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Computation of the Unreliable M^x / M /1 Model for Multiple Working Vacations Queuing System with Encouraged Arrivals

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Abstract

This research analyzed queuing system with encouraged arrivals under multiple working vacations and server breakdowns. It is based on a $M^x/M/1$ model, where the customers arrive in batches or bulk. In addition, the arrival follows a Poisson process and the service time is distributed exponentially. Moreover, in this queuing system, breakdowns can occur at any time, and it is affecting the server's service time. The repair time is independent. Server resume the service as soon as returns from the service facility. Repair time follows exponential distribution. Encouraged arrival in queuing models involves external factors or incentives that prompt the firms. This strategy for mitigating may include preventing maintenance or minimizing the service disruptions and maintain the system efficiency. Encouraged arrival in queuing model conscious effort to increase customers or other entities to a service system during specific periods. Encouraged arrival improves the system performance and service efficiency of the model. Finally, this research solved the Chapman-Kolmogorov balancing equations for the steady state system, analyzed the queuing model using probability generating function and discussed, the stochastic decomposition property and the expected system size.

Keywords: Breakdowns, Encouraged Arrivals, Multiple Working Vacations, Probability Generating Function, Queuing System, Stochastic Decomposition.

Introduction

Queuing is described as the study of an object or group of individuals moving in a queue. It covers the customer size, the service process, the number of servers, and the arrival procedure. It may include individuals, data packets, a patient waiting at a medical facility, banking sectors, etc. The flow of arrivals may be finite or infinite capacity and server serves towards the capability of utilization. Abdelmawgoud M, Dawood A and Moussa M [1] investigated how long service wait times effect on guests' satisfaction in five-star hotels in Egypt. a single server queuing system with working vacation under two type of server was studied by Agraval P, Jain A and Jain M [2]. Spatiotemporal modeling in wireless communication network theory analyzed by Alqaysi [3]. Bouchentouf [4] *et al.*, studied single server queuing system under single working vacation and multiple working vacations with customers' impatience. Queuing theory was introduced by Erlang [5].

Jain M and Jain A[6] dealt with a single server working vacation queuing model with multiple types of server breakdown. Many of researchers studied the queuing systems, Krishnamoorthy A, Joshua A, and Kozyrev D [7] analyzed bulk arrival and bulk service in

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Markova process. Lakshmi and Kassahun [8] studied M/M/C queuing system with various working vacations. Moreover, Laxmi P, Rajesh P, and Kassahun T [9] discussed bulk arrival queue with customer impatience. Batch arrival multiple working vacations with server breakdowns and multiple working vacations are analyzed by Mary[10] *et al.* Malik and Gupta[11] discussed the concept of single server queuing model of finite size with multiple vacations and encouraged arrivals. Moussa[12] *et al.*, employed an M/M/1 queuing model to analyze the service time characteristics of fast-food outlets in Egypt's tourism sector.

A brief review of working vacation with different frame works given by Patna[13] *et al.* Seenivasan and Chandiraleka[14] have studied the concept of single server queue with multiple working vacation dependent on server breakdowns. Also, Servi and finn[15] have introduced a concept of vacation model. Moreover, Sindhu[16] et al., analysed M/M/1 queuing model with interdependent arrival and independent arrival along with service process under working vacation. Finally, the encouraged arrivals have been discussed by Som and Sath[17].

The crucial idea about encouraged arrivals are discussed. This paper analyzed M/M/c/N queuing system with encouraged arrivals, reneging, retention and feedback customers discussed by Som and Seth[18]. In addition, Batch arrival with working vacations are analyzed by Xu[19], *et al.* Yusofv[20] *et al.*, discussed the analysis of propagation in VHF military tactical communication system. The single server infinite queuing capacity and it may afford infinite number of customers to get served. Yukata baba[21] discussed GI/M/1 queue with vacations under server works with different rates.

The unexpected nature of consumer behavior and the dynamic nature of the fiercely competitive business environment, businesses are motivated to study customer behavior. This study aids companies in developing marketing plans to draw in new clients. Precision is crucial since even the smallest departure from the plan might have an impact on finances or result in decreased earnings. Businesses frequently provide discounts and promotions, whether through physical storefronts or online platforms, to encourage new consumers. Customers actively seek out these attractive deals and discounts made available by many companies, which increases their propensity to select a specific firm. These clients are referred to as encouraged arrivals since they are drawn in by the offerings. The core literature on queuing benefits from the concept of encouraged arrivals. For example, during the summer season, juice shops experience an increase in customer influx due to various promotional activities and incentives

Based on the above, this paper studies the system performance measure of unreliable batch arrival queuing model with multiple working vacations and encouraged arrival rate. By solving the Chapman-Kolmogorov balancing equations for the steady state system, it is possible to derive the probability generating function to the system size. The system performance measures are obtained using probability generating function and stochastic decomposition.

Model Description:

A batch arrival of multiple working vacations queue, the arrivals follow a Poisson process and service time follows an exponential distribution. The size of the arrival group be a random variable X and it has threshold value of $k < \infty$ with the probability g_k .

i.e., $P(X = k) = g_k \quad , k = 1,2,3 \ldots \ldots \ldots$

where the arrival rate is composite of all batches. The first and second moments of random variable X with PGF are denoted by $X(Z)$, E(X) and E(X^2).

i.e. $$X(z) = \sum_{k=1}^{\infty} g_k z^k, \quad E\left(X^k\right) = \sum_{n=1}^{\infty} g_n n^k$$

In general, the server serves the customer to takes average amount of time with exponential distribution. The service rate denoted by μ in regular period at time t. But in a server with encouraged arrivals represents the efficiency of both regular customer arrivals and additional customer arrivals at certain amount of time period. The arrivals occur one by one of time period t with Poisson process along the parameter $\lambda(1+\zeta)$,where ζ represents the encouragement rate during that period.

The server works at a different rate instead of being completely idle during the vacation period, in which system works at slow rate. The vacation duration V follows an exponential distribution with the parameter ϖ. Customers are served at exponential service rate $\mu_v(\leq \mu)$ during working vacation period. Furthermore, server enters another working vacation only if the system is empty, which means server finds the system with no customer. This term is known as multiple working vacation. On the other

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part of working vacation if the server finds the system with customer then system immediately turns to regular service period. The service rate are vary for regular busy period and working vacation period but both are follows exponential with the different parameter of μ and μ_v.

If the server is in working vacation period or regular busy period ,when the system get breakdown or repair while working with a Poisson rate α . Whenever the system fails, the server is sent immediately for repair. The repair time is independent. After the repair system restore the service, meanwhile service time of waiting customer or newly arriving customer have to wait until the service of exiting one. which follows exponential distribution $1-e^{-\beta t}$ with the breakdown parameter β.This type of service continues until the system becomes empty again.

This model is denoted by $M_{MWV}^{BDEr}/M/1$

Results and Discussion

System Size Distribution:

Let $N_s(t)$ denotes the number customers in the system at time t and

$$J(t)=\begin{cases}0 \text{ the system is in a working vacation period with encouraged arrivals period at time } t\\ 1 \text{ the system is in regular busy period with encouraged arrivals period at time } t\\ 2 \text{ the system is in breakdown with encouraged arrivals period at time } t\end{cases}$$

Then $\{N_s(t),J(t)\}$ is a Markov Process

Let $Q_n(t)=Pr\{N_s(t)=n\,;J(t)=0\},n\geq 0$

$P_n(t)=Pr\{N_s(t)=n\,;J(t)=1\},n\geq 1$

$B_n(t)=Pr\{N_s(t)=n,J(t)=2\},n\geq 2$

It represents the system size probability at time t

Assuming the steady state system size probabilities as

$P_n=\lim_{t\to\infty}P_n(t)$ and $Q_n=\lim_{t\to\infty}Q_n(t)$ and $B_n=\lim_{t\to\infty}B_n(t)$ exists

The steady state equations satisfied by P_n 's , Q_n's and B_n's are

$$\lambda(1+\zeta)Q_0=\mu_vQ_1+\mu P_1 \quad (1)$$

$$(\lambda(1+\zeta)+\varpi+\mu_v)Q_n=\lambda(1+\zeta)\sum_{k=1}^{n}Q_{n-k}g_k+\mu_vQ_{n+1},n\geq 1 \quad (2)$$

$$(\lambda(1+\zeta)+\mu+\alpha)P_1=\mu P_2+\varpi Q_1+\beta B_1 \quad (3)$$

$$(\lambda(1+\zeta)+\mu+\alpha)P_n=\beta B_n+\lambda(1+\zeta)\sum_{k=1}^{n-1}P_{n-k}\,g_k+\mu P_{n+1}+\varpi Q_n,n\geq 2 \quad (4)$$

$$(\lambda(1+\zeta)+\beta)B_1=\alpha P_1 \quad (5)$$

$$(\lambda(1+\zeta)+\beta)\,B_n=\alpha P_n+\lambda(1+\zeta)\sum_{k=1}^{n-1}B_{n-k}g_k\,,n\geq 2 \quad (6)$$

The partial probability generating function are defined as follows

$Q(z)=\sum_{n=0}^{\infty}Q_nz^n$, $P(z)=\sum_{n=1}^{\infty}P_nz^n$ and $B(z)=\sum_{n=1}^{\infty}B_nz^n$

Multiplying Eq 2 by z^n and summing up $n\geq 1$, then adding with Eq 1 by using of Rouche's theorem, Q(z) obtain as

$$Q(z)=\frac{\mu_v(z-z_1)Q_0}{[\lambda(1+\zeta)z(1-X(z))+\mu_v(z-1)+\varpi z]z_1} \quad (7)$$

In Eq 3 Multiplying by z and Eq 4 summing up $n\geq 2$ with z^n on both sides , from this P(z) is written as

$$P(z)=\frac{\mu_v\lambda(1+\zeta)z\,Q_0}{z_1}\left(\frac{(z-1)z_1(X(z_1)-1)-(z_1-1)z(X(z)-1)}{\lambda(1+\zeta)z(1-X(z))+\mu(z-1)+\varpi z}\right)\left(\frac{\lambda(1+\zeta)(1-X(z))+\beta}{[\lambda(1+\zeta)z(1-X(z))+\mu(z-1)+\alpha z][\lambda(1+\zeta)(1-X(z))]+\beta[\lambda(1+\zeta)z(1-X(z))+\mu(z-1)]}\right) \quad (8)$$

As proceeding Eq.5 and 6, B(z) becomes,

$$B(z)=\frac{\alpha P(z)}{\lambda(1+\zeta)(1-X(z))+\beta} \quad (9)$$

Thus, total probability generating function of $P^{BDEr}_{MWV}(z)$ is obtained from adding of

Q(z) , P(z) and B(z)

$$P_{MWV}^{BDEr}(z) = Q(z) + P(z) + B(z)$$

$$P_{MWV}^{BDEr}(z) = \frac{\mu_v(z-1)Q_0}{z_1\left(\lambda(1+\zeta)z\big(1-X(z)\big)+\mu(z-1)\right)} \left(\frac{\mu(z-z_1)+\lambda(1+\zeta)zz_1\big(X(z_1)-X(z)\big)}{\lambda(1+\zeta)z\big(1-X(z)\big)+\mu_v(z-1)+\varpi z}\right) \left(\frac{\lambda(1+\zeta)\big(1-X(z)\big)+\beta+\frac{\alpha\lambda(1+\zeta)zz_1\big(X(z_1)-X(z)\big)}{\mu(z-z_1)+\lambda(1+\zeta)zz_1\big(X(z_1)-X(z)\big)}}{\lambda(1+\zeta)\big(1-X(z)\big)+\beta+\frac{\alpha\lambda(1+\zeta)z\big(1-X(z)\big)}{\mu(z-1)+\lambda(1+\zeta)z\big(1-X(z)\big)}}\right) \quad 10$$

Using normalizing condition $P_{MWV}^{BDEr}(1) = 1$ with $X(1) = 1,\ X'(1) = E(X)$,

$$Q_0 = \frac{\varpi z_1}{\mu_v}\left(\frac{\beta\mu(1-\rho)-\alpha\lambda(1+\zeta)E(X)}{\mu\beta(1-z_1)+\lambda(1+\zeta)z_1\big(X(z_1)-1\big)(\alpha+\beta)}\right) \quad 11$$

$$Where \quad \rho = \frac{\lambda(1+\zeta)}{\mu}E(X)$$

Substituting Q_0 in Eq 10 the PGF is

$$P_{MWV}^{BDEr}(z) = \frac{\mu(1-\rho)(z-1)}{\lambda(1+\zeta)z\big(1-X(z)\big)+\mu(z-1)} \cdot \frac{\left(\frac{\mu(z-z_1)+\lambda(1+\zeta)z\big(X(z_1)-X(z)\big)}{\lambda(1+\zeta)z\big(1-X(z)\big)+\mu_v(z-1)+\varpi z}\right)}{\left(\frac{\lambda(1+\zeta)\big(X(z_1)-1\big)+\mu(1-z_1)}{\varpi}\right)} \cdot \frac{1-\frac{\alpha\rho}{\beta(1-\rho)}}{1+\frac{\alpha\lambda(1+\zeta)z_1\big(X(z_1)-1\big)}{\beta[\lambda(1+\zeta)z_1(X(z_1)-1)+\mu(1-z_1)]}} \cdot \frac{\lambda(1+\zeta)\big(1-X(z)\big)+\beta+\frac{\alpha\lambda(1+\zeta)zz_1\big(X(z_1)-X(z)\big)}{\mu(z-z_1)+\lambda(1+\zeta)zz_1\big(X(z_1)-X(z)\big)}}{\lambda(1+\zeta)\big(1-X(z)\big)+\beta+\frac{\alpha\lambda(1+\zeta)z\big(1-X(z)\big)}{\mu(z-1)+\lambda(1+\zeta)z\big(1-X(z)\big)}} \quad 12$$

Hence, the total PGF can be written as

$$P_{MWV}^{BDEr}(z) = P_{\frac{M^X}{M1}\,MWV}^{BDEr}(z)\,\frac{\left(1-\frac{\alpha\rho}{\beta(1-\rho)}\right)}{\left(1+\frac{\alpha\lambda(1+\zeta)z_1\big(X(z_1)-1\big)}{\beta[\lambda(1+\zeta)z_1(X(z_1)-1)+\mu(1-z_1)]}\right)} \cdot \frac{\lambda(1+\zeta)\big(1-X(z)\big)+\beta+\frac{\alpha\lambda(1+\zeta)zz_1\big(X(z_1)-X(z)\big)}{\mu(z-z_1)+\lambda(1+\zeta)zz_1\big(X(z_1)-X(z)\big)}}{\lambda(1+\zeta)\big(1-X(z)\big)+\beta+\frac{\alpha\lambda(1+\zeta)z\big(1-X(z)\big)}{\mu(z-1)+\lambda(1+\zeta)z\big(1-X(z)\big)}}$$

Thus, the total probability generating function is obtained.

Decomposition Property:

To decompose the probability generating function of a batch arrival multiple working vacations of breakdown with an encouraged arrival queuing model into the product of two random variables.one of which is

$$P_{MWV}^{EDEr}(z) = \frac{\mu(1-\rho)(z-1)}{\lambda(1+\zeta)z\big(1-X(z)\big)+\mu(z-1)} \cdot \frac{\left(\frac{\mu(z-z_1)+\lambda(1+\zeta)z\big(X(z_1)-X(z)\big)}{\lambda(1+\zeta)z\big(1-X(z)\big)+\mu_V(z-1)+\varpi z}\right)}{\left(\frac{\lambda(1+\zeta)\big(X(z_1)-1\big)+\mu(1-z_1)}{\varpi}\right)}$$

Which is coincide the system size for the batch arrival multiple working vacation with breakdown queuing model[3] and the another one is

$$\frac{1-\frac{\alpha\rho}{\beta(1-\rho)}}{1+\frac{\alpha\lambda(1+\zeta)z_1\big(X(z_1)-1\big)}{\beta[\lambda(1+\zeta)z_1(X(z_1)-1)+\mu(1-z_1)]}} \cdot \frac{\lambda(1+\zeta)\big(1-X(z)\big)+\beta+\frac{\alpha\lambda(1+\zeta)zz_1\big(X(z_1)-X(z)\big)}{\mu(z-z_1)+\lambda(1+\zeta)zz_1\big(X(z_1)-X(z)\big)}}{\lambda(1+\zeta)\big(1-X(z)\big)+\beta+\frac{\alpha\lambda(1+\zeta)z\big(1-X(z)\big)}{\mu(z-1)+\lambda(1+\zeta)z\big(1-X(z)\big)}}$$

Which gives the PGF of conditional system size distribution during breakdown period with encouraged arrivals.

This decomposition allows to analyses the classical M^x/M/1 /BD multiple working vacations queueing model and the additional queue length with encouraged arrival in the system separately. Which can be useful for performance analysis. This gives the Decomposition Property.

Expected System Size of the Model:

The performance measurers of the system size under steady state condition is obtained.

Now, the expected system size is,

$$L = \frac{d}{dz}\left(P_{MWV}^{BDEr}(z)\right) \ at\ z = 1$$

$$= \frac{1-\frac{\alpha\rho}{\beta(1-\rho)}}{1+\frac{\alpha\lambda(1+\zeta)z_1(X(z_1)-1)}{\beta[\lambda(1+\zeta)z_1(X(z_1)-1)+\mu(1-z_1)]}} \left\{ \begin{array}{l} \frac{\rho\mu}{\beta C^2}\left[C^2+\alpha\left(z_1-\frac{D}{\lambda(1+\zeta)E(X)}\right)(D-C)+\alpha D\left(\frac{C}{\beta}-\frac{1}{\rho}\right)-1\right]+ \\ \left[\frac{\lambda(1+\zeta)\left(E(X)+E(X^2)\right)}{2\mu(1-\rho)}+\frac{\lambda(1+\zeta)E(X)-\mu_V}{\varpi}+\frac{z_1\left(\mu-\lambda(1+\zeta)E(X)\right)}{C}\right]\left(1+\frac{\alpha D}{\beta C}\right) \end{array} \right\} \quad 14$$

Where $C = \mu(1-z_1)+\lambda(1+\zeta)z_1(X(z_1)-1)$ and $D = \lambda(1+\zeta)z_1(X(z_1)-1)$

Thus, the expected system size of the model is evaluated.

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Conclusion

This paper focuses on analyzing a batch arrival breakdown queuing system with multiple working vacations and encouraged arrivals. This paper deals an effectiveness of stochastic decomposition property and the system performance is measured. The steady state probabilities of the queuing system have been obtained, using probability generating function. Furthermore, future research of this queuing model is balking arrival rate as an additional parameter and exploring these techniques to enhance the system's performance with the numerical illustrations.

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Author's Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Sri Krishna College of Technology, Coimbatore, India.

Author's Contribution

S. A. and J. K. designed the model and developed the theoretical framework. S. A. took the lead in derivation and writing the manuscript. S. A. helped to research and shape the manuscript. S. A. supervised the article. Both the authors contributed to the final manuscript.

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حساب نظام M^x / M / 1 غير الموثوق به للاصطفاف في إجازات العمل المتعددة مع تشجيع الوافدين

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الخلاصة

تحلل هذه الورقة نظام قائمة الانتظار مع تشجيع الوافدين في ظل إجازات عمل متعددة وتعطل الخادم . يتبع هذا النموذج M^x / M / 1، حيث يصل العملاء على دفعات .يتبع الوصول عملية Poisson ويتم توزيع وقت الخدمة بشكل أسي .في هذا النظام يمكن أن تحدث أعطال في أي وقت وتؤثر على وقت خدمة الخادم .في هذا البحث تم تحليل النموذج باستخدام دالة توليد الاحتمالات .علاوة على ذلك ، تمت أيضًا مناقشة خاصية التحلل والحالات الخاصة.

الكلمات المفتاحية: الأعطال، تشجيع الوافدين، إجازات العمل المتعددة، وظيفة توليد الاحتمالات، التحلل العشوائي