

Analysing the Performance of $M/M(a, b)/1/ MWV$ Queuing Model with the Busy Period Breakdown

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Received 31/05/2023, Revised 27/04/2023, Accepted 29/04/2024, Published Online First 20/07/2024



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Abstract

This paper aims to analyze the $M/M(a,b)/1$ multiple working vacations queuing model with a breakdown. Instead of the server being fully idle during the vacation period, the server serves at a different rate during multiple working vacations. The system has only one server, and the service rate varies depending on the arrival state. Customers' enter the system to get service with parameter λ_v following the Poisson distribution. The server provides service for customers' in regular busy periods with parameter μ and under multiple working vacations, the server provides service with parameter μ_v with the exponential distribution. In this model, batches of customers are served as a group under the general bulk service rule, which was introduced by Neuts. In the batch service process, the service times for each customer within a batch may be independent and random variables. The number of customers' in each batch can also vary. Thus, each batch of service contains a minimum of 'a' units and a maximum of 'b' units of customers'. Suppose that the number of customers waiting in the queue is less than 'a' server begins a vacation random variable V with parameter η , the breakdown β_v occurs during the busy state. This paper analyzed the steady-state equation, steady-state solutions, and measures of system performance. Specifically, various performance analyses, namely the mean length and other characteristics like the probability that the server is idle, regular busy and working vacation periods are analyzed. Finally, this paper computed the results with the working vacation and the classical multiple working vacation models.

Keywords: Breakdown, Busy State, Idle, Multiple Working vacation (MWV), Working State.

Introduction

In the past, numerous papers have been published on queuing models with server breakdowns and vacations. For example, Servi and Finn¹ introduced the working vacation policy, in which instead of the server fully stopping the service during the vacation period, the server serves at a different service rate during the working vacation. In addition, Tian *et al*², developed a $M/M/1$ queue model with a single

working vacation. Jain³ analyzed the queuing models of working vacations with multiple types of server breakdowns. Choudhury⁴ analyzed a batch service single vacation $Mx/G/1$ queuing model with a single server policy. Finally, to analyse the service time characteristics of fast-food establishments, Moussa *et al*⁵, used an $M/M/S$ queuing model. Abdelmawgoud *et al*⁶, examined the impact of

lengthy wait times on customer satisfaction at five-star hotels. Azmi and Namh⁷ derived the M/E_r/1/N queuing system, which was considered in equilibrium based on the Erlang study.

Berdjoudj *et al*⁸., developed the sensitivity analysis of the M/M/1 retrial queue with working vacations and vacation interruptions. Chakravarthy and Rakhee⁹ analyzed a queuing model with server breakdowns, repairs, vacations, and backup servers. Seenivasan *et al*¹⁰., developed the performance analysis of two heterogeneous server queuing model with intermittently obtainable server using a matrix geometric method. A Discrete-Time GIX/Geo/1 Queue with multiple working vacations under late and early arrival system was presented by Barbhuiya and Gupta¹¹. In addition, the cost optimization of single server retrial queuing model with Bernoulli schedule working vacation, vacation interruption and balking developed by Kumar *et al*¹².; the performance analysis of retrial queuing model with working vacation, interruption, waiting server, breakdown and repair analyzed by Gupta and Kumar¹³; and Agrawal *et al*¹⁴., suggested the M/M/1 queuing model with working vacation and two type of server breakdown. Moreover, Markovian queueing model with single working vacation and catastrophic is presented by Seenivasan and Abinaya¹⁵.

Ayyappan & Meena¹⁶ analyzed server vacation, repair with breakdown in Phase type queuing model. Agarwal *et al*¹⁷., studied the working breakdown queuing model for heterogeneous servers using PSO. Working Breakdown queue with vacation analyzed by Somasundaram *et al*¹⁸. Mathew *et al*¹⁹., discussed the server breakdown and impatience customer model. MAP/PH/1 queuing system with breakdown, setup time and repair derived by arulmozhi²⁰. Algorithm using circular queue data structure developed by Ibraheem and Hasan²¹, which is applicable in personal information security and network communication security. Khan & Paramasivam²² analyzed the encouraged arrival vacation queuing model with breakdown. Working vacation queuing system with impatient customers with breakdown developed by Manoharan &

Raman²³. Su *et al*²⁴., presented the traffic breakdown and human driven vehicles. Working vacation and server breakdown with Markovian queue analyzed by Liu *et al*²⁵. Setup time, Repair and breakdown in retrial queue analyzed by Tian *et al*²⁶.

Finally, in this paper, the multiple working vacation customers are served according to the general bulk service rule, and there was a breakdown in busy periods. This paper aims to analyze the steady-state probability equation and the measures of system performance.

Methodology

In this model, this paper assumes the arrival Poisson process with parameter λ_v . The exponential service process μ and the service offered during the vacation is μ_v . When the vacation is over, server switches his service μ_v to μ . The model denoted as M/M(a,b)/1/MWV with breakdown. In addition, batches of customers are served under the general bulk service rule. Thus, each batch of service contains minimum 'a' units of and maximum 'b' units. Moreover, this model supposes that the number of customers waiting in the queue is less than 'a' server begins a vacation random variable V with parameter η . Finally, the steady states are analyzed and the measures of system performance are derived, considering that the breakdown denoted as β_v occurs in the busy period.

Let $N_Q(t)$ = the number of customers in the queue at time t and $J(t) = 0, 1 \text{ or } 2$ according to whether the server is idle, working vacation or regular busy on vacation state respectively.

$$R_n^I(t) = Pr\{N_Q(t) = n, J(t) = 0\}; 0 \leq n \leq a - 1$$

$$Q_n^V(t) = Pr\{N_Q(t) = n, J(t) = 1\}; n \geq 0$$

$$P_n^B(t) = Pr\{N_Q(t) = n, J(t) = 2\}; n \geq 0$$

$J(t) = 0$, the size of the queue and the system are same. $J(t) = 1 \text{ or } 2$, the size of the queue and systems are $a \leq x \leq b$ customers.

Probabilities of the steady state are;

$$Q_n^V = \lim_{t \rightarrow \infty} Q_n^V(t); R_n^I = \lim_{t \rightarrow \infty} R_n^I(t);$$

$$P_n^B = \lim_{t \rightarrow \infty} P_n^B(t)$$

Exist, the Chapman Kolmogrove equations satisfied by them in the steady state are given by;

$$\lambda_v R_0^I = \mu P_0^B + \mu_v Q_0^V \quad 1$$

$$\lambda_v R_n^I = \lambda_v R_{n-1}^I + \mu P_n^B + \mu_v Q_n^V; 1 \leq n \leq a-1 \quad 2$$

$$(\lambda_v + \eta + \mu_v) Q_0^V = \lambda_v R_{a-1}^I + \mu_v \sum_{n=a}^b Q_n^V \quad 3$$

$$(\lambda_v + \eta + \mu_v) Q_n^V = \lambda_v Q_{n-1}^V + \mu_v Q_{n+b}^V; n \geq 1 \quad 4$$

$$(\lambda_v + \mu + \beta_v) P_0^B = \mu \sum_{n=a}^b P_n^B + \eta Q_0^V \quad 5$$

$$(\lambda_v + \mu + \beta_v) P_n^B = (\lambda_v + \beta_v) P_{n-1}^B + \mu P_{n+b}^B + \eta Q_n^V; n \geq 1 \quad 6$$

Steady State Solution

To solve the steady state equation, the forward shifting operator E on P_n^B and Q_n^V are introduced as;

$$E(P_n^B) = P_{n+1}^B; E(Q_n^V) = Q_{n+1}^V; \text{for } n \geq 0$$

Thus the Eq 4 gives homogeneous difference equation;

$$[\lambda_v + \mu_v Q_{b+1}^V - (\lambda_v + \mu_v + \eta)E]Q_n^V = 0; n \geq 0 \quad 7$$

The characteristic equation of the difference equation is given by;

$$h(z) = \lambda_v + \mu_v z^{b+1} - (\lambda_v + \mu_v + \eta)z = 0$$

By taking $f(z) = (\lambda_v + \mu_v + \eta)z$ & $g(z) = \lambda_v + \mu_v z^{b+1}$, here $|g(z)| < |f(z)|$ on $|z| = 1$. By Rouche's theorem $h(z)$ has unique root r_v inside the contour $|z| = 1$. The solution of the homogeneous difference Eq 7 is given by;

$$Q_n^V = (r_v^n) Q_0^V \quad 8$$

From Eq 6 can be written as;

$$[\lambda_v + \beta_v + \mu E^{b+1} - (\lambda_v + \mu + \beta_v)E]P_n^B = -\eta r_v^{n+1} Q_0^V \quad 9$$

Again by Rouche's theorem, the equation $\lambda_v + \beta_v + \mu z^{b+1} - (\lambda_v + \mu + \beta_v)z = 0$ has a unique root r with $|r| < 1$ provide $\frac{\lambda_v}{b\mu} < 1$.

The solution of the non-homogeneous difference Eq 9 is given by;

$$P_n^B = (Zr^n + Z^* r_v^n) Q_0^V \quad 10$$

Where;

$$Z^* = \frac{\eta r_v}{\lambda_v(r_v-1) + \beta(r_v-1) + \mu r_v(1-r_v^b)} \quad \text{if } r_v \neq r \quad 11$$

The expression for R_n^I is obtained by adding Eqs 1 and 2 and substitute P_n^B and Q_0^V value to get;

$$R_n^I = \left[\frac{\mu}{\lambda_v} \left(\frac{Z(1-r^{n+1})}{(1-r)} + \frac{Z^*(1-r_v^{n+1})}{(1-r_v)} \right) + \frac{\mu_v(1-r_v^{n+1})}{\lambda_v(1-r_v)} \right] Q_0^V \quad 12$$

Now to calculate Z , consider Eq 5 and substitute P_n^B and Q_n^V value to get;

$$\frac{Z\mu(1-r^a)}{(1-r)} = \frac{\eta}{(1-r_v)} - \frac{Z^*\mu(1-r_v^a)}{(1-r_v)} \quad 13$$

Hence the steady state queue size probability of the model are expressed in terms of Q_0^V and are given by;

$$Q_n^V = (r_v^n) Q_0^V; n \geq 0 \quad 14$$

$$P_n^B = (Zr^n + Z^* r_v^n) Q_0^V; n \geq 0 \quad 15$$

Where;

$$Z = \frac{(1-r)}{\mu(1-r^a)} \left[\frac{\eta}{(1-r_v)} - \frac{Z^*\mu(1-r_v^a)}{(1-r_v)} \right] \quad 16$$

$$Z^* = \frac{\eta r_v}{\lambda_v(r_v-1) + \beta(r_v-1) + \mu r_v(1-r_v^b)} \quad 17$$

And;

$$R_n^I = \left[\frac{\mu}{\lambda_v} \left(\frac{Z(1-r^{n+1})}{(1-r)} + \frac{Z^*(1-r_v^{n+1})}{(1-r_v)} \right) + \frac{\mu_v(1-r_v^{n+1})}{\lambda_v(1-r_v)} \right] Q_0^V = 0; 0 \leq n \leq a-1 \quad 18$$

By using normalizing condition and calculate the value of Q_0^V

$$\sum_{n=0}^{\infty} Q_n^V + \sum_{n=0}^{\infty} P_n^B + \sum_{n=0}^{a-1} R_n^I = 1.$$

It obtain, $(Q_0^V)^{-1} = \omega(r_v, \mu_v) + Z\omega(r, \mu) + Z^*\omega(r_v, \mu)$.

Where, $\omega(x, y) = \frac{1}{(1-x)} \left(1 + \frac{y}{\lambda_v} \left(c - \frac{x(1-x^a)}{(1-x)}\right)\right)$.

Performance Measures

Mean Queue Length

The expected queue length is given by,

$$L_q = \sum_{n=1}^{\infty} n(Q_n^V + P_n^B) + \sum_{n=1}^{a-1} nR_n^I \quad 19$$

Substituting the values of Q_n^V , P_n^B and R_n^I to get;

$$L_q = Z\omega^*(r, \mu) + Z^*\omega^*(r_v, \mu) + \omega^*(r_v, \mu_v)$$

Where, $\omega^*(x, y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda_v(1-x)} \left\{ \frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2} \right\}$.

and Z & Z^* are given by Eqs 16 & 17.

Other Characteristics

If $Pr_{(wv)}$, $Pr_{(busy)}$ and $Pr_{(idle)}$ denote the probability that the server in idle, regular busy and working vacation period then;

$$Pr_{(idle)} = \sum_{n=0}^{a-1} R_n^I$$

Where the R_n^I is given by Eq 18.

$$Pr_{(busy)} = \sum_{n=0}^{\infty} P_n^B = \left(\frac{Z}{(1-r)} + \frac{Z^*}{(1-r_v)} \right) Q_0^V$$

$$Pr_{(wv)} = \sum_{n=0}^{\infty} Q_n^V = \frac{Q_0^V}{(1-r_v)}$$

Where Z & Z^* are given by Eqs 16 & 17.

Particular Cases

Case 1: M/M/1 Model

Letting $a=b=1$ and $\beta_v = 0$ in Eqs 14 to 18. It obtain,

$$Q_n^V = (r_v^n) Q_0^V; n \geq 0$$

$$P_n^B = \frac{Z^*}{r_v} (r_v^{n+1} - r^{n+1}) Q_0^V; n \geq 0$$

and $R_0^I = \frac{Q_0^V}{r_v}$

Where $r = \frac{\lambda_v}{\mu} = \rho_v$, $Z = -\frac{Z^*\rho_v}{r_v}$

And $Z^* = \frac{\eta r_v}{\mu(1-r_v)(r_v-\rho_v)}$.

The above equations gives the probabilities of $M/M/1$ working vacation queuing model analyzed by Liu *et al*²⁷.

Case 2: Heterogeneous Arrival M/M(a, b)/1/MWV Model with Breakdown in Busy Period

In this study considered homogeneous arrival, suppose arrival process is heterogeneous. Heterogeneous arrival refers each stage has different arrival rate, that is $\lambda_v = \lambda_{iv}$ in idle state, $\lambda_v = \lambda_{wv}$ in working vacation state, and $\lambda_v = \lambda_{bv}$ in busy state. The probabilities of the queue of the form;

$$Q_0^V = (r_v^n) Q_0^V \text{ and}$$

$$P_n^B = (Zr^n + Z^*r_v^n) Q_0^V; n \geq 0$$

$$R_n^I = \left[\frac{\mu}{\lambda_{iv}} \left(\frac{Z(1-r^{n+1})}{(1-r)} + \frac{Z^*(1-r_v^{n+1})}{(1-r_v)} \right) + \frac{\mu_v}{\lambda_{iv}} \frac{(1-r_v^{n+1})}{(1-r_v)} \right] Q_0^V = 0$$

Where, $Z = \frac{(1-r)}{\mu(1-r^a)} \left[\frac{\eta}{(1-r_v)} - \frac{Z^*\mu(1-r_v^a)}{(1-r_v)} \right]$

$$\text{and } Z^* = \frac{\eta r_v}{\lambda_{bv}(r_v-1) + \mu r_v(1-r_v^b)}$$

Further;

$$(Q_0^V)^{-1} = \omega(r_v, \mu_v) + Z\omega(r, \mu) + Z^*\omega(r_v, \mu)$$

where, $\omega(x, y) = \frac{1}{(1-x)} \left(1 + \frac{y}{\lambda_{iv}} \left(c - \frac{x(1-x^a)}{(1-x)}\right)\right)$ and

$$L_q = Z\omega^*(r, \mu) + Z^*\omega^*(r_v, \mu) + \omega^*(r_v, \mu_v)$$

where, $\omega^*(x, y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda_{iv}(1-x)} \left\{ \frac{a(a-1)}{2} + \frac{ax^{a-1}(1-x) - x^2(1-x^a)}{(1-x)^2} \right\}$

These above equation gives the queue probabilities of heterogeneous arrival on $M/M(a, b)/1/MWV$ queuing model with breakdown that analysed by Lidiya and Mary²⁸.

Results and Discussion

Here the $M/M(a,b)/1/MWV$ queuing system with breakdown in the busy period is analyzed, for this model the steady state equations and mean queue length are calculated. When $a=b=1$ and $\beta_v = 0$ this model deduces to $M/M/1$ working vacation model and when $\beta_v = 0$ this model coincides with the classical $M/M(a,b)/1/MWV$ model. When a

Conclusion

This paper analyzed the $M/M(a,b)/1/MWV$ model with a breakdown. The arrival rate varies depending on the server's state. Whereas, the breakdown occurs during the busy state. In addition, this paper developed the steady-state probability equation and the measures of system performance. In specific, various performance

breakdown occurs during a busy period it can have an impact on the system's operations and it may increase the waiting time. It leads to customers losing trust in the system. Understanding the queuing model with breakdown it may predict the customer's behaviour and improve the system's performance.

analysis namely the mean length and other characteristics like the probability that the server is idle, regular busy and busy vacation periods are analyzed. Finally, it computed the results with the working vacation and classical multiple working vacation models.

Acknowledgment

The authors would like to extend their sincere gratitude to the editors, associate editors and reviewers for their valuable commands and

suggestions which have improved the quality of the work.

Authors' Declaration

- Conflicts of Interest: None.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.

- Ethical Clearance: The project was approved by the local ethical committee at Bharathiar University, Coimbatore, Tamil Nadu, India.

Authors' Contribution Statement

This work was carried out in collaboration with both authors. L.P. designed the paper, analyzed the study data, wrote the manuscript, and planned the publication. J.R.M. provided insightful suggestions

and revisions throughout the research process, ensuring the accuracy and clarity of the final manuscript. Both authors read and approved the final manuscript.

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دراسة أداء M/M (أ، ب) $MWV/1$ مع التقسيم في فترة العمل

ليديا ف ، جوليا روز ماري ك

قسم الرياضيات، كلية نيرمالا للبنات، جامعة بهاراتيار، كويمباتور، تاميل نادو ، الهند.

الخلاصة

في هذه الورقة ، نقوم بتحليل M/M (أ ، ب) $MWV/1$ (مع الانهيار بدلاً من أن يكون الخادم خاملًا تمامًا خلال فترة الإجازة ، يخدم الخادم بمعدل مختلف أثناء إجازات العمل المتعددة. يختلف سعر الخدمة حسب حالة الوصول. يصل العميل إلى النظام مع المعلمة μ_v ليتبع خادم توزيع Poisson الذي يوفر الخدمة مع المعلمة μ وتحت خادم الإجازات المتعددة العامل يوفر الخدمة مع المعلمة μ_v مع التوزيع الأسي. في هذا النموذج ، يتم تقديم دفعات العملاء بموجب القاعدة العامة للخدمة المجمعة. وبالتالي تحتوي كل دفعات من الخدمة على وحدات "أ" كحد أدنى وحد أقصى "ب". لنفترض أن العملاء المنتظرين في قائمة الانتظار أقل من خادم يبدأ متغيرًا عشوائيًا V للعلطة مع المعلمة η . هنا يحدث الانهيار β_v أثناء حالة الانشغال. قمنا بتحليل معادلة الحالة المستقرة ومقاييس الأداء للنظام. تحليل أداء مختلف ، أي متوسط الطول وخصائص أخرى مثل احتمال تحليل الخادم في فترة الخمول ، والانشغال المنتظم ، والإجازة المزدحمة. نقوم أيضًا بحساب النتائج باستخدام نماذج الإجازات المتعددة الكلاسيكية وعطلات العمل المتعددة.

الكلمات المفتاحية: الانهيار ، حالة الانشغال ، الخمول ، إجازة عمل متعددة (MWV) ، حالة العمل.