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# Analysis of Multiple Working Vacations Queuing System With Encouraged Arrival Using M/M (a,b)/1 Model

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## Abstract

Generally, it is very common in our daily life to meet up with queuing systems. This study examines the concept of encouraged arrival in M/M (a,b)/1/Multiple Working Vacations queuing model that follows the General Bulk Service Rule. The considered queuing model consists of three states namely idle, working vacation and regular busy period. The single server available in the system usually goes on vacation whenever the system is idle, that is, when it is empty. This model deals with multiple working vacations of the server which are exponentially distributed and the concept of encouraged arrival of the customers is examined particularly in the regular busy period. Specifically, the term encouraged arrival is a recent addition to the queue with the existing customers following Poisson distribution and are served in batches. The main objective of the study is to calculate the mean queue length (Lq) and various other performance measures of the discussed queuing model. Moreover, a real life application of the studied model has been implemented in a case study and the expected queue length has been discussed.

**Keywords:** Encouraged Arrival, Expected Queue length, General Bulk Service Rule, Multiple working Vacations, Queuing model.

## Introduction

The queuing process has an impact on all of our daily activities. Customers have to wait in queue every time as there aren't enough servers or the servers' service rate isn't fast enough to keep up with the rate at which customers are joining the queue. The queuing theory evaluates this phenomenon mathematically to determine the optimal solution, ensuring that no one has to wait in queue for an extended amount of time to receive service. Numerous industries including telecommunications, city traffic, health sector, inventory control, etc., find extensive use for queuing theory. Due to his early work in the field, Erlang AK is regarded as the founder of queuing theory.

A vacation is a period spent away from the main service location, and it can be caused by a variety of circumstances in vacations queuing scheme, a server from a primary service center may be unavailable for an arbitrary period. Working vacation (WV) is one kind of vacation policy under which the server provides service at a lower speed during the vacation period rather than stopping service completely. Furthermore, during the WV period, if there are customers at a service completion instant, the server can stop the vacation and come back to the regular busy state. This policy is called vacation interruption.

As the concept of uncertainty in queuing, fuzzy logic can be implemented. Thus Aarthi S. and Shanmugasundari M mainly focused on the use of intuitionistic fuzzy queuing theory to demonstrate and compare the performance of a single server queuing model with infinite capacity[1]. In Tourism

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industry, seasonal changes may lead to an increase in the arrival of customers and thus people need to wait in longer queues to get served. Abdelmawgoud et al., investigated the impact of prolonged waiting time of food service on customers' satisfaction in five-star hotels in Egypt[2]. By using the method of stages introduced by Erlang NA Abid and AK Al-Madi considered (M/Er/1/N) queuing system in equilibrium and calculated the probability for the empty system in the explicit form[3].

In recent days, lots of algorithms and techniques have been available for data security. Ibraheem N. and Hasan M. discussed a cryptosystem that combines several Substitution Cipher Algorithms along with the Circular queue data which can be applied efficiently for personal information security and network communication security as well [4]. In the realm of queuing theory, a bulk service queuing system introduces an interesting twist to the conventional waiting process.

In General, a bulk arrival queuing system involving a single server providing second optional service under bi-level control policy and the single servers' single vacation was analyzed by K. Julia Rose Mary, M. I. Afthab Begum and M. Jemila Parveen obtained with server vacations and obtained with the closed form. And the study also provided insights into queueing systems with bulk service and working vacations[5].

To observe that with many new concepts and conditions, numerous researchers are providing their results in the field of queuing theory but initially, Y. Levy and U. Yechiali discussed the vacations in queuing model which has laid the foundation for all its developed and upcoming findings in the field[6].

Later, Liu et al. stated the queue size probabilities of M/M/1 Multiple Working Vacation Queuing model with N-policy and different arrival rates. Using the method of matrix geometry solution, the conditional stochastic decomposition structures of the queue length as well as the waiting time were calculated[7]. The explicit expressions of M/M/1 queue sizes were derived by Majid et al. for both the server's normal busy time and its working vacation period by deriving the probability generating functions of the steady state probabilities. Numerous performance indicators were calculated, including the expected system size, the rate of customer loss as a result of reneging and balking, and the expected waiting time for a client served[8].

In general the characteristics of any queue help to evaluate the efficiency of that model, thus Malik S. and Gupta R. calculated some of the operating characteristics of a finite capacity queuing model with multiple vacations and encouraged customers. The steady-state solution is obtained by using the recursive technique. Some of the operating characteristics of the system like expected queue length, sojourn time, and probabilities of different states of the server were derived via recursive technique[9]. Also in tourism industry, Moussa et al., used an M/M/1 queuing model to measure the service time characteristics in fast food restaurants in Egypt. The research displayed an effective method to measure service time characteristics in fast food restaurants in Cairo[10]. The concept of a general type of bulk queue was discussed early by Neuts, who also examined the length of the queue and its busiest times[11].

A class of semi-vacation policies was first presented by L.D Servi and S.G Finn in which the servers operate at a reduced pace rather than suspending all primary service altogether while on vacations and presented Simple explicit formulae for the mean, variance, and distribution of the number and time in the system[12]. As many Researchers have contributed their findings in queuing theory, a single server finite capacity Markovian queuing model was proposed by Bhupender Kumar Som and Sunny Seth by merging the two present challenges of encouraged arrivals and the prevalent reverse reneging[13].

An independent ongoing analysis of the stock for two commodities in a queuing model was offered by K. Lakshmanan et al. under steady state conditions and the joint limiting distributions of the random variables were examined. Measures of system performance and long-term predicted total cost rate were also derived[14]. Again Srivaastava with other researchers considered a Markovian model of bulk arrival queuing with two service tiers—first come, first served and bulk service and by utilizing the supplement variable technique, determined the distribution of queue sizes for the model under consideration as well as performance

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metrics such server idle time, queue length, and busy period[15].

By using bivariate probability generating function (PGF) method, Tamrakar G. K. and Banerjee A. dealt with a queuing model and in which the steady-state joint distributions of the queue content and server content (when server is busy) and joint distribution of the queue content and type of the vacation taken by the server (when server is in vacation) have been obtained[16]. Wang J et al. developed a precise numerical approach to assess different performance indicators of a tandem queuing network with abandonment which can be used to analyze queuing networks with abandonment in a variety of contexts and service disciplines[17].

Based on the above literature review, this paper discusses the concept of encouraged arrival in M/M(a,b)/1/MWV and obtained with the mean queue length and other performance measures which can be applied to any queuing model in real life with batch service in the fields of transport vehicles, shuttle-bus services, freight trains, express elevators, tour guides and so on.

## Materials and Methods

An M/M(a,b)/1 queuing model of multiple working vacations with encouraged arrival is considered with the following assumptions: The arrival process in this model is a Poisson with parameter $\lambda_w$, whereas the encouraged arrival process also follows Poisson distribution with parameter $\lambda_w(1+\delta)$. According to Neuts' general bulk service rule (GBSR), the server processes the clients in batches. This rule states that the server will only begin providing service if at least ‘a’ customer is present. After completing a service, if the server discovers ‘a’ (or) more customers but not more than b clients in the system, then he serves them all at once; if he discovers more than b, he serves the first b-customers in turn while the others wait. As a result, there are a minimum of "a" units and a maximum of "b" units in each batch for service. The assumption is that the service time of batches of size s’$(a \le s` \le b)$ is an independent random variable with identical distribution and a parameter with an exponential distribution ‘$\mu_w$’.

When a service is finished and there are fewer than 'a' clients in the queue, the server starts for vacation(s), which is exponentially distributed and denoted with parameter ή. If the system length is still less than "a" after finishing one vacation, the server takes another vacation, and so on, until the server detects at least "a" customers in the queue (i.e., multiple vacations are used). If a server begins providing service during a vacation at a service rate ($\mu_{wv}$) that is different from the regular service rate ($\mu_w$) if the queue size reaches at least "a". The size of the batch in service is ‘k’ with $a \le k \le b$ and the service rates are independent of the size of the batch in service, thus when the vacations are over the server will shift the service rate from $\mu_{wv}$ to $\mu_w$, when the server is operating in a busy period.

In this model, it is assumed that there is an increase in the arrival rate, i.e., encouraged arrival occurs in the regular busy period and then the server continues to serve following GBSR. The above queuing model with assumed conditions is denoted as M/M(a,b)/1/MWV with encouraged arrival which can be employed in the fields of transport vehicles like cabs, taxis, shuttle-bus services, freight trains ,express elevators, tour guides and so on. Any concept where the service is attained in batches and with the mentioned conditions can be expressed in the state of queuing model as below:

### State System Size Equations

Let $N_Q(t)$ = number of customers or the products to be produced waiting at the time, ‘t’ and $J`(t)$ = 0,1 or 2 denotes that the server is idle during the vacations or working during vacations or in the regular busy period respectively.

Let $IR_n(t) = Pr\{N_Q(t) = n, J`(t) = 0\}$ ; $0 \le n \le a-1$

$VQ_n(t) = Pr\{N_Q(t) = n, J`(t) = 1\}$ ; $n \ge 0$

$BP_n(t) = Pr\{N_Q(t) = n, J`(t) = 2\}$ ; $n \ge 0$

When $J`(t) = 0$, the size of the queue and the system are same,

When $J`(t)$ = 1 or 2, then the size of the system is the sum of total number of customers waiting in the queue or the size of the service batch containing a

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$\leq x \leq$ b customers or products to be produced. The Steady State Probabilities satisfying the Chapman Kolmogrov equations are assumed as follows:

$VQ_n = \lim_{t\to\infty} VQ_n(t);$

$IR_n = \lim_{t\to\infty} IR_n(t);$

$BP_n = \lim_{t\to\infty} BP_n(t);$

The Steady State equations are expressed below:

$\lambda_w IR_0 = \mu_w BP0 + \mu_{wv} VQ_0$ 1

$\lambda_w IR_n = \lambda_w IR_{n-1} + \mu_w BP_n + \mu_{wv} VQ_n$ for $1 \leq n \leq a-1$ 2

$(\lambda_w + ή + \mu_{wv})\, VQ_0 = \lambda_w IR_{a-1} + \mu_{wv} VQ_n$ 3

$(\lambda_w + ή + \mu_{wv})\, VQ_n = \lambda_w VQ_{n-1} + \mu_{wv} VQ_{n+b}$ for $n \geq 1$ 4

$(\lambda_w(1+\delta) + \mu_w)\, BP_0 = \mu_w BP_n + ήVQ_0$ 5

$(\lambda_w(1+\delta) + \mu_w)\, BP_n = \lambda_w(1+\delta)\, BP_{n-1} + \mu_w BP_{n+b} + ήVQ_n$ for $n \geq 1$ 6

In the above mentioned steady state equations, Eq.1 & Eq.2 denotes the idle state while Eq.3 & Eq.4 denote the working vacation state and encouraged arrival occurs during the regular busy period i.e., in Eq 5 & Eq 6

## Steady State Solution

The concept of forward shifting operator (E) is introduced on $BP_n$ and $VQ_n$ to solve the above defined steady state equations,

$E(BP_n) = BP_{n+1}$ ; $E(VQ_n) = VQ_{n+1}$ ; for $(n \geq 0)$

The homogeneous difference equation is obtained from Eq 4

$(\mu_{wv} E^{(b+1)} - (\lambda w + ή + \mu_{wv})\, E + \lambda_w)\, VQ_n = 0$ ; $n \geq 0$ 7

The characteristic equation of the difference equation is expressed as follows

$h(w) = (\mu_{wv} w^{(b+1)} - (\lambda_w + ή + \mu_{wv}) w + \lambda_w) = 0$

By assuming $f(w) = (\lambda_w + ή + \mu_{wv}) w$ and $g(w) = \mu_{wv} w^{(b+1)} + \lambda_w$ , it is obvious that if on =1, then by Rouche's theorem, h(w) has only one root $z_v$ inside the contour. As the root is real, solution of the homogeneous difference equation is obtained as

$VQ_n = VQ_0$ ; $n \geq 0$ 8

Similarly, Eq 6 can be written as

$[\mu_w E^{(b+1)} - (\lambda_w(1+\delta) + \mu_w)E + \lambda_w(1+\delta)]\, BP_n - ή = VQ_0$ ; $n \geq 0$
9

Again by Rouche's Theorem, the equation

$[\mu_w w^{(b+1)} - (\lambda_w(1+\delta) + \mu_w) w + \lambda_w(1+\delta)] = 0$ has a unique root 'z' with $|z|<1$ provided $\frac{\lambda_w(1+\delta)}{b\mu} < 1$

The non-homogeneous difference equation is solved and the solution obtained is given by

$BP_n = (Xz^n + Yz_v^n)\, VQ_0$ 10

where $Y = \frac{ήz_v}{\lambda_w(1+\delta)(z_v-1) + \mu_w z_v (1-z_v^b)}$ if $z_v \neq z$ 11

by adding Eq 1 & Eq 2 over 0 to n, and substituting $VQ_n$ is obtained $BP_n$ , $IR_n$ is obtained as follows

$IR_n = [\frac{\mu_w}{\lambda_w}\left(\frac{X(1-z^{n+1})}{(1-z)} + \frac{Y(1-z_v^{n+1})}{(1-z_v)}\right) + \frac{\mu_{wv}}{\lambda_w}\frac{(1-z_v^{n+1})}{(1-z_v)}]\, VQ_0$

Hence, the steady state queue size probabilities are expressed in terms of the unknowns X and $VQ_0$ , in order to calculate X, consider Eq.5 and substituting the value of $BP_n$, it is found that

$X(\lambda_w(1+\delta) + \mu_w) - \frac{\mu_w(z - z^{b+1})}{(1-z)}) = ή - Y\left(\lambda_w(1+\delta) + \mu_w - \frac{\mu_w(z_v^a - z_v^{b+1})}{(1-z_v)}\right)$ 12

Which can be simplified as $\frac{X\mu_w(1-z^a)}{(1-z)} = \left(\left(\frac{ή}{(1-z_v)}\right) - \frac{Y\mu_w(1-z_v^a)}{(1-z_v)}\right)$ 13

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Eq 3 is also verified and the steady state queue size probabilities are expressed in terms of $VQ_0$ and are mentioned as below

$$VQ_n = z_v^n\, VQ_0 \quad ; \quad n \geq 0 \qquad 14$$

$$BP_n = (Xz^n + Yz_v^n)VQ_0 \quad ; \quad n \geq 0 \qquad 15$$

$$IR_n = \left[\frac{\mu_w}{\lambda_w}\left(\frac{X(1-z^{n+1})}{(1-z)} + \frac{Y(1-z_v^{n+1})}{(1-z_v)}\right) + \frac{\mu_{wv}}{\lambda_w}\frac{(1-z_v^{n+1})}{(1-z_v)}\right]VQ_0;\ 0 \leq n \leq a-1 \qquad 16$$

$$\text{Where}\quad X = \frac{(1-z)}{\mu_w\,(1-z^a)}\left(\left(\frac{ή}{(1-z_v)}\right) - \frac{Y\mu_w(1-z_v^a)}{(1-z_v)}\right) \qquad 17$$

The value for $VQ_0$ is obtained by using the normalizing condition

$$\sum_{n=0}^{\infty}(VQ_n + BP_n) + \sum_{n=0}^{a-1} IR_n = 1$$

Thus $(VQ_0^{-1}) = F(z_v, \mu_{wv}) + X\,F(z, \mu_w) + Y\,F(z_v, \mu_w)$ ;

$$\text{Where } F(r,t) = \frac{1}{1-r}\left(1 + \frac{t}{\lambda_w}\left(a - \frac{r(1-r^a)}{(1-r)}\right)\right)$$

## Performance Measures

### 1. Mean Queue Length

The expected queue length $(L_q)$ is calculated as

$$(L_q) = \sum_{n=1}^{\infty} n(VQ_n + BP_n) + \sum_{n=0}^{a-1} n\, IR_n \qquad 18$$

by substituting the values of $VQ_n$, $BP_n$ and $IR_n$, $(L_q)$ is simplified as

$$(L_q) = X\,H(z, \mu_w) + YH(z_v, \mu_w) + H(z_v, \mu_{wv})$$

$$\text{where } H(r,t) = \frac{r}{(1-r)^2} + \frac{t}{\lambda_w(1-r)}\left(\frac{a(a-1)}{2} + \frac{ar^{a+1}(1-r) - r^2(1-r^a)}{(1-r)^2}\right)$$

### 2. Other Performance Measures

If $P_v$, $P_{busy}$ and $P_{idle}$ respectively denote the probability that the server is in vacations state, in regular busy and is idle in state, then

$$P_v = \sum_{n=0}^{\infty}(VQ_n) = \frac{VQ^0}{(1-z_v)}$$

$$P_{busy} = \sum_{n=0}^{\infty}(BP_n) = \left(\frac{X}{(1-z)} + \frac{Y}{(1-z_v)}\right)VQ_0$$

$$P_{idle} = \sum_{n=0}^{a-1} nIR_n$$

## Particular Cases

### Case 1: M/M/1 Multiple Working Vacations Model

The M/M/1 multiple working vacations queuing model's steady state queue size probabilities are calculated

Eq 14 to Eq 16 at a = b = 1 and $\lambda_w(1+\delta) = \lambda_w$ imply

$$VQ_n = z_v^n VQ_0 \quad ; \quad n \geq 0$$

$$BP_n = \frac{Y}{z_v}(z_v^{n+1} - z^{n+1}) \ ; \quad n \geq 0$$

$IR_0 = \frac{VQ^0}{z_v}$ where 'z' = $\frac{\lambda_w}{\mu_w}$ , $X = -\frac{Y\rho}{z_v}$ and $Y = \frac{ήz_v}{\mu_w(1-z_v)+(z_v-\rho)}$

Which results in the queue size probabilities of M/M/1 Multiple Working Vacations queueing model

### Case 2: M/M(a,b)/1 Multiple working vacations model

In Eq 16 if $\lambda_w(1+\delta) = \lambda_w$ , with $\beta=0$ then the queue size probabilities is given as

$$VQ_n = z_v^n\, Q_0 \quad ; \quad n \geq 0$$

$$BP_n = (Xz^n + Yz_v^n)VQ_0 \quad ; \quad n \geq 0$$

$$IR_n = \left[\frac{\mu_w}{\lambda_w}\left(\frac{X(1-z^{n+1})}{(1-z)} + \frac{Y(1-z_v^{n+1})}{(1-z_v)}\right) + \frac{\mu_{wv}}{\lambda_w}\frac{(1-z_v^{n+1})}{(1-z_v)}\right] VQ_0 \quad ; \quad 0 \leq n \leq a-1$$

By substituting the values of $VQ_n$, $BP_n$ and $IR_n$ in $(L_q)$, it is observed that

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$(L_q) = X\,H(z,\mu_w) + YH(z_v, \mu_w) + H(z_v, \mu_{wv})$

As $H(s,t) = \frac{s}{(1-s)^2} + \frac{t}{\lambda_w(1-s)}\left(\frac{a(a-1)}{2} + \frac{as^{a+1}(1-s)-s^2(1-s^a)}{(1-s)^2}\right)$ ;

where $X = \frac{(1-z)}{\mu_w\,(1-z^a)}\left(\left(\frac{ή}{(1-z_v)}\right) - \frac{Y\mu_{wv}(1-z_v^a)}{(1-z_v)}\right)$

$Y = \frac{ήz_v}{\lambda_w(z_v-1)+\mu_w z_v(1-z_v^b\ )}$ if $z_v \neq z$

Which coincides with the queue size probabilities of M/M(a,b)/1 Multiple Working Vacations

**Case Study**

The above studied queuing model can be implemented in all real life scenarios, where a single server serves the customers in batches with minimum and maximum limits. Share autos, food processing units are some of the situations where this model can be implemented. Winch Train and cable cars are most widely used in many of the countries in tourist spots and most crowded visited places. In India, many such winch trains and cable cars are being operated in many of the places. Palani Temple located in Tamil Nadu is one of the most visited temples in the state and to make it easier for the devotees to climb up the hill, a winch train is being operated. The discussed scenario can be mathematically modeled into M/M(a,b)/1 queuing model with the following assumptions:

As only one winch is being operated, no server is assumed to be single and it is assumed that the minimum and maximum limit for the winch to serve is 5 and 10 persons respectively. The arrival rate of the customers $\lambda_w = 4$ during idle and vacation state. Specifically, as encouraged arrival occurs during the regular busy period with ($\delta = 0.005$)), the arrival rate in busy periods ($\lambda_w(1+\ \delta)$) is 4.02. Moreover, the service rate ($\mu_w$) during regular busy period is 0.**9** and the service during working vacation ($\mu_{wv}$) varies from 0.05 to 0.2 for which the vacation parameter (˙η) differs from 0.02 to 0.1 respectively. With the above consideration, the expected queue length ($L_q$) of M/M(a,b)/1 with encouraged arrival under multiple working vacations is compared with only M/M(a,b)/1 under multiple working vacations tabulated in Table 1:

**Table 1. (Lq) of M/M(a,b)/1 with encouraged arrival under multiple working vacations and M/M(a,b)/1 with multiple working vacations**

| **$\mu_{wv}$** | **ή** | **Lq (M/M(a,b)/1 MWV with EA** | **Lq (M/M(a,b)/1 MWV** |
|---|---|---|---|
| **0.05** | 0.02 | 211.4884 | 211.4792 |
| | 0.04 | 159.6256 | 159.5174 |
| | 0.06 | 90.03724 | 89.93257 |
| | 0.08 | 64.7256 | 64.62661 |
| | 0.1 | 48.8218 | 48.7279 |
| **0.1** | 0.02 | 127.1215 | 127.1157 |
| | 0.04 | 105.6186 | 105.3954 |
| | 0.06 | 74.5771 | 74.37234 |
| | 0.08 | 53.15424 | 52.96209 |
| | 0.1 | 46.24617 | 46.06819 |
| **0.15** | 0.02 | 100.1694 | 100.1646 |
| | 0.04 | 83.77327 | 83.42952 |
| | 0.06 | 64.5224 | 64.21326 |
| | 0.08 | 45.53549 | 45.24713 |
| | 0.1 | 41.82879 | 41.56427 |
| **0.2** | 0.02 | 75.20602 | 75.20226 |
| | 0.04 | 72.6344 | 72.16986 |
| | 0.06 | 53.17009 | 52.75088 |
| | 0.08 | 42.28379 | 41.90016 |
| | 0.1 | 39.73958 | 39.38889 |

From Table 1, it is observed that when $\mu_{wv} = 0.05$ and ή = 0.02, the queue length is maximum (211.4884) in M/M(a,b)/1 Multiple Working Vacations with encouraged arrival when compared with M/M(a,b)/1 Multiple Working Vacation.

Also, it is observed when $\mu_{wv} = 0.2$ and ή = 0.1, the queue length is minimum in M/M(a,b)/1 Multiple Working Vacation (39.73958) when compared with M/M(a,b)/1 Multiple Working Vacations under encouraged arrival (39.38889) .

The above tabulations are represented graphically in Fig 1 and Fig 2, respectively.

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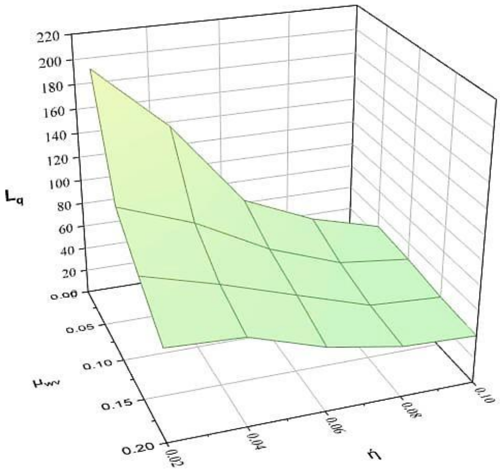


**Figure 1. Lq of M/M (a,b)/1 MWV with Encouraged Arrival**

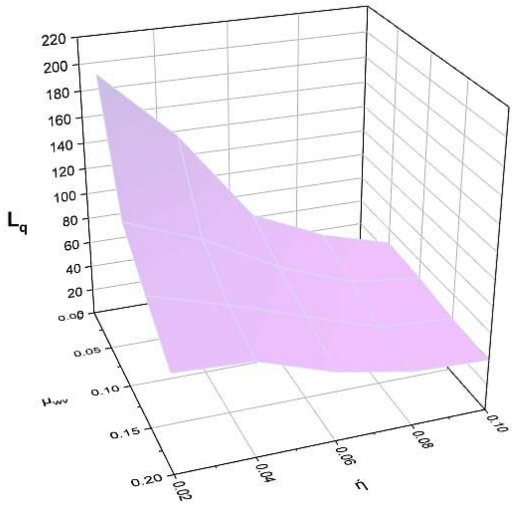


**Figure 2. Lq of M/M (a,b)/1 MWV**

The graphical representation demonstrates that with the increased vacation parameter $\dot\eta$, the mean queue length decreases gradually. Thus in each service rate during vacation, with vacation parameters ranging from 0.02 to 0.1, the mean queue length reaches its peak at the minimum values of $\dot\eta$ &$\mu_{wv}$ and the mean queue length reduces at a great margin level when $\dot\eta$ & $\mu_{wv}$ are maximum.

From the above computations, it is observed that the encouraged arrival is an impact factor in deciding the queue length. Thus on varying the encouraged arrival rate during the regular busy period as $\lambda_w(1 + \delta)$ = 4.02, 4.04, 4.2 and also by varying $\mu_{wv}$ from 0.05 to 0.2 ,other parameters are assumed to be constant as arrival rate in idle and working vacation ($\lambda_w$) is 4.0, $\mu_w$ = 0.9 and $\dot\eta$ = 0.02 , the mean queue length is computed and tabulated in Table 2:

**Table 2. Mean Queue length of Varied Encouraged arrival rate of M/M (a,b)/1 MWV Queuing model**

| $\mu_{wv}$ | Lq $\lambda w(1+\delta)$ = 4.02 ; $\delta$ = 0.001 | Lq $\lambda w(1+\delta)$ = 4.04; $\delta$ = 0.01 | Lq $\lambda w(1+\delta)$ = 4.2 ; $\delta$ =0.05 |
|---|---|---|---|
| **0.05** | 211.4884 | 211.4975 | 211.5721 |
| **0.1** | 127.1215 | 127.1273 | 127.1751 |
| **0.15** | 100.1693 | 100.1741 | 100.2131 |
| **0.2** | 75.2060 | 75.2098 | 75.2406 |

The tabulated values are graphically represented in Fig 3.

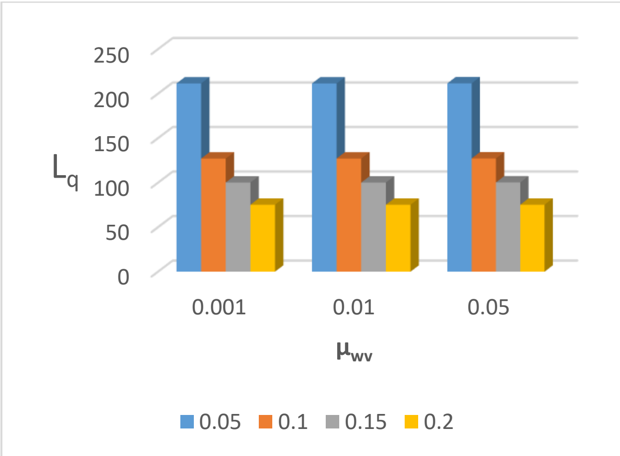


**Figure 3. Mean Queue length of Varied Encouraged arrival rate of M/M (a,b)/1 MWV Queuing model**

From Fig 3, it is clear that as the working vacation service rate ($\mu_{wv}$) varies from 0.05 to 0.2 the queue length decreases gradually irrespective of the encouraged arrival rate.

For the varied encouraged arrival rate $\lambda_w(1+\delta)$ = 4.02, 4.04 and 4.2, the queue length of the system increases as 211.4884, 211.4975 and 211.5721 respectively as $\mu_{wv}$ = 0.05. The graph illustrates that when $\mu_{wv}$ = 0.2, the queue length reaches its minimum as 75.2060, 75.2098 and 75.2406 for corresponding encouraged arrival rates of $\lambda_w(1+\delta)$ = 4.02, 4.04 and 4.2 respectively.

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Hence queue length of the system increases with a higher encouraged arrival rate and lesser working vacation service rate. Thus to reduce the queue length, the service rate of the winch train during working vacation can be increased.

## Results and discussion

As the discussed model follows multiple working vacations, the server works during different vacations with varied vacation parameters and working vacations' service rates. The M/M(a,b)/1/MWV queuing model with encouraged arrival is discussed and it is observed that while letting the batch size as $a = b = 1$ and with the absence of encouraged arrival, i.e., $\lambda_w(1+\delta) = \lambda_w$, the considered queuing model resembles in the queue size probabilities of M/M/1 Multiple Working Vacations queueing model[7]. Also, when there is no encouraged arrival rate, the queuing model coincides with M/M(a,b)/1 Multiple Working Vacations queueing model. Additionally, the considered model is analyzed and studied with a case study of calculating the mean queue length of a winch train during encouraged arrival.

## Conclusion

An M/M(a,b)/1/MWV queuing model with encouraged arrival of customers is studied with steady state. The mean queue length and various other performance measures were calculated. Specifically, the considered model is discussed in the case of a single server Winch train which undergoes batch service during encouraged arrival. The case study has been examined with differing encouraged arrival rates and working vacation service rates. It can be suggested that increased service rates may reduce the queue length in M/M(a,b)/1/MWV queuing model with encouraged arrival. Generally, the considered queuing model with the obtained mean queue length and other characteristic measures helps in determining the efficiency of the queue. In future work, it is planned to examine the M/M(a,b)/1/MWV of encouraged arrival with customer impatient behaviors like balking and the heterogeneous condition of encouraged arrival in the M/M(a,b)/1/MWV queuing model.

## Author's Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript. Ethical Clearance: The project was approved by the local ethical committee at Bharathiar University, Coimbatore, India.

## Authors' contribution

This work was carried out in collaboration between both the authors. P.P. diagnosed and derived Expected Queue Length of the considered model. K.J.R.M. deduced the particular cases. The manuscript was written by P.P. and K.J.R.M. edited the manuscript with revisions idea. All the authors read and approved the final manuscript.

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# تحليل نظام انتظار إجازات العمل المتعددة مع تشجيع الوصول باستخدام نموذج M/M(a,b)/1

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**الخلاصة**

تتناول هذه الدراسة مفهوم الوصول المشجع في نموذج قائمة الانتظار M/M(a,b)/1/MWV وتتبع القاعدة العامة للخدمة السائبة. يستمر الخادم في أخذ إجازة عندما يكون النظام خاملاً، أي عندما يكون فارغًا. يتعامل هذا النموذج مع إجازات العمل المتعددة التي يتم توزيعها بشكل كبير ويتم تقديم مفهوم الوصول المشجع في الفترة المزدحمة العادية. يعد مصطلح الوصول المشجع إضافة حديثة إلى المفاهيم الحالية لنظرية الانتظار لسلوك المستهلك ويتبع توزيع بواسون. تهدف هذه الدراسة إلى حساب متوسط طول قائمة الانتظار (Lq) ومقاييس الأداء الأخرى المختلفة لنموذج قائمة الانتظار الذي تمت مناقشته.

**الكلمات المفتاحية:** الوصول المشجع، طول قائمة طابور الانتظار المتوقع، قاعدة الخدمة العامة المجمعة، إجازات العمل المتعدد، نموذج طابور الانتظار.