Published Online First: January, 2024
https://dx.doi.org/10.21123/bsj.2024.9126
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Analyzing the Two-Phase Heterogeneous and Batch Service Queuing System with Breakdown in Two-Phases, Feedback, and Vacation

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Received 25/05/2023, Revised 22/12/2023, Accepted 24/12/2023, Published Online First 20/01/2024

Abstract

Customers are arriving at the system in bulk according to the Poisson process with rate λ_1. The batch service process is split into two phases called first essential service (FES) and second essential service (SES) with minimum server capacity $'a'$ and maximum server capacity $'b'$. Customer who requires feedback after SES completion will be taken immediately for service by the server with probability β. After completion of SES, if there is no feedback with probability $1-\beta$, the queue length is greater than or equal to $'a'$ then the server begins FES, or if the queue length is less than $'a'$ then the server leaves for vacation. Here we address utilization of renewal period of the server after completion of FES or SES with probability δ or α respectively. When there is no server failure after FES completion, the server will provide SES with probability$1-\delta$. Similarly, if there is no server failure after SES the server may go for FES or vacation with probability$1-\ \alpha$. After completing a vacation if the queue length is greater than or equal to $'a'$ then the server begins FES. On the other hand, if the number of customers waiting in the queue is smaller than$'a'$, then the server remains idle until the queue length reaches the value$'a'$. For the designed model probability-generating function of the queue size at an arbitrary time is obtained by using the supplementary variable technique and suitable analytical results are derived with numerical examples.

Keywords: Batch Service, Breakdown, Bulk Arrival, Feedback, Two Phase Service.

Introduction

Several academicians have tested queue systems with vacations and their numerous combos. Some of those studies are queue structures with vacation queue fashions via Tian & Zhang [1]. In many actual applications, there can be a couple of arrivals into the machine, these types of systems are labeled as bulk arrival queue structures. For batch carrier queue structures with multiple vacations, Arumuganathan & Jeyakumar [2] obtained consistent nation conditions for numerous performance measures and value optimization. The steady-state queue size distribution for the M^X/G (a, b)/1 queue machine with numerous operating vacations changed into lately acquired by way of Jeyakumar & Senthilnathan [3]. In all queue models with vacations that have been studied inside the beyond, the server remains on vacation. Even, whilst the queue period is excessive enough to initiate the primary provider. The modeling of actual-time systems can benefit from those vacation fashions. Baba [4] examined the M/PH/1 line with working vacations and interruptions. M^X/G (a, b)/1 queue model with vacation interruption changed into researched through Haridass & Arumuganathan [5].

Through the real-time application, the researchers were in a position to deduce diverse queue device functions. Gao & Liu [6] investigated the M/G/1 queue under a Bernoulli agenda with working vacation and vacation interruption. When looking back at 2013, Tao *et al.*,[7] blanketed the GI/M/1 queue with Bernoulli-scheduled vacation and vacation interruption. A countless-buffer batch-arrival queue

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with a batch-size-dependent provider was tested with the aid of Pradhan & Gupta[8]. The consistent state takes a look at two M^X/M (a, b)/1 queue models with random breakdowns changed into examined using Madan *et al.*,[9] they took into account the reality that the repair time is deterministic for one version and exponential for every other. It has been cited inside the literature on queue fashions with server breakdown that, excluding Madan *et al*. each creator who discusses server breakdown inside the context of a bulk provider queue version deals with a server that can simply serve one customer at a time. The server may also interrupt right away if trouble arises. Yet, in the majority of instances, it is impossible to disturb the server before it has completed providing its batch of services. Jeyakumar & Senthilnathan [10] also look at breakdown without carrier disruption in a batch carrier queue model and a bulk arrival queue model. They developed a model using closedown time and constructed probability-generating functions for the completion epochs of services, vacations, and renovations. An M/G/1 queue with an N-policy, a single vacation, an unreliable service station, and a replaceable repair facility was examined by Wu *et al.*,[11].

Sama *et al.*,[12] introduced an unstable server and delayed repair for a bulk arrival Markovian queueing system with state-dependent arrival and N-policy. The M/M/1 vacation two-phase queueing model with server start-up, time-out, and breakdowns was examined by Rao *et al.*, [13]. Enogwe *et al.*, [14] analysed a non-Markovian queue with two types of service balking and Bernoulli server vacation. The model is referred to as a bivariate Markov process and includes the elapsed service time and vacation time as supplementary variables. In the article of Sekar & Kandaiyan [15] investigated a single-server retrial queue with delayed repair and feedback under working breaks and vacations. If the essential and sufficient requirements are viable, the system can be stabilized. Server breakdown based on service modes was introduced by Niranjan[16]. In this paper, the researcher used supplementary variable techniques to derive important performance measures. Blondia[17] analyzed energy harvesting in the queueing model for a wireless sensor node. Merit & Haridass [18] worked on a simulation analysis of the bulk queueing system with a working breakdown. An application of queueing system in 4G/5G networks was given by Deepa *et al.*, [19]. Niranjan *et al.*,[20] introduced two types of breakdowns with two Phases of service in a bulk queueing system. A M^x/G (a, b)/1 queuing system with optional additional services, numerous vacations, and setup time is examined by Ayyappan & Deepa[21].

Niranjan *et al.*,[22] analysed a non-markovian bulk queueing system with renewal and start-up/shutdown times. Maximum entropy analysis of the control of the arrival and batch service queueing system with breakdown and multiple vacations was discussed by Nithya & Haridass[23]. The impacts of reneging, server breakdown, and server vacation on the various stages of the batch arrivals queueing system with a single server serving clients in three varying modes are investigated in the article by Enogwe & Obiora-Ilouno[24]. Khan & Paramasivam[25] analyzed the quality control policy for the Markovian model with feedback, balking, and maintaining reneged clients using an iterative method to the *n*th customer in the system. Ammar & Rajadurai[26] analyzed an innovative type of retry queueing system with functional breakdown services presented in this inquiry. Priority and regular clients are two different categories that are taken into consideration. The following article presents that combination with a cryptosystem which has several Substitution Cipher Algorithms along with the Circular queue data structure. Homophonic Substitution Cipher and Polyalphabetic Substitution Cipher are the two different substitution techniques Ibraheem & Hasan[27]. The Public Network Channel Transferring their data by Secured Based Steganography and Cryptography Techniques, Naser *et al.*,[28]. In business management, Moussa *et al.*,[29] analyzed the service time characteristics of fast-food outlets using an M/M/1 queuing model. In addition, Abdelmawgoud *et al.*,[30] determined how long service wait times affect customers' satisfaction in five-star hotels.

Motivation

Cloud computing has played a vital role in many practical applications. A type of cloud computing that allows to access applications on the internet. Cloud computing takes a minimum of 20 documents and a maximum of 100 documents. Also, iCloud takes the data from the cloud and transfers it directly without any interruption.

Simple Mail Transfer Protocol (SMTP) is used to deliver data across the internet using file transfer data to clients. The next process of attachments is to transfer the data to clients. During the data transfer sometimes the system may be affected by the virus, the service will not be interrupted immediately, and

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it will be continued for the current batch of data attached or else transferred by doing some technical precaution arrangements. After the service is completed, the antivirus is activated and detects the problems in the system. If there are no issues the process would be continued. After data transmission, if there is no data available for processing the server will be assigned to some secondary works such as system updating, and cleaning temporary files etc., This can be modeled as 'Analysis of two-phase heterogeneous batch service queue system with a breakdown in two phases, feedback and vacation'.

Model Description

Customers are arriving at the system in bulk according to the Poisson process with rate λ_1. The batch service process is split into two phases called FES and SES with minimum server capacity $'a'$ and maximum server capacity $'b'$ according to the general bulk service rule[31]. When the server fails, during FES or SES the service process will not be interrupted. It performs continuously for the current batch by doing some technical precaution arrangements. The server will be repaired after the service completion of FES or SES with probability δ or α respectively during the renewal period of the server. When there is no server failure after FES completion, the server will provide SES with probability$1-\delta$. Simi\larly, if there is no server failure after SES the server may go for FES or vacation with probability$1-\alpha$. Customer who requires feedback after SES completion will be taken immediately for service by the server with probabilityβ. After completion of SES, if there is no feedback with probability $1-\beta$ queue length is greater than or equal to $'a'$ then the server begins FES, or if the queue length is less than $'a'$ then the server leaves for vacation. On SES completion, if the queue length is greater than or equal to $'a'$ then the server goes for FES. After completing a vacation if the queue length is greater than or equal to $'a'$ then the server begins FES. On the other hand, if the number of customers waiting in the queue is smaller than $'a'$.The server remains idle after the vacation completion if the queue size is smaller than $'a'$. This proposed system is schematically represented in Fig 1

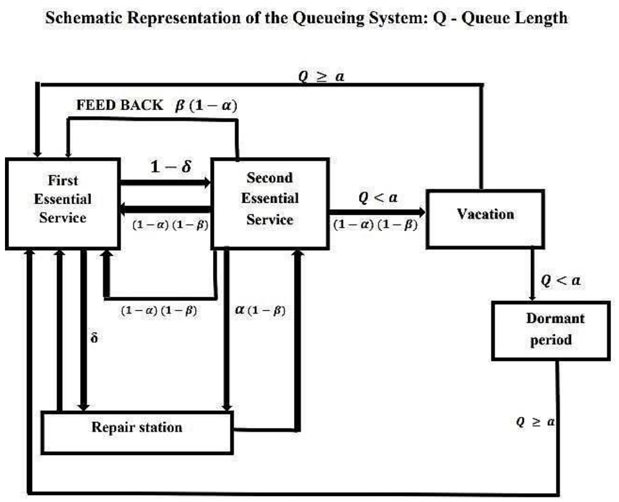

Figure 1. Schematic Representation of the Queue system

Notations

Let X be the group size random variable of the arrival, λ_1 be the Poisson arrival rate, g_k be the probability that k customers arrive in a batch, $X(z)$ be the probability generating function (PGF) of X. Let $N_s\ (t)$ be the number of customers under the service at time t and $N_q\ (t)$ be the number of customers waiting for service at time t.δ, α be the breakdown probability of FES and SES respectively, β be the feedback probability. Notations are given in Table 1 below.

Table 1. Notations

	Cumulative Distribution Function	Probability Distribution Function	Laplace Stieltjes Transform	Remaining Service Time
First essential service (FES)	S(x)	$s(x)$	$\tilde{S}(\Theta)$	$S^0(x)$
Second essential service (SES)	$S_1(x)$	$s_1(x)$	$\tilde{S}_1(\Theta)$	$s_1{}^0(x)$
Vacation	Q(x)	q(x)	$\tilde{Q}(\Theta)$	$Q^0(x)$
Repair time	R(x)	r(x)	$\tilde{R}(\Theta)$	$R^0(x)$

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$C(t)$= 0-when the server is busy with FES,1-when the server is busy with SES, 2-when the server is on vacation, 3-when the server is on repair, 4-when the server is on a dormant period.

The State probabilities are defined as follows;

$$F_{ij}(x,t)dt = P_r\left\{\begin{matrix} N_s(t) = i,\ N_q(t) = j, \\ x \le S^0(t) \le x + dt, C(t) = 0 \end{matrix}\right\}$$

$$S_{ij}(x,t)dt = P_r\left\{\begin{matrix} N_s(t) = i,\ N_q(t) = j, \\ x \le S_1{}^0((t) \le x + dt, C(t) = 1 \end{matrix}\right\}$$

$$Q_n(x,t)dt = P_r\left\{\begin{matrix} N_q(t) = n,\ x \le Q^0(t) \le x+dt \\ C(t) = 2,\ 0 \le n \le a\text{-}1 \end{matrix}\right\}$$

$$R_n(x,t)dt = P_r\left\{\begin{matrix} N_q(t) = n,\ x \le R^0(t) \le x+dt \\ C(t) = 3,\ n \ge 1 \end{matrix}\right\}$$

$$T_n(t) = P_r\{N_q(t) = n,\ \ C(t) = 4\},\ 0 \le n \le a\text{-}1$$

Steady State Analysis

The successive equations are obtained by using the supplementary variable technique[32].

$$-\frac{d}{dx}F_{i0}(x) = -\lambda_1 F_{i0}(x) + \beta(1\alpha)S_{i0}(0)s(x) + 2(1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{mi}(0)s(x) + \sum_{m=0}^{a-1} T_m\ \lambda_1 g_{i-m} s(x) + R_i(0)s(x) + Q_i(0)s(x),\ a \le i \le\ b \quad 1$$

$$-\frac{d}{dx}F_{ij}(x) = -\lambda_1 F_{ij}(x) + \sum_{k=1}^{j} F_{i\ j-k}(x)\lambda_1 g_k + \beta(1-\alpha)S_{ij}(0)s(x) \quad a \le i \le b\text{-}1\ ,\ j \ge 1 \quad 2$$

$$-\frac{d}{dx}F_{bj}(x) = -\lambda_1 F_{bj}(x) + \sum_{k=1}^{j} F_{b\ j-k}(x)\lambda_1 g_k + \beta(1-\alpha)S_{bj}(0)s(x) + 2(1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{mb+j}(0)s(x) + R_{b+j}(0)s(x) + Q_{b+j}(0)s(x) + \sum_{m=0}^{a-1} T_m\ \lambda_1 g_{b+j-m} s(x),\ j \ge 1 \quad 3$$

$$-\frac{d}{dx}S_{i0}(x) = -\lambda_1 S_{i0}(x) + (1-\delta)\sum_{m=a}^{b} F_{mi}(0)s_1(x) + R_i(0)s_1(x)\ \ a \le i \le b \quad 4$$

$$-\frac{d}{dx}S_{ij}(x) = -\lambda_1 S_{ij}(x) + \sum_{k=1}^{j} S_{i\ j-k}(x)\lambda_1 g_k \quad 5$$

$$-\frac{d}{dx}S_{bj}(x) = -\lambda_1 S_{bj}(x) + \sum_{k=1}^{j} S_{b\ j-k}(x)\lambda_1 g_k + (1-\delta)\sum_{m=a}^{b} F_{mb+j}(0)s_1(x) + R_{b+j}(0)s_1(x) \quad 6$$

$$-\frac{d}{dx}R_0(x) = -\lambda_1 R_0(x) + \delta\sum_{m=a}^{b} F_{m0}(0)\ r(x) + \alpha(1-\beta)\sum_{m=a}^{b} S_{m0}(0)r(x) \quad 7$$

$$-\frac{d}{dx}R_n(x) = -\lambda_1 R_n(x) + \delta\sum_{m=a}^{b} F_{mn}(0)r(x) + \alpha(1-\beta)\sum_{m=a}^{b} S_{mn}(0)r(x) + \sum_{k=1}^{n} R_{n-k}(x)\lambda_1 g_k,\ a \le n \le b \quad 8$$

$$-\frac{d}{dx}Q_0(x) = -\lambda_1 Q_0(x) + (1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{m0}(0)q(x) \quad 9$$

$$-\frac{d}{dx}Q_n(x) = -\lambda_1 Q_n(x) + (1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{mn}(0)q(x) + \sum_{k=1}^{n} Q_{n-k}(x)\lambda_1 g_k, 1 \le n \le a\text{-}1 \quad 10$$

$$0 = -\lambda_1 T_0 + Q_0(0) \quad 11$$

$$0 = -\lambda_1 T_n + Q_n(0) + \sum_{k=1}^{n} T_{n-k}\lambda_1 g_k, 1 \le n \le a\text{-}1 \quad 12$$

Laplace-Stieltjes transform of $\tilde{F}_{in}(\theta)$,$\tilde{S}_{in}(\theta)$,$\tilde{R}_n(\theta)$ and $\tilde{Q}_n(\theta)$ are defined as

$$\tilde{F}_{in}(\theta) = \int_0^\infty e^{-\theta x} F_{in}(x)dx$$
$$\tilde{S}_{in}(\theta) = \int_0^\infty e^{-\theta x} S_{in}(x)dx$$
$$\tilde{R}_n(\theta) = \int_0^\infty e^{-\theta x} R_n(x)dx$$
$$\tilde{Q}_n(\theta) = \int_0^\infty e^{-\theta x} Q_n(x)dx$$

Taking Laplace-Stieltjes transform on both sides from Eq.1 to Eq.10,

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$$\theta\tilde{F}_{i0}(\theta) - F_{i0}(0) = \lambda_1\tilde{F}_{io}(\theta) - \tilde{S}(\theta)\begin{bmatrix}\sum_{m=0}^{a-1} T_m\,\lambda_1 g_{i-m} + \beta(1-\alpha)S_{i0}(0) + Q_i(0)\\ +2(1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{mi}(0) + R_i(0)\end{bmatrix} a \le i \le b \quad 13$$

$$\theta\tilde{F}_{ij}(\theta) - F_{ij}(0) = \lambda_1\tilde{F}_{ij}(\theta) - \sum_{k=1}^{j}\tilde{F}_{ij-k}(\theta)\,\lambda_1 g_k - \beta(1-\alpha)S_{ij}(0)\tilde{S}(\theta) \quad , \quad a \le i \le b\text{-}1 \quad j \ge 1 \quad 14$$

$$\theta\tilde{F}_{bj}(\theta) - F_{bj}(0) = \lambda_1\tilde{F}_{bj}(\theta) - \tilde{S}(\theta)\begin{bmatrix}\sum_{k=1}^{j}\tilde{F}_{bj-k}(\theta)\,\lambda_1 g_k + \sum_{m=0}^{a-1} T_m\,\lambda_1 g_{b+j-m} + \beta(1-\alpha)S_{bj}(0) + Q_{b+j}(0)\\ +2(1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{mb+j}(0) + R_{b+j}(0), j \ge 1\end{bmatrix} \quad 15$$

$$\theta\tilde{S}_{i0}(\theta) - S_{i0}(0) = \lambda_1\tilde{S}_{i0}(\theta) - \tilde{S}_1(\theta)[(1-\delta)\sum_{m=a}^{b} F_{mi}(0) + R_i(0)] , a \le i \le b \quad 16$$

$$\theta\tilde{S}_{ij}(\theta) - S_{ij}(0) = \lambda_1\tilde{S}_{ij}(\theta) - \sum_{k=1}^{j}\tilde{S}_{i\,j-k}(\theta)\lambda_1 g_k \quad , a \le i \le b\text{-}1 \quad j \ge 1 \quad 17$$

$$\theta\tilde{S}_{bj}(\theta) - S_{bj}(0) = \lambda_1\tilde{S}_{bj}(\theta) - \sum_{k=1}^{j} S_{b\,j-k}(\theta)\,\lambda_1 g_k - \tilde{S}_1(\theta)\left[(1-\delta)\sum_{m=a}^{b} F_{mb+j}(0) + +R_{b+j}(0)\right] j \ge 1 \quad 18$$

$$\theta\tilde{R}_0(\theta) - R_o(0) = \lambda_1\tilde{R}_0(\theta) - \tilde{R}(\theta)[\delta\sum_{m=a}^{b} F_{m0}(0) + \alpha(1-\beta)\sum_{m=a}^{b} S_{m0}(0)] \quad 19$$

$$\theta\tilde{R}_n(\theta) - R_n(0) = \lambda_1\tilde{R}_n(\theta) - \tilde{R}(\theta)\left[\delta\sum_{m=a}^{b} F_{mn}(0) + \alpha(1-\beta)\sum_{m=a}^{b} S_{mn}(0)\right] - \sum_{k=1}^{n}\tilde{R}_{n-k}(\theta)\,\lambda_1 g_k \quad 20$$

$$\theta\tilde{Q}_0(\theta) - Q_o(0) = \lambda_1\tilde{Q}_0(\theta) - \tilde{Q}(\theta)[(1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{m0}(0)] \quad 21$$

$$\theta\tilde{Q}_n(\theta) - Q_n(0) = \lambda_1\tilde{Q}_n(\theta) - \sum_{k=1}^{n}\tilde{Q}_{n-k}(\theta)\lambda_1 g_k - \tilde{Q}(\theta)(1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{mn}(0), \quad 0 \le n \le a\text{-}1 \quad 22$$

Probability Generating Function (PGF)

To obtain the PGF of the queue size at an arbitrary time epoch, the following PGF are defined.

$$\tilde{F}_i(z,\theta) = \sum_{j=0}^{\infty}\tilde{F}_{ij}(\theta)z^j \quad F_i(z,0) = \sum_{j=0}^{\infty} F_{ij}(0)\,z^j$$

$$\tilde{S}_i(z,\theta) = \sum_{j=0}^{\infty}\tilde{S}_{ij}(\theta)z^j \quad S_i(z,0) = \sum_{j=0}^{\infty} S_{ij}(0)\,z^j$$

$$\tilde{R}(z,\Theta) = \sum_{n=0}^{\infty}\tilde{R}_n(\theta)\,z^n \quad R(z,0) = \sum_{n=0}^{\infty} R_n(0)\,z^n$$

$$\tilde{Q}(z,\Theta) = \sum_{n=0}^{a-1}\tilde{Q}_n(\theta)\,z^n \quad Q(z,0) = \sum_{n=0}^{a-1} Q_n(0)\,z^n$$

$$T(z) = \sum_{n=0}^{a-1} T_n z^n \qquad a \le i \le b \quad 23$$

By multiplying from Eq.13 to Eq.22 with suitable powers of z^n and summing over j=0 to ∞, then by using Eq.23,

$$(\theta - \lambda_1 + \lambda_1 X(z))\tilde{F}_i(Z,\theta) = F_i(Z,0) - \tilde{S}(\theta)\begin{bmatrix}\sum_{m=0}^{a-1} T_m\,\lambda_1 g_{i-m} + \beta(1-\alpha)S_{i0}(0) + Q_i(0)\\ +2(1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{mi}(0) + R_i(0)\end{bmatrix} a \le i \le b \quad 24$$

$$z^b(\theta - \lambda_1 + \lambda_1 X(z))\tilde{F}_b(Z,\theta) = z^b F_b(Z,0) - \tilde{S}(\theta)\begin{bmatrix} 2(1-\alpha)(1-\beta)\left(\sum_{m=a}^{b-1} S_m(z,0) - \sum_{m=a}^{b}\sum_{j=0}^{b-1} S_{mj}(0)z^j\right) \\ +S_b(Z,0)\,(\beta(1-\alpha)z^b - 2(1-\alpha)(1-\beta)) \\ +Q(z,0) - \sum_{n=0}^{b-1} Q_n(0)\,z^n \\ +R(z,0) - \sum_{n=0}^{b-1} R_n(0)\,z^n \\ +\lambda_1\left(T(z)X(z) - \sum_{m=0}^{a-1}\left(T_m z^m \sum_{j=1}^{b-m-1} g_j z^j\right)\right) \end{bmatrix} \quad 25$$

$$(\theta - \lambda_1 + \lambda_1 X(z))\tilde{S}_i(z,\theta) = S_i(z,0) - \tilde{S}_1(\theta)\left[[(1-\delta)\sum_{m=a}^{b} F_{mi}(0) + R_i(0)]\right] \quad 26$$

$$z^b(\theta - \lambda_1 + \lambda_1 X(z))\tilde{S}_b(Z,\theta) = z^b S_b(Z,0) - (1-\delta)F_j\tilde{S}_1(\theta) - (1-\delta)\tilde{S}_1(\theta)\left(\sum_{m=a}^{b-1} F_m(z,0) - \sum_{m=a}^{b}\sum_{j=0}^{b-1} F_{mj}(0)z^j\right) - \tilde{S}_1(\theta)\left[R(z,0) - \sum_{n=0}^{b-1} R_n(0)\,z^n\right] \quad 27$$

$$(\theta - \lambda_1 + \lambda_1 X(z))\tilde{R}(z,\theta) = R(z,0) - \tilde{R}(\theta)\ [\delta\, F_m(z,0) + \alpha(1-\beta)S_m(z,0)]\ , a \le n \le b \quad 28$$

$$(\theta - \lambda_1 + \lambda_1 X(z))\tilde{Q}(z,\theta) = Q(z,0) - \tilde{Q}(\theta)[(1-\alpha)\,(1-\beta)S_m(z,0)] \qquad 1 \le n \le a\text{-}1 \quad 29$$

Let $f_j = \sum_{j=0}^{b-1} F_{mj}(0)$, $s_j = \sum_{j=0}^{b-1} S_{mj}(0)$, $q_i = \sum_{n=0}^{b-1} Q_n(0)$, $r_i = \sum_{n=0}^{b-1} R_n(0)$,

Let P (z) be the probability-generating function of the queue size at an arbitrary time epoch. Then

$$P(z) = \sum_{m=a}^{b-1} \tilde{F}_m(z,0) + \tilde{F}_b(z,0) + \sum_{m=a}^{b-1} \tilde{S}_m(z,0) + \tilde{S}_b(z,0) + \tilde{Q}(z,0) + \tilde{R}(z,0) + T(z) \quad 30$$

$$P(z) = \frac{\tilde{S}_1(\lambda_1-\lambda_1 X(z)-1)s_i M_1 M_2 + (\tilde{S}_1(\lambda_1-\lambda_1 X(z))-1)M_2 f(z) + \tilde{S}\left((\lambda_1-\lambda_1 X(z))-1\right)M_1 M_2 c_i + \tilde{S}\left((\lambda_1-\lambda_1 X(z))-1\right)g(z) + \tilde{Q}\left((\lambda_1-\lambda_1 X(z))-1\right)M_7 M_1 M_2 + \tilde{R}\left((\lambda_1-\lambda_1 X(z))-1\right)M_8 M_1 M_2 + + \sum_{n=0}^{a-1} T_n z^n(-\lambda_1+\lambda_1 X(z))M_1 M_2}{M_1 M_2(-\lambda_1+\lambda_1 X(z))} \quad 31$$

Where

$$M_1 = z^b[1 - \tilde{S}_1(\lambda_1 - \lambda_1 X(z))\tilde{R}(\lambda_1 - \lambda_1 X(z))\alpha(1-\beta)]$$

$$M_2 = z^b - \delta\tilde{S}(\lambda_1 - \lambda_1 X(z))\tilde{R}(\lambda_1 - \lambda_1 X(z))$$

$$M_3 = [(1-\delta)\tilde{S}_1(\lambda_1 - \lambda_1 X(z)) - \delta\tilde{S}_1(\lambda_1 - \lambda_1 X(z))\tilde{R}(\lambda_1 - \lambda_1 X(z))]F_j$$

$$M_4 = (1-\delta)\tilde{S}_1(\theta)\left(\sum_{m=a}^{b-1} F_m(z,0) - \sum_{m=a}^{b} f_i z^j\right)$$

$$M_5 = \tilde{S}_1(\lambda_1 - \lambda_1 X(z))\tilde{R}(\lambda_1 - \lambda_1 X(z))\left(\delta\sum_{m=a}^{b-1} F_m(z,0) + \alpha(1-\beta)\sum_{m=a}^{b-1} S_m(z,0)\right)$$

$$M_6 = \tilde{S}(\lambda_1 - \lambda_1 X(z))\{(\beta(1-\alpha)z^b + 2(1-\alpha)(1-\beta) + + \tilde{R}(\lambda_1 - \lambda_1 X(z))\alpha(1-\beta))\}$$

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$$c_i = \beta(1-\alpha)\mathrm{S_i}(z,0) + 2(1-\alpha)(1-\beta)\sum_{m=a}^{b} S_{mi}(0) + Q_i(0) + R_i(0) + \sum_{m=0}^{a-1} T_m \lambda_1 g_{i-m}$$

$$g(z) = [M_3 - M_4 - M_5 - \tilde{S}_1(\lambda_1 - \lambda_1 X(z))R_i \mathrm{z^n}]\, M_6 - M_1 2(1-\alpha)(1-\beta) \tilde{S}(\lambda_1 - \lambda_1 X(z))(\textstyle\sum_{m=a}^{b-1} \mathrm{S_m}(z,0) - \mathrm{S_i z^j}) - M_1 \tilde{S}(\lambda_1 - \lambda_1 X(z)) \{\tilde{R}(\lambda_1 - \lambda_1 X(z)) \left[\left(\delta \sum_{m=a}^{b-1} \mathrm{F_m}(z,0) + \alpha(1-\beta)\left(\sum_{m=a}^{b-1} \mathrm{S_m}(z,0)\right)\right) - r_i \mathrm{z^n}\right] + \tilde{Q}(\lambda_1 - \lambda_1 X(z))[(1-\alpha)(1-\beta)\mathrm{S_m}(z,0) - q_i \mathrm{z^n}] + \lambda_1\left(T(z)X(z) - \sum_{m=0}^{a-1}\left(\mathrm{T_m z^m} \sum_{j=1}^{b-m-1} \mathrm{g_j\, z^j}\right)\right)\}$$

$$s_i = \left[(1-\delta)\sum_{i=a}^{b} \mathrm{F_{mi}}(0) + R_i(0)\right]$$

$$M_7 = [(1-\alpha)(1-\beta)\mathrm{S_m}(z,0)]$$

$$M_8 = [\delta\, \mathrm{F_m}(z,0) + \alpha(1-\beta)\mathrm{S_m}(z,0)]$$

Theorem 1:
The unknown constants Q_n involved in T_n are expressed in terms of d_n as, $q_n = \sum_{i=0}^{n} d_{n-i}\beta_i$, n =0, 1, 2… a-1, where β_i is the probability that 'i' customers arrive during the vacation.

Theorem 2:
Let B_j be the collection of a set of positive integers (not necessarily distinct) A, such that, the sum of elements in A is j, then, $T_n = \frac{1}{\lambda}\sum_{j=0}^{n} L_{n-j} \sum_{j=1}^{n(B_j)} \prod_{l\in A} g_l$

Proof:
From the Eq 11 and Eq 12,
$\lambda_1 T_0 = q_0(0) = q_0$
$\lambda_1 T_n = q_n(0) + \lambda_1 \sum_{k=1}^{n} T_{n-k}\, g_k;\ 1 \le n \le a-1$
When=1, $\lambda_1 T_1 = L_1 + L_0 g_1$
When=2, $\lambda_1 T_2 = L_2(0) + \lambda_1 T_1 g_1 + \lambda_1 T_0 g_2$
$= L_2 + L_1 g_1 + q_0({g_1}^2 + g_2)$
When n=3, $\lambda_1 T_3 = L_3 + L_2 g_1 + q_1({g_1}^2 + g_2) + q_0({g_1}^3 + 2 g_1 g_2 + g_3)$
$T_3 = \frac{1}{\lambda_1}\left(\sum_{j=0}^{3} L_{3-j} \sum_{j=1}^{n(B_j)} \prod_{l\in A} g_l\right)$
Where $B_1 = \{\{1\}\}$, $B_2 = \{\{1,1\},\{2\}\}$, and $B_3 = \{\{3\},\{1,1,1\}\{1,2\},\{2,1\}\}$
By induction,
Therefore, $T_n = \frac{1}{\lambda_1}\left(\sum_{j=0}^{n} L_{n-j} \sum_{j=1}^{n(B_i)} \prod_{l\in A} g_l\right)$
Hence the proof.

Steady State Condition

The probability-generating property P (z) must satisfy P (1) = 1. Using the L Hospitals rule to evaluate limz→1 P (z) and equating the expression to one, it can be deduced that 1 is the condition to be satisfied for the existence of a consistent nation for the model under study, in which

$$\rho = \frac{s_i E(S_1)\lambda_1 E(X) + (1-\delta)a_i\lambda_1 E(S_1)E(X) + (1-\delta)C_i\big(1-\alpha(1-\beta)\big)E(S)\lambda_1 E(X) + E(S)\lambda_1 E(X) g(z) + (1-\alpha)(1-\beta)b_i\big(1-\alpha(1-\beta)\big)(1-\delta)E(Q)\lambda_1 E(X) + \big(1-\alpha(1-\beta)\big)(1-\delta)d_i E(R)\lambda_1 E(X) + e_i\big(1-\alpha(1-\beta)\big)(1-\delta)\lambda_1 E(X)}{\big(1-\alpha(1-\beta)\big)(1-\delta)\lambda_1 E(X)}$$

Performance Measures

Expected queue length

$E(Q) = \lim_{z\to 1} P'(z)$

$$E(Q) = \frac{\begin{pmatrix} M_1 M_2 {M_3}'({W_1}'' + {W_2}'' + {W_3}'' + {W_4}'' + {W_5}'') \\ -W'(M_1 M_2 {M_3}'' + 2M_1 {M_2}' {M_3}' + 2{M_1}' M_2 {M_3}') \end{pmatrix}}{2(M_1)^2(M_2)^2({M_3}')^2}$$

Where $M_1 = \mathrm{z^b}[1 - \tilde{S}_1(\lambda_1 - \lambda_1 X(z)) \tilde{R}(\lambda_1 - \lambda_1 X(z))\alpha(1-\beta)]$

$$M_2 = \mathrm{z^b} - \delta\tilde{S}(\lambda_1 - \lambda_1 X(z))\tilde{R}(\lambda_1 - \lambda_1 X(z))$$

$$M_3 = (-\lambda_1 + \lambda_1 X(z))$$

$$W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7$$

Published Online First: January, 2024
https://dx.doi.org/10.21123/bsj.2024.9126
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
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$$W_1 = \tilde{S}_1(\lambda_1 - \lambda_1 X(z) - 1)s_i M_1 M_2, \qquad W_2 = \left(\tilde{S}_1\left(\lambda_1 - \lambda_1 X(z)\right) - 1\right) M_2 f(z)$$
$$W_3 = \tilde{S}\left(\left(\lambda_1 - \lambda_1 X(z)\right) - 1\right) M_1 M_2 c_i, W_4 = \tilde{S}\left(\left(\lambda_1 - \lambda_1 X(z)\right) - 1\right) g(z)$$
$$W_5 = \tilde{Q}\left(\left(\lambda_1 - \lambda_1 X(z)\right) - 1\right) M_8 M_1 M_2, \qquad W_6 = \tilde{R}\left(\left(\lambda_1 - \lambda_1 X(z)\right) - 1\right) M_9 M_1 M_2$$
$$W_7 = \sum_{n=0}^{a-1} T_n z^n M_1 M_2 M_3$$

Expected waiting time in the queue

$$\text{E (W)} = \frac{E(Q)}{\lambda_1 E(X)}$$

Expected length of busy period

Let B be the busy period random variable. Let T be the residence time that the server is rendering (FES) or under repair and M the residence time that the server is rendering (SES) or under renewal. Then

E (T) = E (F) +δ E(R) E (M) = E (S) + αE(R)

Where E (F) is the expected FES time
E(S) is the expected SES time
E(R) is the expected repair time during FES
E (B) is the expected length of a busy period

Define a random variable J as

$$J = \begin{cases} 0, & \textit{if the server finds less than 'a' customers after SES} \\ 1, & \textit{if the serve atleast 'a' customers after SES} \end{cases}$$

Then Expected length of the busy period is given by

$$E(B) = E(B/J=0)P(J=0) + E(B/J=1)P(J=1) = E(T)P(J=0) + \left(E(T)+E(B)\right)P(J=1)$$
$$E(B)\left(1 - P(J=1)\right) = E(T)\left(P(J=0) + P(J=1)\right)$$
$$\text{E (B)} = \frac{E(T)}{P(J=0)}$$
$$\text{E (B)} = \frac{E(T)}{\sum_{I=0}^{a-1}((1-\delta)\mathrm{f_i} + (1-\alpha)\mathrm{s_i} + \mathrm{r_i})}$$

Expected length of idle period

Between the start of a vacation and the start of a busy period is the definition of an idle period. For the idle period, let I serve as the random variable. The likelihood that the system state visits 'i' during an idle period is $\alpha_{i,}$ where i=0, 1, 2...a-1.

$$I_i = \begin{cases} 1 \textit{ if the state i during an idle period.} \\ 0 \textit{ otherwise} \end{cases}$$

Conditioning on the queue size at service completion epoch,$\alpha_0 = \pi_0$

$$\alpha_i = P(I_i = 1) = \pi_i + \sum_{k=0}^{c-1} \pi_k\, P\left({I_{i-k}}^1 = 1\right); \qquad i = 1, 2, \ldots a\text{-}1$$

Thus, the expected length of the idle period is obtained from a

$$E(I) = \frac{1}{\lambda_1}\sum_{i=0}^{a-1} \alpha_{i,}$$

Where $\frac{1}{\lambda_1}$ is the expected staying time in the state 'i' during an idle period.

The probability that the server is busy with FES

Let P (F) be the probability that the server is busy with FES at time t.

Therefore,

$$\text{P (F)} = \lim_{z \to 1} \tilde{F}(z, 0)$$
$$\text{P (F)} = \lim_{z \to 1}\left(\sum_{i=a}^{b-1} \tilde{F}_i(z, 0) + \tilde{F}_b(z, 0)\right)$$
$$\text{P (F)} = \frac{E(S)c_i M_1(1) M_2(1) + E(S)g(1)}{M_1(1) M_2(1)}$$

The probability that the server is busy with SES

Let P (S) be the probability that the server is busy with SES at time t.

Therefore,

$$\text{P (S)} = \lim_{z \to 1} \tilde{S}(z, 0)$$

$$P(S) = \lim_{z\to 1}\left(\sum_{i=a}^{b-1}\tilde{S}_i(z,0)+\tilde{S}_b(z,0)\right)$$

$$P(S) = \frac{E(S_1)s_i M_1(1)+E(S_1)f(1)}{M_1(1)}$$

The probability that the server is down during FES and SES

$$P(R) = \lim_{z\to 1}\tilde{R}(z,0)$$

$$= \lim_{z\to 1}\frac{\tilde{R}\left(\left(\lambda_1-\lambda_1 X(z)\right)-1\right)[\delta\, F_m(z,0)+\alpha(1-\beta)S_m(z,0)]}{-\lambda_1+\lambda_1 X(z)}$$

$$P(R) = \frac{E(R)\lambda_1 E(X)(M_4(1)+M_5(1)}{\lambda_1 E(X)}$$

The probability that the server is on vacation

$$P(V) = \lim_{z\to 1}\tilde{Q}(z,0)$$

$$= \lim_{z\to 1}\frac{\tilde{Q}\left(\left(\lambda_1-\lambda_1 X(z)\right)-1\right)[(1-\alpha)(1-\beta)S_m(z,0)]}{-\lambda_1+\lambda_1 X(z)}$$

$$P(V) = \frac{E(V)\lambda_1 E(X)M_6(1)}{\lambda_1 E(X)}$$

Numerical Illustrations:

The theoretical results obtained for the proposed model are justified numerically with the following assumptions and notations:

FES time distribution is 2-Erlang with parameter μ_1

SES time distribution is 2-Erlang with parameter μ_2

The batch size distribution of the arrival is geometric with a mean 2

Vacation time is exponential with parameter ε

Repair time is exponential with parameter η

The minimum threshold value *a*

Maximum threshold value *b*

Effects of various parameters on performance measures:

Effects of arrival rate on various performance measures are presented in Table 2 and Fig 2.
Effects of breakdown probability on various performance measures are presented in Table 3 and Fig 3.
Effects of renovation rate on various performance measures are presented in Table 4 and Fig 4.

Impacts of various performance measures for fixed threshold values are presented. From Tables 2 and Fig 2, it is clear that, if λ_1 increases, E (Q), E (B), and E (W) increase whereas E (I) decreases. In Table 3 and 4 and Fig.3 and 4, an effect of performance measures for different failure rates and repair times are presented, it is observed that E(Q) and E(W) will be increased whenever the failure rate increases and E(Q) and E(W) will be decreased whenever the repair time increases.

Table 2. Arrival rate versus performance measures

λ_1	E(Q)	E(B)	E(I)	E(W)
4.0	6.3654	5.3451	2.5691	3.4576
4.5	7.4563	6.2315	1.8433	3.5673
5.0	9.4346	8.2341	1.3421	4.1293
5.5	11.3492	9.7312	1.0212	5.3452
6.0	12.5632	11.6313	0.4363	6.2321

(For *a*=2, *b*=4, ε=10, η=8, δ=0.2, β = 0.5)
E (Q) – Expected queue length E (B) - Expected length of busy period
E (W) - Expected waiting time in the queue E (I) - Expected length of idle period

Table 3. Breakdown probability versus Performance measures

δ	E(Q)	E(B)	E(I)	E(W)
0.1	7.3238	6.2312	1.5431	5.4534
0.2	8.3423	6.9821	1.4234	6.4312
0.3	9.2342	7.1231	1.3487	7.4313
0.4	11.3483	8.1231	1.0291	9.4221
0.5	12.3421	9.1231	0.8742	11.2124

Published Online First: January, 2024
https://dx.doi.org/10.21123/bsj.2024.9126
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Table 4. Renovation rate versus performance measures

Renovation rate (α)	E(Q)	E(B)	E(I)	E(W)
3	6.932	3.912	0.224	1.832
4	6.134	3.516	0.236	1.762
5	5.513	3.089	0.243	1.654

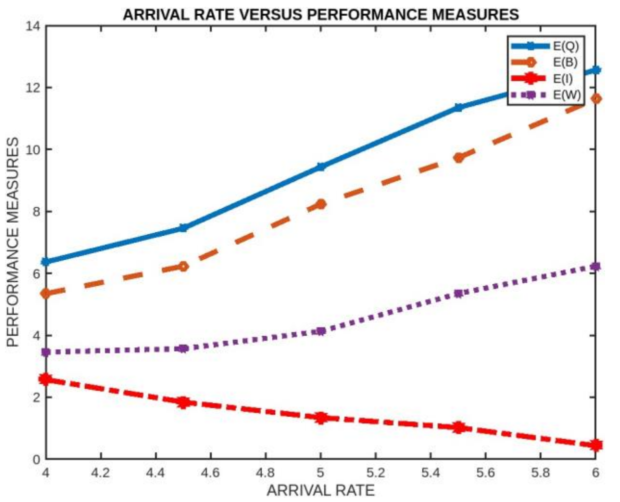

Figure 2. Arrival Rate versus Performance Measures

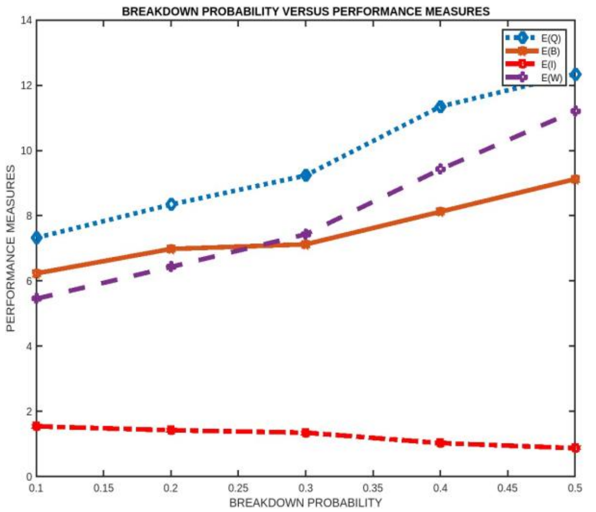

Figure 3. Breakdown Probability versus Performance Measures

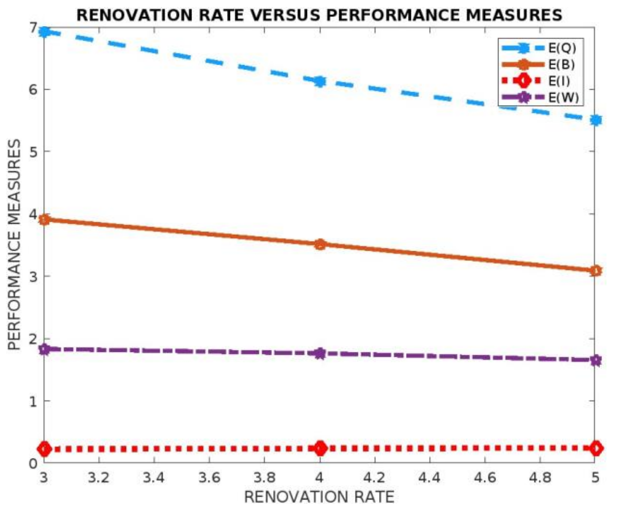

Figure 4. Renovation Rate versus Performance Measures

Conclusion

In this paper, M^X/G (a, b)/1 queueing model with mandatory two stages of service, vacation, and common renewal station two stages of service are considered. The uniqueness of the model is based on introducing server loss in two phases of service and common renewal of service stations for the queueing system. Generating function for queue size was obtained by imparting supplementary variable techniques. Some important performance characteristic of this model is computed with appropriate numerical results. All the above-discussed results and conclusions have played a significant role in managerial decision-making. It is possible to extend this model with multiple vacation and vacation interruptions in the future.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai, India.

Authors' Contribution Statement

This work was carried out in collaboration between all authors. Model description, Mathematical modeling, Probability Generating Function, and Steady State analysis were carried out by S. P. N. and he validated analytical results using Mat Lab. S. D. L. wrote and edited the manuscript with a revised idea. All authors read and approved the final manuscript.

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Published Online First: January, 2024
https://dx.doi.org/10.21123/bsj.2024.9126
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

تحليل نظام انتظار الخدمة غير المتجانس والمتعدد المراحل مع التقسيم إلى مرحلتين، والملاحظات، والإجازة

إس بي نيرانجان ، إس ديفي لاثا

قسم الرياضيات، معهد فيل تيك رانجاراجان د. ساجونثالا للبحث والتطوير للعلوم والتكنولوجيا، تشيناي، الهند.

الخلاصة

يصل العملاء إلى النظام بكميات كبيرة وفقًا لعملية بواسون بمعدل $\alpha 1$. تنقسم عملية الخدمة المجمعة إلى مرحلتين تسمى الخدمة الأساسية الأولى (FES) والخدمة الأساسية الثانية (SES) مع الحد الأدنى لسعة الخادم "a" والحد الأقصى لسعة الخادم "b" عندما يفشل الخادم، أثناء FES أو SES، لن تتم مقاطعة عملية الخدمة. يتم تنفيذه بشكل مستمر للدفعة الحالية عن طريق القيام ببعض الترتيبات الاحترازية الفنية. العميل الذي يحتاج إلى تعليقات بعد اكتمال SES سيتم نقله فورًا للخدمة بواسطة الخادم مع احتمال β. بعد الانتهاء من SES، إذا لم تكن هناك ردود فعل مع الاحتمال $1-\beta$، فإن طول قائمة الانتظار أكبر من أو يساوي a' ثم يبدأ الخادم الخدمة الأساسية الأولى، أو إذا كان طول قائمة الانتظار أقل من a' ثم الخادم يغادر لقضاء اجازة. سيتم إصلاح الخادم بعد انتهاء خدمة FES أو SES باحتمال δ أو α على التوالي خلال فترة تجديد الخادم. عندما لا يكون هناك فشل في الخادم بعد اكتمال FES، سيزود الخادم SES بالاحتمال $1-\delta$. وبالمثل، إذا لم يكن هناك فشل في الخادم بعد SES، فقد يذهب الخادم إلى FES أو إلى إجازة مع احتمال $1-\alpha$. عند اكتمال SES، إذا كان طول قائمة الانتظار أكبر من أو يساوي a'، فسينتقل الخادم إلى FES. بعد الانتهاء من الإجازة، إذا كان طول قائمة الانتظار أكبر من أو يساوي a'، فسيبدأ الخادم في أول خدمة أساسية. ومن ناحية أخرى، إذا كان عدد العملاء المنتظرين في قائمة الانتظار أقل من a'، فسيظل الخادم خاملاً حتى يصل طول قائمة الانتظار إلى القيمة a'. بالنسبة للنموذج المصمم، يتم الحصول على دالة توليد الاحتمالية لحجم قائمة الانتظار في وقت عشوائي باستخدام تقنية المتغير التكميلي. يتم أيضًا حساب مقاييس الأداء المختلفة باستخدام الرسوم التوضيحية الرقمية المناسبة.

الكلمات المفتاحية: خدمة الدفعات، الأعطال، الوصول بالجملة، التعليقات، الخدمة على مرحلتين..