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Oscillation of the Impulsive Hematopoiesis Model with Positive and Negative Coefficients

Iman Sabeeh Hadeed [*1], ***Hussain Ali Mohamad*** [2]

[1]Department of Mathematics, College of Science, University of Baghdad, Baghdad, Iraq
[2]Department of Mathematics, College of Science for Woman, University of Baghdad, Baghdad, Iraq.
*Corresponding Author.

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Abstract

In this paper, the problem of oscillating solutions for an impulsive hematopoiesis model with positive and negative coefficients is investigated. There are several evolutionary processes, which frequently encounter dramatic shifts at specific times and are sensitive to short-term perturbations. As a result, we construct several oscillation criteria that are either brand-new or enhance many of recent findings in the literature. We also give illustrations of how impulsiveness affects the oscillating solutions of the hematopoiesis model.

Keywords: Delay differential equations, Hematopoiesis model, Impulsive, Oscillation, Sufficient conditions.

Introduction

The impulsive differential equations (IDE) are always defined as ordinary differential equations in addition to the impulsive condition. The differential equation is a powerful tool for modeling the continuous events and processes seen in biology, population dynamics, biotechnologies, industrial robots, etc. It can be used, for instance, to describe some physiological phenomenon. The phenomenon of oscillations is observed in biological models for instance hematopoiesis model. Under some conditions, the solutions of the delay differential equation would exhibit non-oscillatory or oscillatory properties[1-3]. Since those phenomena can be conceived of as impulses, the IDE is more suitable than the conventional differential equation for simulating those discontinuous processes brought on by impulses[4-7].

Many authors looked for sufficient conditions to ensure oscillatory property for different differential equations. As a result, they established a lot of papers for oscillatory theory for ordinary [8-11] and delay differential equations [12-15] The oscillation criteria of various IDE, including super-half-linear IDE, half-linear impulsive differential equations, and mixed nonlinear differential equations, were obtained by researchers Ozbekler and Zafer, who published the impulsive differential equations as well [16]. The methodology for analyzing impulsive differential equations were supplied by Agarwal, Karakoc, and Zafer, who also offered a summary of various findings on the oscillation of IDE up to 2010 (see [7, 17]). If the effects of impulses are taken into account, the question is whether or not the non-oscillatory or

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oscillatory features of delay differential equations remain invariant.
The most of parameters in real-world events are not fixed constants; instead, they are estimated using specific statistical techniques, and the estimates get better with time. Additionally, the environment's variability has a significant impact on some ecological and biological dynamical systems. Since the selection pressures acting on systems in an oscillating environment are different from those acting on systems in a stable environment, the impacts of a periodically fluctuating environment are particularly crucial for evolutionary theory. However, some dynamical systems exhibit rapid changes at specific points in their evolution, which serves to distinguish them from others. The weather, the availability of resources, the use of drugs or radiation in the treatment of hematological illnesses, food supplies, pharmacological factors, mating behaviors, and other seasonal influences are the main causes of this. Figure 1 illustrates how those events are based on the so-called IDE [18].

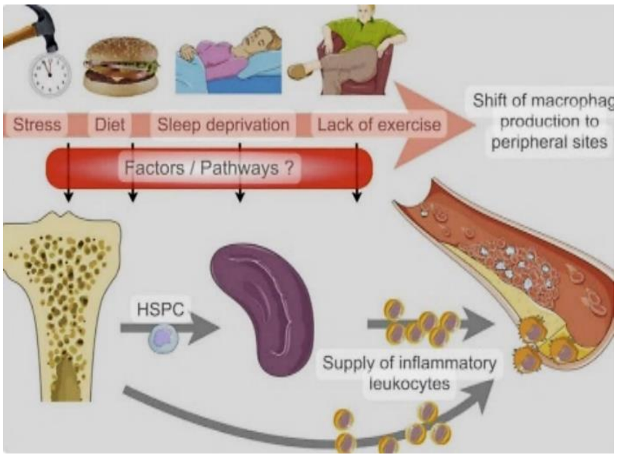

Figure 1. Effects of lifestyle on hematopoiesis [19].

Impulsive hematopoiesis model is an extension of the traditional hematopoiesis model that incorporates the concept of rapid bursts or "impulses" of blood cell production in response to certain stimuli or conditions. This model suggests that under specific circumstances such as infection, injury, or stress, there can be a rapid expansion of certain blood cell lineages to meet increased demand or replenish depleted populations[20]. This study introduces an impulsive hematopoiesis model that oscillates with both positive and negative coefficients. Our research for the impulsive hematopoiesis model introduced linear impulses that are consistent with the use of drugs or radiation to treat hematological illnesses when the necessary circumstances for its oscillation are present. The term "impulsive hematopoiesis model" refers to stem cells that temporarily halt producing new blood cells. We focus on researching fixed moment type oscillation requirements for the impulsive hematopoiesis model with positive and negative coefficients to achieve this goal. Regarding the suggested model, new conclusions for requirements for oscillatory behavior of specific kinds of the first order were established.

Mathematical Tools

The impulsive hematopoiesis model of the positive and negative coefficients:

$$\left.\begin{array}{ll} \hbar'(t)+\delta(t)\hbar(t)-\beta(t)V(\hbar(\tau(t)))=0, & t\neq t_k \\ \hbar(t_k^+)+b_k\hbar(t_k)=a_k\,\hbar(t_k) & , t=t_k \end{array}\right\} k=1,2,\ldots \quad 1$$

where $V\left(\hbar(\tau(t))\right)=\frac{1}{1+\left(\hbar(\tau(t))\right)^n}$ is the flux function, t_k^+ is the impulsive moment points and $\delta,\beta\in C([t_0,\infty);R^+)$, and $\tau(t)\in C([t_0,\infty);R)$, $\lim_{t\to\infty}\tau(t)=\infty$, when τ is a strictly increasing function. The function $\tau^{-1}(t)$ is the inverse of the function $\tau(t)$. Let's introduce an invariant oscillation transformation based on the analogous procedure in [4].

$$\hbar(t)=H(t)-K, \quad 2$$

when K is the unique positive equilibrium point of Eq 1. $\hbar(t)$ oscillates about K if and only if $H(t)$ oscillates about zero. Then, Eq 1 can be reduced to

$$\left.\begin{array}{ll} H'(t)+\delta(t)H(t)-\eta(t)G(H(\tau(t)))=0, & t\neq t_k \\ H(t_k^+)+b_kH(t_k)=a_k\,H(t_k) & , t=t_k \end{array}\right\} k=1,2,\ldots \quad 3$$

where $\eta(t)G\left(H(\tau(t))\right)=K\delta(t)+\beta(t)V(\hbar(\tau(t)))$.

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To obtain the desired outcomes, the following lemmas are essential[6].

Lemma 1: (i) Suppose that $g, h: [t_0, \infty) \to R$ are continuous functions, $h(t) \geq t$ and $h'(t) \geq 0, g(t) \geq 0$ eventually for $t \geq t_0$. If

$$\liminf_{t\to\infty} \int_t^{h(t)} g(s)ds > \frac{1}{e}, \qquad 4$$

then the inequality $x'(t) - g(t)x(h(t)) \geq 0$ has no eventually positive solutions.

(ii). Assume that $g, h \in C[t_0, \infty) \to [0, \infty)$, $g(t) \geq 0$, $\lim_{t\to\infty} h(t) = \infty$ and $h(t) \leq t$. If

$$\liminf_{t\to\infty} \int_{h(t)}^{t} g(s)ds > \frac{1}{e}, \qquad 5$$

then the inequality $x'(t) + g(t)x(h(t)) \leq 0$ has no eventually positive solutions.

Lemma 2: Assume that

I. $f, g, y, \tau, \gamma \in C([t_0, \infty); \mathbb{R})$, $f(t) < 0$, $\lim_{t\to\infty} f(t)$ exist, $0 < g(t) \leq 1$,

$$t > \tau(t), \gamma(t) \geq t, t_0 \leq t, \lim_{t\to\infty} \tau(t) = \infty$$

and

$$y(t) \leq f(t) + g(t) \max\{x(s): \tau(t) \leq s \leq \gamma(t)\}, \quad t \geq t_0 \qquad 6$$

then $y(t)$ cannot be nonnegative for $t \geq t_1 \geq t_0$.

II. Assume that $f, g, x, \tau, \gamma \in C\big[[t_0, \infty); \mathbb{R}\big]$, $f(t) > 0$, $\lim_{t\to\infty} f(t)$ exist,

$g(t) \geq 1, \tau(t) < t, \gamma(t) \geq t,\ t \geq t_0,\ \lim_{t\to\infty} \tau(t) = \infty$ and

$$y(t) \geq f(t) + g(t) \min\{y(s): \tau(t) \leq s \leq \gamma(t)\}, t \geq t_0. \qquad 7$$

then $y(t)$ cannot be nonpositive for $t \geq t_1 \geq t_0$.

Results and Discussion

In this work, the first order impulsive hematopoiesis model was investigated, to obtain sufficient conditions for oscillation of all solutions of the hematopoiesis model.

The following lemmas are proven to have the main results.

Lemma 3: Suppose that $H(t)$ is an eventually positive solution to Eq 3 such that:

$$W(t) = H(t) - \int_t^{\tau(t)} \delta(u)H(u)du, \qquad 8$$

where $t_k < t < \tau(t) \leq t_{k+1}$ and $\tau(t_k)$ is not an impulsive point, in addition to:

H_1: $G(H(u)) \geq \gamma_1 H(u), \gamma_1 > 0$,
H_2: $[\gamma_1\eta(t) - \delta(\tau(t))(\tau(t))'] \geq 0,\ t \in (t_k, t_{k+1}]$,

H_3: There exist two sequences of positive real numbers a_k and b_k such that $(a_k - b_k) \geq 1$, $k = 1,2,\ldots$ where $a_k = a_i, b_k = b_i$ and $\tau(t_k) = t_i, i < k$,

H_4: $\limsup_{t\to\infty} \left(\int_{\tau(t_k)}^{t_k} \delta(u)du\right) \leq 1, t \in (t_k, t_{k+1}]$, $k = 1,2,3 \ldots$. Then $W(t)$ is eventually positive and nondecreasing function.

Proof: Let $H(t)$ be an eventually positive solution of Eq 3 which is $H(t) > 0, H(\tau(t)) > 0,$ *and*, $t \geq t_0$. Differentiate Eq 8 for every interval $(t_k, t_{k+1}]$ where $k = 1,2,3 \ldots$ and use Eq 3, we obtain

$$\begin{aligned} W'(t) &= H'(t) - \delta(\tau(t))(\tau(t))'H(\tau(t)) \\ &\quad + \delta(t)H(t) \\ &= -\delta(t)H(t) + \eta(t)G(H(\tau(t))) \\ &\quad - \delta(\tau(t))(\tau(t))'H(\tau(t)) \\ &\quad + \delta(t)H(t) \end{aligned}$$

$$= \eta(t)G\left(H(\tau(t))\right) - \delta(\tau(t))(\tau(t))'H(\tau(t))$$

$$\geq \gamma_1\eta(t)H(\tau(t)) - \delta(\tau(t))(\tau(t))'H(\tau(t))$$

$$\geq [\gamma_1\eta(t) - \delta(\tau(t))(\tau(t))']H(\tau(t)). \qquad 9$$

Hence, $W(t)$ is a nondecreasing function for $\in (t_k, t_{k+1}]$, $k = 1,2,3 \ldots$.

To demonstrate that $W(t_k^+) \geq W(t_k)$ for $k = 1,2,3 \ldots$ In view of $0 < a_k - b_k \leq 1$,

$k = 1,2,\dots$, we have from Eq 8 concerning the condition H_3 when $\tau(t_k) = t_i$, $i < k$, then:

$$W(t_k^+) = H(t_k^+) - \int_{t_k}^{\tau(t_k)} \delta(u)H\big((u)\big)du$$
$$= (a_k - b_k)H(t_k) - \int_{t_k}^{\tau(t_k)} \delta(u)H\big((u)\big)du$$
$$\geq H(t_k) - \int_{t_k}^{\tau(t_k)} \delta(u)H\big((u)\big)du$$
$$= W(t_k)$$

Therefore, $W(t)$ is nondecreasing on $[t_0,\infty)$, $-\infty < \lim_{t\to\infty} W(t) = l \leq \infty$. We claim that $W(t) \geq 0$ for $t \in [t_0,\infty)$, otherwise $W(t) < 0$. Since $W(t)$ is nonincreasing on $[t_0,\infty)$, then $\lim_{t\to\infty} W(t) = l < 0$ and $W(t) \leq l$. From Eq 8, we have

$$H(t) \leq l + \int_{t_k}^{\tau(t_k)} \delta(u)H\big((u)\big)du$$
$$\leq l + \max_{t_k \leq s \leq \tau(t_k)} H(s) \int_{t_k}^{\tau(t_k)} \delta(u)du$$
$$\leq l + \max_{t_k \leq s \leq \tau(t_k)} H(s) . \qquad 10$$

According to Lemma 2, the above inequality cannot ultimately have a positive solution. This contradiction shows that $W(t) > 0$ for $t \neq t_k$. Since $W(t)$ is nonincreasing, so $W(t_k) > W(t) \geq 0$ for $t \in (t_k, t_{k+1}]$.To prove $W(t) > 0$ for t=t_k. First, we demonstrate, $W(t_k) > 0$ for $k = 1,2,\dots$. If that's not the case, then there are some $m \geq 0$ such that $W(t_m) = 0$, and $W(t_m^+) \leq 0$. Then $W(t_{m+1}) = 0$, integrating Eq 9 on $(t_m , t_{m+1}]$ yield:

$$W(t_{m+1}) = W(t_m^+) + \int_{t_m}^{t_{m+1}} [\gamma_1\eta(t) - \delta\big(\tau(t)\big)\big(\tau(t)\big)']H\big(\tau(t)\big)\,du$$
$$> W(t_m^+) \geq W(t_m) = 0 .$$

This contradiction demonstrates that $W(t_k) > 0$ for $k = 1,2 \dots$, as well as $W(t) \geq W(t_{k+1}) > 0$, for $t \in (t_k, t_{k+1}]$, $k = 1,2,\dots$. Thus, $W(t) > 0$ for $t \geq t_0$. And the proof is finished.

Lemma 4: Suppose that $H(t)$ is an eventually positive bounded solution to Eq 3 and let

$$W(t) = H(t) + \int_{\tau(t)}^{t} \delta(u)H\big((u)\big)du , \qquad 11$$

where $t_k < \tau(t) < t \leq t_{k+1}$and $\tau(t_k)$ is not impulsive point, let H_4 hold, in addition to:

H_5: $G\big(H(u)\big) \leq \gamma_1 H(u), \gamma_1 > 0$,
H_6: $[\gamma_1\eta(t) - \delta\big(\tau(t)\big)\big(\tau(t)\big)'] \leq 0,\ t \in (t_k, t_{k+1}]$,

H_7: there exist two sequences of positive real numbers a_k and b_k such that $0 < a_k - b_k \leq 1$, $k = 1,2,\dots$ where $a_k = a_i$, $b_k = b_i$ and $\tau(t_k) = t_i, i < k$. Then $W(t)$ is nonincreasing.

Proof: Assume that $H(t) > 0, H(\tau(t)) > 0 , t \in (t_k, t_{k+1}]$, Differentiate Eq. 11and use Eq. 3 for every interval $(t_k, t_{k+1}]$ where $k = 1,2,\dots$

$$W'(t) = H'(t) + \delta(t)H(t) - \delta\big(\tau(t)\big)\big(\tau(t)\big)'H\big(\tau(t)\big)$$
$$= \eta(t)G(H\big(\tau(t)\big)) - \delta(t)H(t) + \delta(t)H(t) - \delta\big(\tau(t)\big)(\tau(t))'\, H\big(\tau(t)\big)$$
$$= \eta(t)G\left(H\big(\tau(t)\big)\right) - \delta\big(\tau(t)\big)\big(\tau(t)\big)'H\big(\tau(t)\big)$$
$$\leq \gamma_1\eta(t)H\big(\tau(t)\big) - \delta\big(\tau(t)\big)\big(\tau(t)\big)'H\big(\tau(t)\big)$$
$$\leq [\gamma_1\eta(t) - \delta\big(\tau(t)\big)\big(\tau(t)\big)']H\big(\tau(t)\big) \leq 0. \qquad 12$$

Hence, $W(t)$ is nonincreasing for $t_k < t \leq t_{k+1}$ for $k = 1,2,3 \dots$.To demonstrate $W(t_k^+) \leq W(t_k)$ for $k = 1,2,\dots$, we have $0 < a_k \leq 1 \leq b_k$ and $\tau(t_k) \neq t_k$, $k = 1,2,\dots$ then:

$$W(t_k^+) = H(t_k^+) + \int_{\tau(t_k)}^{t_k} \delta(u)H\big((u)\big)du$$

$$\leq (a_k - b_k)H(t_k) + \int_{\tau(t_k)}^{t_k} \delta(u)H((u))du$$

$$\leq H(t_k) + \int_{t_k}^{\tau(t_k)} \delta(u)H((u))du$$
$$= W(t_k).$$

Thus, $W(t)$ is nonincreasing on $[t_0, \infty)$.

Lemma 5: Suppose that $H(t)$ is an eventually positive solution to Eq 3 and

$$w(t) = H(t) - \int_{\tau^{-1}(t)}^{t} \eta(u)G(H(\tau(u)))du, \qquad 13$$

where $t_k < \tau^{-1}(t) < t \leq t_{k+1}$, let H_1 and H_3 hold, as well as the following presumptions:

H_8: $[\gamma_1\eta(\tau^{-1}(t))(\tau^{-1}(t))' - \delta(t)] \geq 0,$
$t \in (t_k, t_{k+1}], \qquad k = 1,2, \ldots$

H_9: $\lim_{t\to\infty} \sup[\gamma_1 \int_{\tau^{-1}(t)}^{t} \eta(u)du \leq 1,$

where $t \in (t_k, t_{k+1}]$. Then $W(t)$ is eventually positive and nondecreasing function.

Proof: Let $H(t)$ be an eventually positive solution of Eq 3 that is $H(t) > 0, H(\tau(t)) > 0,$ *and*, $t \geq t_0$. Differentiate Eq 13 for every interval $(t_k, t_{k+1}]$ where $k = 1,2, \ldots$ and use Eq 13, we get

$$W'(t)$$
$$= H'(t) - \eta(t)G\left(H(\tau(t))\right)$$
$$+ \eta(\tau^{-1}(t))G(H(t))(\tau^{-1}(t))'$$
$$= -\delta(t)H(t) + \eta(t)G(H(\tau(t)))$$
$$- \eta(t)G(H(\tau(t)))$$
$$+ \eta(\tau^{-1}(t))G(H(t))(\tau^{-1}(t))'$$
$$= -\delta(t)H(t)$$
$$+ \eta(\tau^{-1}(t))G(H(t))(\tau^{-1}(t))'$$
$$\geq \gamma_1\eta(\tau^{-1}(t))H((t)(\tau^{-1}(t))'$$
$$- \delta(t)H(t)$$

$$\geq \left[\gamma_1\eta(\tau^{-1}(t))(\tau^{-1}(t))' - \delta(t)\right]H(t). \qquad 14$$

Hence, $W(t)$ is nondecreasing function for $t \in (t_k, t_{k+1}]$, $k = 1,2,3, \ldots$ .To prove that $W(t_k^+) \geq W(t_k)$ for $k = 1,2, \ldots$ In view of $a_k - b_k \geq 1$, regarding the condition H_3, and from Eq 13 when $\tau(t_k) = t_i$, $i < k$, then:

$$W(t_k^+) = H(t_k^+) - \int_{\tau^{-1}(t)}^{t_k} \eta(u)G(H((\tau(u)))du$$
$$= (a_k - b_k)H(t_k)$$
$$- \int_{\tau^{-1}(t_k)}^{t_k} \eta(u)G(H((\tau(u)))du$$
$$\geq H(t_k) - \int_{\tau^{-1}(t)}^{t_k} \eta(u)G(H((\tau(u)))du$$
$$= W(t_k)$$

Thus, $W(t)$ is nonincreasing on $[t_0, \infty)$, hence $-\infty \leq \lim_{t\to\infty} W(t) = l < \infty$.

We claim that $W(t) \geq 0$ for $t \in [t_0, \infty)$. Otherwise $W(t) < 0$. Since $W(t)$ is nonincreasing on $[t_0, \infty)$, then $\lim_{t\to\infty} W(t) = l < 0$ and $W(t) \leq l$. From Eq 13, we get

$$H(t) \leq l + \int_{t_k}^{\tau(t_k)} \eta(u)G(H((u)))du$$

$$\leq l + \max_{t_k \leq s \leq \tau(t_k)} \{H(s)\}\gamma_1 \int_{t_k}^{\tau(t_k)} \eta(u)du$$

$$\leq l + \max_{t_k \leq s \leq \tau(t_k)} H(s). \qquad 15$$

According to Lemma 2, the above inequality cannot ultimately have a positive solution. This contradiction shows that $W(t) > 0$ for $t \neq t_k$. Since $W(t)$ is nonincreasing, so $W(t_k) > W(t) \geq 0$ for $t \in (t_k, t_{k+1}]$.To prove $W(t) > 0$ for t=t_k, we first prove that $W(t_k) > 0$ for $k = 1,2,3, \ldots$. If this statement is incorrect, then some $m \geq 0$ exist such that $W(t_m) = 0$, and $W(t_m^+) \leq 0$. Then $W(t_{m+1}) = 0$, integrating Eq. 14 on $(t_m, t_{m+1}]$ yield:

$$W(t_{m+1}) = W(t_m^+)$$
$$+ \int_{t_m}^{t_{m+1}} [\gamma_1\eta(\tau^{-1}(t))(\tau^{-1}(t))'$$
$$- \delta(t)]H(t)\,dt$$

$$> W(t_m^+) \geq W(t_m) = 0.$$

This contradiction demonstrates that $W(t_k) > 0$ for $k = 1,2,3 \ldots$, as well as $W(t) \geq W(t_{k+1}) > 0$,

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for $t \in (t_k, t_{k+1}]$, $k = 1,2,\dots$. Thus $W(t) > 0$ for $t \geq t_0$. And that concludes the evidence.

Theorem 1: Let $W(t)$ be specified as in Eq 8 and that the hypotheses $H_1 - H_3$ are satisfied, in addition to:

$$\limsup_{t\to\infty} \int_t^{\tau(t)} \left[\gamma_1 \eta(s) - \delta\left(\left(\tau(s)\right)\right)\left(\tau(s)\right)'\right] d(s) > 1, \qquad 16$$

where $, t_k < t < \tau(t) \leq t_{k+1}$, $k = 1,2,\dots$. Then every solution of Eq 3 oscillates.

Proof: Suppose that $H(t)$ is the eventually positive solution of Eq 3, Lemma 3 demonstrates that $W(t)$ is a positive nonincreasing function since $w(t) < H(t)$and from Eq 9, we get

$$W'(t) \geq \left[\gamma_1 \eta(t) - \delta\left(\left(\tau(t)\right)\right)\left(\tau(t)\right)'\right] W\left(\tau(t)\right) \geq 0. \qquad 17$$

Integrating inequality Eq 17 from t to $\tau(t)$, we get

$$w\left(\tau(t)\right) - w(t) \geq w\left(\tau(t)\right) \int_t^{\tau(t)} \left[\gamma_1 \eta(s) - \delta\left(\left(\tau(s)\right)\right)\left(\tau(s)\right)'\right] d(s),$$

$$w\left(\tau(t)\right) \geq w\left(\tau(t)\right) \int_t^{\tau(t)} \left[\gamma_1 \eta(s) - \delta\left(\left(\tau(s)\right)\right)\left(\tau(s)\right)'\right] d(s),$$

$$\int_t^{\tau(t)} \left[\gamma_1 \eta(s) - \delta\left(\left(\tau(s)\right)\right)\left(\tau(s)\right)'\right] d(s) \leq 1,$$

which is a contradiction with the condition Eq 16.

Theorem 2: Let $W(t)$ be specified as in Eq 8 and that the hypotheses $H_1 - H_3$ are satisfied, in addition to:

$$\liminf_{t\to\infty} \int_t^{\tau(t)} \left[\gamma_1 \eta(s) - \delta\left(\tau(s)\right)\left(\tau(s)\right)'\right] d(s) > \frac{1}{e}, \qquad 18$$

where $, t_k < t < \tau(t) \leq t_{k+1}$, $k = 1,2,\dots$. Then every solution of Eq. 3 oscillates.

Proof: Suppose that $H(t)$ is eventually positive solution of Eq 3, Lemma 3 demonstrates that $W(t)$ is a positive nonincreasing function. Since $W(t) \leq H(t)$, and from Eq 9, we get

$$W'(t) \geq \left[\gamma_1 \eta(t) - \delta\left(\tau(t)\right)\left(\tau(t)\right)'\right] H \geq \left[\gamma_1 \eta(t) - \delta\left(\tau(t)\right)\left(\tau(t)\right)'\right] W\left(\tau(t)\right).$$

So,

$$W'(t) - \left[\gamma_1 \eta(s) - \delta\left(\left(\tau(s)\right)\right)\left(\tau(s)\right)'\right] W\left(\tau(t)\right) \geq 0.$$

The latest inequality cannot have an eventually positive solution, which is a contradiction, according to Lemma1, Lemma 3, and condition Eq 18.

Theorem 3: Let $W(t)$ defined as in Eq 13 and the assumptions $H_1 - H_3, H_8, H_9$ hold, in addition to:

$$\liminf_{t\to\infty} \int_t^{\tau(t)} \left[\gamma_1 \eta\left(\tau^{-1}(t)\right)\left(\tau^{-1}(t)\right)' - \delta(t)\right] d(s) > \frac{1}{e}, \qquad 19$$

where $, t_k < \tau^{-1}(t) < t \leq t_{k+1}$, $k = 1,2,3\dots$. Then every solution of Eq. 3 oscillates.

Proof: Suppose that $H(t)$ is eventually positive solution of Eq 3, Lemma 5 demonstrates that $W(t)$ is a positive nonincreasing function. Since $W(t) \leq H(t)$, and from Eq 14, we get

$$W'(t) \geq \left[\gamma_1 \eta\left(\tau^{-1}(t)\right)\left(\tau^{-1}(t)\right)' - \delta(t)\right] H(t) \geq \left[\gamma_1 \eta\left(\tau^{-1}(t)\right)\left(\tau^{-1}(t)\right)' - \delta(t)\right] W(t) \geq \left[\gamma_1 \eta\left(\tau^{-1}(t)\right)\left(\tau^{-1}(t)\right)' - \delta(t)\right] W\left(\tau(t)\right).$$

So,

$$W'(t) - \left[\gamma_1 \eta\left(\tau^{-1}(t)\right)\left(\tau^{-1}(t)\right)' - \delta(t)\right] W\left(\tau(t)\right) \geq 0. \qquad 20$$

The latest inequality cannot have an eventually positive solution, which is a contradiction, according to Lemma 1, Lemma 5, and condition Eq 19.

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To discussion our results, we provide examples in this area to demonstrate the reliability of the findings from the earlier results.

Example 1: Consider the impulsive hematopoiesis equation of the form:

$$\hbar(t) + \frac{1}{9}\, e^{-t}\hbar(t) - \frac{\frac{1}{9}(8-e^{-t})}{1+\hbar(t-2\pi)} = 0,\ t \neq \qquad 21$$

$$\hbar(t_k^+) = \frac{2k+1}{k}\hbar(t_k),\quad t = t_k \text{ and } \quad k = 1,2,\ldots$$

Let $\tau(t) = t - 2\pi, \tau^{-1}(t) = t + 2\pi,\ \gamma_1 = 2, where\ , \delta(t) = \frac{1}{9}\ e^{-t}\ ,\beta(t) = \frac{1}{9}(8 - e^{-t}),$

$$\delta(\tau(t))\ = \frac{1}{9}\left(e^{-(t-2\pi)}\right), (\ \tau(t))' = 1.$$

Let's introduce an invariant oscillation transformation: $\hbar(\mathrm{t}) = H(t) - \mathrm{K}$ we find

$$H'(t) + \frac{1}{9}\ e^{-t}H(t) - \left[K\frac{1}{9}\ e^{-t} + \frac{\frac{1}{9}(8-e^{-t})}{1+H(t-2\pi)-K}\right] = 0,\ t \neq t_k \qquad 22$$

$$H(t_k^+) = \frac{2k+1}{k}H(t_k),\quad t = t_k \text{ and } \quad k = 1,2,\ldots$$

Thus, $\eta(t)G(H(\tau(t))) = K\frac{1}{9}\ e^{-t} + \frac{\frac{1}{9}(8-e^{-t})}{1+H(t-2\pi)-K}$

To apply conditions $H_2\ and\ H_3$

$$[\gamma_1\eta(t) - \delta\left(\tau(t)\right)\left(\tau(t)\right)'] \geq 0.$$

Let $t_k = k, t \geq 0,\ \ a_k\ = 2$ and $b_k = \frac{1}{k}$, we can see that

$$a_k\ - b_k = \frac{2k+1}{k} > 1$$

And the conditions H_4 leads to $\limsup_{t\to\infty}\left(\int_{\tau(t_k)}^{t_k}\delta(u)du\right) \leq 1.$

Finally, the condition of Eq 11 leads to

$$\left(\limsup_{t\to\infty}\int_{\tau^{-1}(t)}^{\tau(t)}\left[\gamma_1\eta(s) - \delta\left(\left(\tau(s)\right)\right)\left(\tau(s)\right)'\right]d(s) > 1\right)$$

Hence, all conditions of Theorem 2 hold, so all solutions are oscillatory. For instance,

$$H(t) = \begin{cases} sint, & t \neq t_k \\ 2 + \frac{1}{k}, & t = t_k \end{cases} \text{ is such a solution.}$$

The solution of Eq 22 oscillates about zero we can see that in Fig 2, hence the solution of Eq 21 oscillates about equilibrium K.

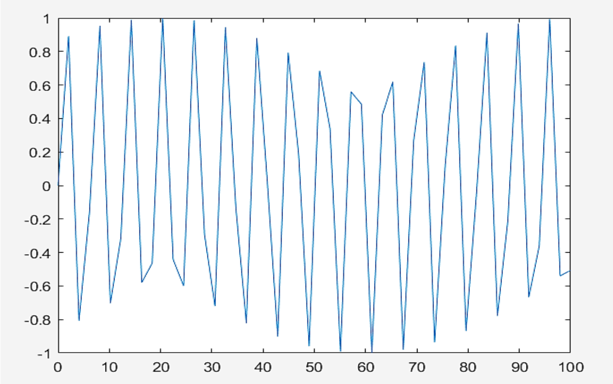

Figure 2. The solution of (22) is oscillates.

Example.2: Consider the impulsive hematopoiesis equation of the form:

$$\hbar'(t) + \left(1 - e^{-\frac{\pi}{2}} - e^{-\frac{\pi}{2}}e^{-t}\right)\hbar(t) - \frac{e^{-\frac{\pi}{2}}e^{-t}}{1 + \hbar\left(t - \frac{5\pi}{2}\right)} = 0, t \neq k \qquad 23$$

$$\hbar(k^+) = \frac{2k}{3k+2}\hbar(k),\ \ t = k,\ k = 1,2,\ldots\ \ .\text{where } t \geq 0.$$

Let $\tau(t) = t - \frac{5\pi}{2}, \tau^{-1}(t) = t + \frac{5\pi}{2},\ \gamma_1 = 2, where\ \beta(t) = e^{-\frac{\pi}{2}}e^{-t}\ ,\quad \delta(t) = \left(1 - e^{-\frac{\pi}{2}} - e^{-\frac{\pi}{2}}e^{-t}\right),\ \beta(\tau^{-1}(t))\ = e^{-\frac{\pi}{2}}e^{-(t+\frac{5\pi}{2})}, (\ \tau(t))' = 1.$

Let's introduce an invariant oscillation transformation: $\hbar(\mathrm{t}) = H(t) - \mathrm{K}$,we find

$$H'(t) + \left(1 - e^{-\frac{\pi}{2}} - e^{-\frac{\pi}{2}}e^{-t}\right)H(t) - \left[K\left(1 - e^{-\frac{\pi}{2}} - e^{-\frac{\pi}{2}}e^{-t}\right) + \frac{e^{-\frac{\pi}{2}}e^{-t}}{1 + H\left(t - \frac{5\pi}{2}\right) - K}\right] = 0, \quad t \neq k,$$

$H(k^+) = \frac{2k}{3k+2}H(k),\ \ t = k,\ k = 1,2,\dots$, where $t \geq 0$. 24

Thus, $\eta(t)G(H(\tau(t))) = K\left(1 - e^{-\frac{\pi}{2}} - e^{-\frac{\pi}{2}}e^{-t}\right) + \frac{e^{-\frac{\pi}{2}}e^{-t}}{1+H\left(t-\frac{5\pi}{2}\right)-K}$

To apply conditions H_3 *and* H_8:

$a_k = \frac{2k+1}{3k+2}$, $b_k = \frac{k+1}{3k+2}$, thus $a_k - b_k = \frac{2k+1}{3k+2} - \frac{k+1}{3k+2} = \frac{2k}{3k+2} > 1$

And $[\gamma_1\eta(\tau^{-1}(t))(\tau^{-1}(t))' - \delta(t)] \geq 0,\ t \in (t_k, t_{k+1}],\ k = 1,2,\dots$.

Apply condition H_9: $\lim_{t\to\infty} \sup[\gamma_1 \int_{\tau^{-1}(t)}^{t} \eta(u)du \leq 1$.

Therefore, all conditions of Theorem 3 hold, and all solutions of Eq 24 are oscillatory about zero so all solutions of Eq 23 are oscillatory about equilibrium K.

Conclusion

The blood maintains homeostasis, which is a relatively constant internal state of physical and chemical circumstances that is managed by living systems through a self-regulating mechanism despite the alterations required for existence. Negative feedback loops are a part of this process, which helps us adapt to changes and maintain life. Homeostasis can be defined mathematically as the constancy of an equilibrium or oscillation state. An a priori objective is to identify sufficient conditions for the oscillation of a positive solution for the impulsive hematopoiesis model Eq. 1 with positive and negative coefficients. So, we study the linear impulses-added impulsive hematopoiesis model Eq. 1, which is consistent with the administration of drugs or radiation in the management of hematological illnesses. Its oscillation is ensured by sufficient conditions, which is a new finding for the proposed model's oscillatory behavior and an improvement over several findings in the nonlinear case in literature. We concluded that the impulse conditions play an essential role in taking into account the qualitative features of solutions for the hematopoiesis model after creating the necessary impulsive requirements.

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Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Baghdad.

Authors' Contributions Statement

Both authors worked together to complete this project. The manuscript was written and edited by I. S. H. with. H. A. M. looked over the findings and offered corrections. The last draft was read by the authors and got their approval.

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تذبذب نموذج تكون الدم النبضي مع معاملات موجبة وسالبة

ايمان صبيح حديد[1]، حسين علي محمد[2]

[1]قسم الرياضيات، كلية العلوم، جامعة بغداد، بغداد، العراق.
[2]قسم الرياضيات، كلية العلوم للبنات، جامعة بغداد، بغداد، العراق.

الخلاصة

في هذا البحث تم بحث مشكلة الحلول المتذبذبة لنموذج تكون الدم النبضي ذات المعاملات الموجبة والسالبة. هناك العديد من العمليات التطورية ، والتي كثيرًا ما تواجه تحولات دراماتيكية في أوقات محددة وتكون حساسة للاضطرابات قصيرة المدى. نتيجة لذلك ، نقوم ببناء العديد من معايير التذبذب التي تكون إما جديدة تمامًا أو تعزز العديد من النتائج الحديثة في الأدبيات. نقدم أيضًا أمثلة توضيحية لكيفية تأثير النبضات على الحلول المتذبذبة لنموذج تكوين الدم.

الكلمات المفتاحية: المعادلات التفاضلية التباطؤية ، نموذج تكوين الدم ، التذبذب ، النبضات ، الشروط الكافية.