6,\dots,n-1$, $S(u_{12}) = 3 \neq 6 = S(u_{13})$ and $S(u_{12}) = 3 \neq 8 = S(u_{22})$

When $i = 2,4,6,\dots,m-1$ and $j = 1$, $S(u_{21}) = 3 \neq 8 = S(u_{22})$ and $S(u_{21}) = 3 \neq 6 = S(u_{13})$

When i and j are of same even parity, $S(u_{22}) = 8 \neq 4 = S(u_{23})$ and $S(u_{22}) = 8 \neq 4 = S(u_{32})$

When $i = 3,5,7,\dots,m-2$ and $j = 1$, $S(u_{31}) = 6 \neq 4 = S(u_{32})$ and $S(u_{31}) = 6 \neq 3 = S(u_{41})$

When i and j are of same odd parity, $S(u_{33}) = 8 \neq 4 = S(u_{34})$ and $S(u_{33}) = 8 \neq 4 = S(u_{43})$

Case 2: Both m and n are even

Then $S(u_{11}) = S(u_{mn}) = 4$ and $S(u_{1n}) = S(u_{m1}) = 2$

$S(u_{1j}) = \begin{cases} 3 \text{ for even values of j from 2 to } n-1 \\ 6 \text{ for odd values of j from 3 to } n-2 \end{cases}$

$S(u_{mj}) = \begin{cases} 6 \text{ for even values of j from 2 to } n-1 \\ 3 \text{ for odd values of j from 3 to } n-2 \end{cases}$

$S(u_{i1}) = 3$ for even values of i from 2 to $m-2$

$S(u_{in}) = 6$ for even values of i from 2 to $m-2$

$S(u_{ij}) = \begin{cases} 8 \text{ for even values of } i \text{ from 2 to } m-2 \text{ and for even values of j from 2 to } n-2 \\ 4 \text{ for even values of i from 2 to } m-2 \text{ and for odd values of j from 3 to } n-1 \end{cases}$

$S(u_{i1}) = 6$ for odd values of i from 3 to $m-1$

$S(u_{in}) = 3$ for odd values of i from 3 to $m-1$

$S(u_{ij}) = \begin{cases} 4 \text{ for even values of i from 2 to } m-2 \text{ and for even values of j from 2 to } n-2 \\ 8 \text{ for odd values of i from 3 to } m-2 \text{ and for odd values of j from 3 to } n-1 \end{cases}$

For $i = 1$, $S(u_{11}) = 4 \neq 3 = S(u_{12})$ and $S(u_{11}) = 4 \neq 3 = S(u_{21})$

When $i = 1$ and $j = 2,4,6,\dots,n-1$, $S(u_{12}) = 3 \neq 6 = S(u_{13})$ and $S(u_{12}) = 3 \neq 8 = S(u_{22})$

When $i = 2,4,6,\dots,m-1$ and $j = 1$, $S(u_{21}) = 3 \neq 8 = S(u_{22})$ and $S(u_{21}) = 3 \neq 6 = S(u_{13})$

When i and j are of same even parity, $S(u_{22}) = 8 \neq 4 = S(u_{23})$ and $S(u_{22}) = 8 \neq 4 = S(u_{32})$

When $i = 3,5,7,\dots,m-2$ and $j = 1$, $S(u_{31}) = 6 \neq 4 = S(u_{32})$ and $S(u_{31}) = 6 \neq 3 = S(u_{41})$

When i and j are of same odd parity, $S(u_{33}) = 8 \neq 4 = S(u_{34})$ and $S(u_{33}) = 8 \neq 4 = S(u_{43})$

The proof is similar for the other cases.

Illustration: The lucky labeling of $P_2 \times P_3$ (Fig.4) is given below

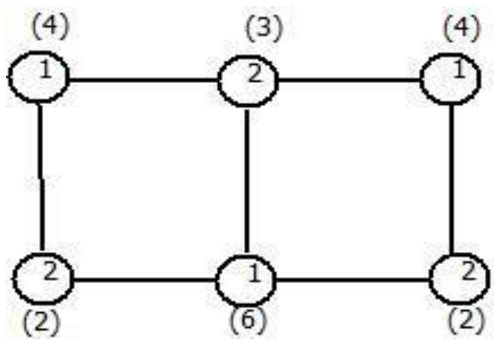

Figure 4. Lucky Labeling of $P_2 \times P_3$

Theorem. 8: $\eta(K_n \sim e) = n - 2$ for $n > 3$.

Proof: Let $V = \{v_1, v_2, \dots, v_n\}$

Consider $K_n \sim e$ where $E = \{e_i e_j / i < j\}$

Let $l(v_k) = \begin{cases} k \text{ for } k = 1,2,\ldots,i-1 \\ n-2 \text{ for } k = i \\ k-1 \text{ for } k = (i+1),(i+2),\ldots,(j-1) \\ n-3 \text{ for } k = j \\ k-2 \text{ for } k = (j+1),(j+2),\ldots,n \end{cases}$

Take $S_0 = \frac{(n-2)(n-1)}{2} + (2n-5)$

Then $A = \{S(v_k): k = 1,2,\ldots,i-1\} = \{S_0 - i + 1, S_0 - i + 2, \ldots, S_0 - 1\}$

$S(v_i) = S_0 - 2n + 5$

$B = \{S(v_k): k = (i+1),(i+2),\ldots,(j-1)\} = \{S_0 - j + 2, S_0 - j + 3, \ldots, S_0 - i\}$

$S(v_j) = S_0 - 2n + 5$

$C = \{S(v_k): k = (j+1),(j+2),\ldots,n\} = \{S_0 - n + 2, S_0 - n + 3, \ldots, S_0 - j + 1\}$

$\min A = S_0 - i + 1 > S_0 - i = \max B$

$\min B = S_0 - j + 2 > S_0 - j + 1 = \max C$

In all cases, $S(v_k) \neq S(v_k + 1)$

Therefore $\eta(K_n \sim e) = n - 2$ for $n > 3$.

Conclusion:

While working on Complete graphs and Complete Bipartite graphs, it has been observed that most of the families have lucky number less than or equal to two. This motivates to attempt the characterization problems on the graphs with lucky number 1 in our previous paper. The characterization of graphs with lucky number 2 seems to be the next interesting research problem.

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Authors' Declaration:

- Conflicts of Interest: None
- We hereby confirm that all the Figures in the manuscript are ours. Besides, the Figures which are not ours have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at The Madurai Kamaraj University, India.

Authors' Contribution Statement:

This work was carried out in collaboration between all authors. J.A., S.P.S. and R.B.G. contributed to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript.

Ethical Clearance: The project was approved by the local ethical committee in The Standard Fireworks Rajaratnam College for Women, Sivakasi, India.

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بعض النتائج الجديدة على وضع العلامات المحظوظة

ج .أشويني[1,3] س .بيثاناتشي سيلفام [2] ر.ب.غناجوثي[3]

[1]كلية الرياضيات، جامعة مادوراي كاماراج، مادوراي، تاميل نادو ، الهند.
[2] قسم الرياضيات، كلية راجاراتنام القياسية للألعاب النارية للبنات، سيفاكاسي، تاميل نادو، الهند.
[3] قسم الرياضيات، كلية V.V.Vanniaperuma للبنات، فيرودوناغار، تاميل نادو، الهند.

الخلاصة:
قدم .Czerwi'nski et al وضع العلامات المحظوظة في عام 2009 وقام أكبري وآخرون والدكتور A.Nellai Murugan et al بدراسته بشكل أكبر .عرف Czerwi'nski رقم الحظ للرسم البياني على النحو التالي :يطلق على تسمية رؤوس الرسم البياني G تسمية الحظ إذا كانت $S(u) \neq S(v)$لكل زوج من الرؤوس المجاورة u و v في G حيث $S(v) = \sum_{u \in N(v)} l(u)$. قد يسمح الرسم البياني G بأي عدد من علامات الحظ .أقل عدد صحيح k الذي يحتوي الرسم البياني G 1,2,...,k هو رقم الحظ ل G المشار إليه ب η (G). يهدف هذا البحث إلى تحديد عدد الحظ للرسم البياني الكامل Kn والرسم البياني الثنائي الكامل $K_{m,n}$ والرسم البياني الثلاثي الكامل $K_{l,m,n}$.. تمت دراسة أيضا كيفية تغير رقم الحظ أثناء إضافة رسم بياني G مع Kn وحذف حافة e من .Kn

الكلمات المفتاحية: رسم بياني كامل، رسم بياني ثنائي كامل، الرسم البياني الثلاثي الكامل، وضع العلامات المحظوظة، رقم الحظ.