2024, 21(10): 3205-3213
https://doi.org/10.21123/bsj.2024.9288
P-ISSN: 2078-8665 - E-ISSN: 2411-7986

Baghdad Science Journal

Fixed Point Results for Almost Contraction Mappings in Fuzzy Metric Space

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Received 08/09/2023, Revised 11/12/2023, Accepted 13/12/2023, Published Online First 20/03/2024, Published 01/10/2024

Abstract

In certain mathematical, computing, economic, and modeling issues, the presence of a solution to a theoretical or real-world problem is synonymous with the presence of a fixed point (Fp) for an appropriate mapping. Consequently, Fp plays an essential role in a wide variety of mathematical and scientific contexts. In its own right, the theory is a stunning amalgamation of analysis (both pure and applied), geometry, and topology. Recent years have shown the theory of Fps is a highly strong and useful tool in the study of nonlinear events. Fp theorems are concerned with mappings f of a set X into itself that, under particular conditions, permit a Fp, that is, a point $x \in X$ such that $f(x) = x$. This work introduces and proves the Fp theorem for various kinds of contraction mappings in a fuzzy metric space ($\mathcal{FM}$-space) namely almost $\widehat{\boldsymbol{Z}}$-contraction mapping and $(\widetilde{\Psi}, \widetilde{\Phi})$- almost weakly contraction mapping. At first, the concept of $\mathcal{FM}$-space and the terms used in the fuzzy setting are recalled. Then the concept of simulation function is given. The concept of simulation function is used to present the notion of almost $\widehat{\boldsymbol{Z}}$-contraction mapping. In addition, this notion is used to prove the existence and uniqueness of the Fp for this kind of mapping. After that the notion of $(\widetilde{\Psi}, \widetilde{\Phi})$-almost weakly contraction mapping is introduced in the framework of $\mathcal{FM}$-space, as well as the Fp theorem for this kind of mapping. At the end of the paper, some examples are given to support the results.

Keywords: Almost contraction mappings, Almost $\widehat{\boldsymbol{Z}}$-contraction mapping, $(\widetilde{\Psi}, \widetilde{\Phi})$almost weakly contraction mapping, Fixed point, Fuzzy metric space.

Introduction

Functional analysis is a discipline of mathematics that evolved from classical analysis. Presently, functional analytic methods and outcomes are crucial in numerous mathematical disciplines and their applications [1-4].

Fp theory has its origins in Banach's well-known study, which was published a century ago. Since the discovery of this fundamental yet very effective nonlinear analytical conclusion, the subject of Fp theory has advanced in several different ways. Ciri [5] established the concept of quasi-contraction and suggested a modification of the Banach contraction principle in 1974.

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https://doi.org/10.21123/bsj.2024.9288
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

On the other hand, in 1965 Zadeh's groundbreaking work in 1965 established and analyzed the notion of a fuzzy set. In 1975, Kramosil and Michalek initially presented the definition of $F\mathcal{M}$-space. A large number of papers on $F\mathcal{M}$-space have been published; see [6-9]. Beginning with the specification of $F\mathcal{M}$-space, Sihag et al [10]. employed the new idea of α-series contraction to develop Fp theorems for a sequence of mappings. Chauhan et al [11]. established unified Fp theorems in $F\mathcal{M}$-spaces. Further significant results on the Fp within the $F\mathcal{M}$-spaces may be seen in [12-14].

This paper aims to investigate the existence of the Fps in a $F\mathcal{M}$-space, providing an approach to expanding and fuzzifying results in metric spaces. To that goal, the definitions of almost $\widehat{\boldsymbol{Z}}$-contraction mapping and $(\widetilde{\Psi}\,;\widetilde{\Phi})$-almost weakly contraction mapping is presented and the existence of Fps is established with regard to these contraction forms.

Preliminaries

The terminologies and outcomes used throughout the paper are provided in this section. The terminology employed in the fuzzy context will be reviewed first.

Definition 1: [15] A binary operation $\odot: [0,1] \times [0,1] \to [0,1]$ is called t-norm if it meets the requirements below for any $u_1, u_2, u_3, u_4 \in [0,1]$:

(1) $1 \odot u_2 = u_2$,

(2) $u_2 \odot u_4 = u_4 \odot u_2$,

(3) $u_2 \odot (u_3 \odot u_4) = (u_2 \odot u_3) \odot u_4$

(4) If $u_2 \le u_4$ and $u_3 \le u_1$ then $u_2 \odot u_3 \le u_4 \odot u_1$,

Definition 2: [16] If Ω is an arbitrary set, $\odot$ is a continuous t-norm, and $\mathfrak{F}_{\mathcal{M}}$ is a fuzzy set on $\Omega^2 \times (0,\infty)$ meet the criteria listed below for every $\mathscr{b}, q, \vartheta \in \Omega$ and $\omega, \tau > 0$:

(1) $\mathfrak{F}_{\mathcal{M}}(\mathscr{b}, q, \omega) > 0$ and $\mathfrak{F}_{\mathcal{M}}(\mathscr{b}, q, \omega) = 1$ if and only if $\mathscr{b} = q$.

(2) $\mathfrak{F}_{\mathcal{M}}(\mathscr{b}, q, \omega) = \mathfrak{F}_{\mathcal{M}}(q, \mathscr{b}, \omega)$;

(3) $\mathfrak{F}_{\mathcal{M}}(\mathscr{b}, \vartheta, \omega) \odot \mathfrak{F}_{\mathcal{M}}(\vartheta, q, \tau) \le \mathfrak{F}_{\mathcal{M}}(\mathscr{b}, q, \omega + \tau)$;

(4) $\mathfrak{F}_{\mathcal{M}}(\mathscr{b}, q, \omega): (0,\infty) \to [0,1]$ is continuous;

Then a 3-tuple $(\Omega, \mathfrak{F}_{\mathcal{M}}, \odot)$ is termed as $F\mathcal{M}$-space.

Definition 3: [16] Let $(\Omega, \mathfrak{F}_{\mathcal{M}}, \odot)$ be $F\mathcal{M}$-space and $\{\mathscr{b}_j\}$ be a sequence in Ω. Then

(i) $\{\mathscr{b}_j\}$ in Ω is termed as convergent to a point $\mathscr{b} \in \Omega$ if $lim\, \mathfrak{F}_{\mathcal{M}}(\mathscr{b}_j, \mathrm{v}, \omega) = 1$ as $j \to \infty$.

(ii) $\{\mathscr{b}_j\}$ is termed as Cauchy, if for each $0 < \xi < 1$ and $\omega > 0$, there is $j_\circ \in \mathbb{N}$ such that $\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_m, \mathscr{b}_j, \omega) > 1 - \xi, \forall m, j \ge j_\circ$

Definition 4: [17] Let $\hat{\boldsymbol{\zeta}} : [0,\infty) \times [0,\infty) \to R$ be function. Then $\hat{\boldsymbol{\zeta}}$ is termed a simulation function if the conditions listed below are fulfilled:

(a) $\hat{\boldsymbol{\zeta}}(0,0) = 0$.

(b) $\hat{\boldsymbol{\zeta}}(\omega, \tau) < \tau - \omega$ for each $\omega, \tau > 0$.

(c) If $\{\omega_j\}, \{\tau_j\}$ are sequences in $(0,\infty)$ such that $\lim\limits_{j\to\infty} \omega_j = \lim\limits_{j\to\infty} \tau_j > 0$, then

$$\lim_{j\to\infty} sup\, \hat{\boldsymbol{\zeta}}(\omega_j, \tau_j) < 0$$
1

(d) If $\{\omega_j\}, \{\tau_j\}$ are sequences in $(0,\infty)$ such that $\lim\limits_{j\to\infty} \omega_j = \lim\limits_{j\to\infty} \tau_j > 0$ and $\omega_j < \tau_j$, then Eq 1 is fulfilled.

Results and Discussion

Main Results

The notion of almost $\widehat{\boldsymbol{Z}}$-contraction mapping is presented then the Fp theorem for this mapping is established.

Consider $\widehat{\boldsymbol{Z}}$ to represent the set of all simulation functions $\hat{\boldsymbol{\zeta}}: [0,\infty) \times [0,\infty) \to R$.

Definition 5: Let $(\Omega, \mathfrak{F}_{\mathcal{M}}, \odot)$ be $F\mathcal{M}$-space and $\hat{\boldsymbol{\zeta}} \in \widehat{\boldsymbol{Z}}$. A mapping $\Gamma: \Omega \to \Omega$ is called an almost

2024, 21(10): 3205-3213
https://doi.org/10.21123/bsj.2024.9288
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
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$\widehat{\boldsymbol{Z}}$-contraction (briefly" al- $\widehat{\boldsymbol{Z}}$C map") if there exists constant β with

$$\hat{\zeta}\,[\mathfrak{F}_{\mathcal{M}}(\Gamma\ell,\Gamma\vartheta,\omega),\mathfrak{F}_{\mathcal{M}}(\ell,\vartheta,\omega)+\beta(1-\mathcal{H}(\ell,\vartheta))]\geq 0 \qquad 2$$

for all $\ell, q, \vartheta \in \Omega$ and $\omega > 0$,

where

$$\mathcal{H}(\ell,\vartheta)=\max\{\mathfrak{F}_{\mathcal{M}}(\ell,\Gamma\ell,\omega),\mathfrak{F}_{\mathcal{M}}(\vartheta,\Gamma\vartheta,\omega),\mathfrak{F}_{\mathcal{M}}(\ell,\Gamma\vartheta,\omega),\mathfrak{F}_{\mathcal{M}}(\vartheta,\Gamma\ell,\omega)\}$$

A Fp of an al- $\widehat{\boldsymbol{Z}}$C map is shown to be unique by the subsequent lemma.

Lemma 1: Let $(\Omega,\mathfrak{F}_{\mathcal{M}},\odot)$ be a $F\mathcal{M}$-space. If al- $\widehat{\boldsymbol{Z}}$C map possesses a Fp in $(\Omega,\mathfrak{F}_{\mathcal{M}},\odot)$ then this point is unique.

Proof: Consider $\Gamma:\Omega\to\Omega$ be an al- $\widehat{\boldsymbol{Z}}$C map with respect to $\hat{\zeta}\in\widehat{\boldsymbol{Z}}$. Assume $d, v\in\Omega$ be two different Fps of Γ. As a result of Eq 2 and (b)

$$0\leq\hat{\zeta}[\mathfrak{F}_{\mathcal{M}}(\Gamma d,\Gamma v,\omega),\mathfrak{F}_{\mathcal{M}}(d,v,\omega)+\beta(1-\mathcal{H}(d,v))]$$

$$=\hat{\zeta}[\mathfrak{F}_{\mathcal{M}}(\Gamma d,\Gamma v,\omega),\mathfrak{F}_{\mathcal{M}}(d,v,\omega)+\beta(1-\max\{\mathfrak{F}_{\mathcal{M}}(d,\Gamma d,\omega),$$

$$\mathfrak{F}_{\mathcal{M}}(v,\Gamma v,\omega),\mathfrak{F}_{\mathcal{M}}(d,\Gamma v,\omega),\mathfrak{F}_{\mathcal{M}}(v,\Gamma d,\omega)\}]$$

$$=\hat{\zeta}[\mathfrak{F}_{\mathcal{M}}(d,v,\omega),\mathfrak{F}_{\mathcal{M}}(d,v,\omega)]$$

$$<\mathfrak{F}_{\mathcal{M}}(d,v,\omega)-\mathfrak{F}_{\mathcal{M}}(d,v,\omega)=0$$

but this is a contradiction. As a result, the Fp of Γ in Ω is unique.

Theorem 1: Suppose that $(\Omega,\mathfrak{F}_{\mathcal{M}},\odot)$ is complete $F\mathcal{M}$-space , $\Gamma:\Omega\to\Omega$ be al- $\widehat{\boldsymbol{Z}}$C map with respect to $\hat{\zeta}\in\widehat{\boldsymbol{Z}}$. Then, Γ possesses Fp.

Proof: Consider $\ell_\circ\in\Omega$ and the sequence $\ell_n=\Gamma^n\ell_\circ=\Gamma\ell_{n-1}$. If $\ell_{n_\circ}=\ell_{n_\circ+1}$ for some $n_\circ$ then $\ell_{n_\circ}$ is Fp for Γ. If $\ell_{n_\circ}\neq\ell_{n_\circ+1}$, the proof is bifurcated into two steps.

Step1: to demonstrate that

$$\lim_{n\to\infty}\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega)=1 \qquad 3$$

Since

$$\mathcal{H}(\ell_{n-1},\ell_n)=\max\{\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\Gamma\ell_{n-1},\omega),\mathfrak{F}_{\mathcal{M}}(\ell_n,\Gamma\ell_n,\omega),\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\Gamma\ell_n,\omega),\mathfrak{F}_{\mathcal{M}}(\ell_n,\Gamma\ell_{n-1},\omega)\}$$

$$\mathcal{H}(\ell_{n-1},\ell_n)=\max\{\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_n,\omega),\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega),$$

$$\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_{n+1},\omega)\,,\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_n,\omega)\}$$

$$=\max\{\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_n,\omega),\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega),$$

$$\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_{n+1},\omega)\,,1\}=1$$

utilizing Eq 2, for each $n\in N$, one gets

$$0\leq\hat{\zeta}[\mathfrak{F}_{\mathcal{M}}(\Gamma\ell_{n-1},\Gamma\ell_n,\omega),\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_n,\omega)+\beta\big(1-\mathcal{H}(\ell_{n-1},\ell_n)\big)]$$

$$=\hat{\zeta}[\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega),\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_n,\omega)]$$

$$<\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_n,\omega)-\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega) \qquad 4$$

It follows that,

$0<\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega)<\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_n,\omega)$ for each $n\in N$. Thus, $\{\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega)\}$ is decreasing, so $r\leq 1$ such that $\lim_{n\to\infty}\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega)=r$. To prove that $r=1$ for each $\omega>0$. Assume that $r<1$. Take $\{\omega_n\},\{\tau_n\}$ as $\omega_n=\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega)$ and $\tau_n=\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_n,\omega)$. Since $\lim_{n\to\infty}\omega_n=\lim_{n\to\infty}\tau_n=r$ and $\omega_n<\tau_n$ for each n, by (d) and Eq 4, deduce

$0\leq\limsup_{n\to\infty}\hat{\zeta}[\mathfrak{F}_{\mathcal{M}}(\ell_n,\ell_{n+1},\omega),\mathfrak{F}_{\mathcal{M}}(\ell_{n-1},\ell_n,\omega)]<0$, but this is a contradiction hence $r=1$.

Step2: To demonstrate that $\{\ell_n\}$ is a Cauchy. Assume that $\{\ell_n\}$ is a Cauchy and considers $\{\mathcal{S}_n\}\subset(0,\infty)$ specified by

$$\mathcal{S}_n=\sup\{\mathfrak{F}_{\mathcal{M}}\big(\ell_i,\ell_j,\omega\big):i,j\geq n\}$$

Then $\{\mathcal{S}_n\}$ is a positive decreasing; consequently, some $\mathcal{S}\geq 1$ exists with $\lim_{n\to\infty}\mathcal{S}_n=\mathcal{S}$. If $\mathcal{S}>1$ Then by definition of $\mathcal{S}_n$ for each $k\in N$, there is n_k and m_k such that $m_k>n_k\geq k$ and

$$\mathcal{S}_k-\frac{1}{k}<\mathfrak{F}_{\mathcal{M}}\big(\ell_{m_k},\ell_{n_k},\omega\big)\leq\mathcal{S}_k$$

Thus

2024, 21(10): 3205-3213
https://doi.org/10.21123/bsj.2024.9288
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
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$$\lim_{k\to\infty}\mathfrak{F}_{\mathcal{M}}(\ell_{m_k},\ell_{n_k},\omega)=\mathcal{S}$$
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Now Eq 2 yields the following results:

$$\mathfrak{F}_{\mathcal{M}}(\ell_{m_k},\ell_{n_k},\omega)\le\mathfrak{F}_{\mathcal{M}}(\ell_{m_{k-1}},\ell_{n_{k-1}},\omega)$$

However

$$\mathfrak{F}_{\mathcal{M}}(\ell_{m_{k-1}},\ell_{n_{k-1}},\omega)\ge\mathfrak{F}_{\mathcal{M}}(\ell_{m_{k-1}},\ell_{n_k},\omega)\odot\mathfrak{F}_{\mathcal{M}}(\ell_{n_k},\ell_{n_{k-1}},\omega)$$

$$\ge\mathfrak{F}_{\mathcal{M}}(\ell_{m_{k-1}},\ell_{n_k},\omega)\odot\mathfrak{F}_{\mathcal{M}}(\ell_{n_k},\ell_{m_k},\omega)\odot\mathfrak{F}_{\mathcal{M}}(\ell_{m_k},\ell_{n_{k-1}},\omega)$$

Taking $k\to\infty$ in the above inequality and using Eq 3 and Eq 5, obtain

$$\lim_{k\to\infty}\mathfrak{F}_{\mathcal{M}}(\ell_{m_{k-1}},\ell_{n_{k-1}},\omega)=\mathcal{S}$$
6

Since Γ is an almost $\widehat{\boldsymbol{Z}}$-contraction then:

$$\mathfrak{F}_{\mathcal{M}}(\ell_{n_k},\ell_{m_k},\omega)<\mathfrak{F}_{\mathcal{M}}(\ell_{m_{k-1}},\ell_{n_{k-1}},\omega)+\beta\left(1-\mathcal{H}(\ell_{m_{k-1}},\ell_{n_{k-1}})\right)\quad 7$$

with the assistance of Eq 3, obtain:

$$\lim_{k\to\infty}\mathcal{H}(\ell_{m_{k-1}},\ell_{n_{k-1}})=1\quad 8$$

Taking the sequences $\{\omega_k=\mathfrak{F}_{\mathcal{M}}(\ell_{n_k},\ell_{m_k},\omega)\}$, $\{\tau_k=\mathfrak{F}_{\mathcal{M}}(\ell_{m_{k-1}},\ell_{n_{k-1}},\omega)+\beta(1-\mathcal{H}(\ell_{m_{k-1}},\ell_{n_{k-1}}))\}$

and considering Eqs.5–8, $\lim_{k\to\infty}\omega_k=\lim_{k\to\infty}\tau_k=\mathcal{S}$ and $\omega_k<\tau_k$ for each k. Then, by Eq 2 and (d), obtain

$$0\le\limsup_{n\to\infty}\hat{\boldsymbol{\zeta}}\begin{bmatrix}\mathfrak{F}_{\mathcal{M}}(\ell_{n_k},\ell_{m_k},\omega),\\ \mathfrak{F}_{\mathcal{M}}(\ell_{m_{k-1}},\ell_{n_{k-1}},\omega)+\\ \beta\left(1-\mathcal{H}(\ell_{m_{k-1}},\ell_{n_{k-1}})\right)\end{bmatrix}<0$$

which is a contradiction and so $\mathcal{S}=1$. As a result $\{\ell_n\}$ is a Cauchy and because $(\Omega,\mathfrak{F}_{\mathcal{M}},\odot)$ is a complete there is $w\in\Omega$ with $\lim_{n\to\infty}\ell_n=w$.

As the last step, it should be shown that the point w is a Fp of Γ. Assume that Γw ≠ w.

Eq 2, (b), and (d) provide the following results,

$$0\le\limsup_{n\to\infty}\hat{\boldsymbol{\zeta}}[\mathfrak{F}_{\mathcal{M}}(\Gamma\ell_n,\Gamma\mathrm{w},\omega),\mathfrak{F}_{\mathcal{M}}(\ell_n,w,\omega)+\beta(1-\mathcal{H}(\ell_n,w))]$$

$$\le\limsup_{n\to\infty}\hat{\boldsymbol{\zeta}}[\mathfrak{F}_{\mathcal{M}}(\ell_n,w,\omega)+\beta(1-\mathcal{H}(\ell_n,w))-\mathfrak{F}_{\mathcal{M}}(\ell_{n+1},\Gamma\mathrm{w},\omega)]$$

$$=1-\mathfrak{F}_{\mathcal{M}}(\ell_{n+1},\Gamma w,\omega)$$

which indicate that $\mathfrak{F}_{\mathcal{M}}(\ell_{n+1},\Gamma\mathrm{w},\omega)=1$. This means w is a Fp of Γ.

Following that, the notion of $(\tilde{\Psi},\tilde{\Phi})$-almost weakly contraction mapping is presented, as well as the Fp theorem for this kind of mapping.

Consider $\tilde{\Phi},\tilde{\Psi}:[0,1]\to[0,\infty)$ be mappings such that $\tilde{\Psi}$ is continuous, monotonically decreasing and $\tilde{\Psi}(\delta)=0$ if and only if $\delta=1$ for each $\delta\in[0,1]$ and $\tilde{\Phi}$ is continuous, $\tilde{\Phi}(\beta)=0$ if and only if $\beta=1$ for each $\beta\in[0,1]$

Definition 6: Let $(\Omega,\mathfrak{F}_{\mathcal{M}},\odot)$ be a $F\mathcal{M}$-space and $\Gamma:\Omega\to\Omega$ satisfying the inequality:

$$\tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(\Gamma\ell,\Gamma q,\omega))\le\tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(\ell,q,\omega))-\tilde{\Phi}(\mathfrak{F}_{\mathcal{M}}(\ell,q,\omega))+\mathfrak{D}(1-\mathcal{A}(\ell,q))\quad 9$$

where $\mathcal{A}(\ell,q)=\max\{\mathfrak{F}_{\mathcal{M}}(\Gamma\ell,\ell,\omega),\mathfrak{F}_{\mathcal{M}}(\Gamma\ell,q,\omega),\sqrt{\mathfrak{F}_{\mathcal{M}}(\Gamma q,\ell,\omega)\mathfrak{F}_{\mathcal{M}}(\Gamma q,q,\omega)}\}$

for all $\ell,q\in\Omega$, $\mathfrak{D}\ge0$, and $\omega>0$, then Γ is termed as $(\tilde{\Psi},\tilde{\Phi})$-almost weakly contraction mapping on Ω.

Theorem 2: Let $(\Omega,\mathfrak{F}_{\mathcal{M}},\odot)$ be complete $F\mathcal{M}$-space and $\Gamma:\Omega\to\Omega$ be $(\tilde{\Psi},\tilde{\Phi})$-almost weakly contraction mapping. Then, Γ possesses a Fp in Ω which is unique.

2024, 21(10): 3205-3213
https://doi.org/10.21123/bsj.2024.9288
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

Proof: Consider $\{\mathscr{b}_n\}$ in Ω such that $\Gamma\mathscr{b}_n = \mathscr{b}_{n+1}$. If $\mathscr{b}_n = \mathscr{b}_{n+1}$ then the theorem is obvious. Suppose that $\mathscr{b}_n \neq \mathscr{b}_{n+1}$.

Now,

$$\widetilde{\Psi}\big(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_{n+1}, \omega)\big) = \widetilde{\Psi}\big(\mathfrak{F}_{\mathcal{M}}(\Gamma\mathscr{b}_{n-1}, \Gamma\mathscr{b}_n, \omega)\big)$$

$$\leq \widetilde{\Psi}\big(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n-1}, \mathscr{b}_n, \omega)\big) - \widetilde{\Phi}\big(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n-1}, \mathscr{b}_n, \omega)\big) + \mathfrak{D}(1 - \mathcal{A}(\mathscr{b}_{n-1}, \mathscr{b}_n)) \quad 10$$

where $\mathcal{A}(\mathscr{b}_{n-1}, \mathscr{b}_n) =$

$$\max\{\mathfrak{F}_{\mathcal{M}}(\Gamma\mathscr{b}_{n-1}, \mathscr{b}_{n-1}, \omega), \mathfrak{F}_{\mathcal{M}}(\Gamma\mathscr{b}_{n-1}, \mathscr{b}_n, \omega), \sqrt{\mathfrak{F}_{\mathcal{M}}(\Gamma\mathscr{b}_n, \mathscr{b}_{n-1}, \omega)\mathfrak{F}_{\mathcal{M}}(\Gamma\mathscr{b}_n, \mathscr{b}_n, \omega)}\}$$

$$= \max\{\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_{n-1}, \omega), \mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_n, \omega), \sqrt{\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n+1}, \mathscr{b}_{n-1}, \omega)\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n+1}, \mathscr{b}_n, \omega)}\}$$

$$= \max\{\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_{n-1}, \omega), 1, \sqrt{\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n+1}, \mathscr{b}_{n-1}, \omega)\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n+1}, \mathscr{b}_n, \omega)}\}$$

$$= 1$$

Thus $\mathcal{A}(\mathscr{b}_{n-1}, \mathscr{b}_n) = 1$. Hence

$$\widetilde{\Psi}\big(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_{n+1}, \omega)\big) \leq \widetilde{\Psi}\big(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n-1}, \mathscr{b}_n, \omega)\big) - \widetilde{\Phi}\big(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n-1}, \mathscr{b}_n, \omega)\big) \quad 11$$

By using the property of $\widetilde{\Phi}$, one gets

$$\widetilde{\Psi}\big(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_{n+1}, \omega)\big) < \widetilde{\Psi}\big(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n-1}, \mathscr{b}_n, \omega)\big) \quad 12$$

Now, using $\widetilde{\Psi}$'s monotonically decreasing property, one gets

$$\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_{n+1}, \omega) > \mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n-1}, \mathscr{b}_n, \omega) \quad 13$$

Hence $\{\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_{n+1}, \omega)\}$is increasing. Then there is $r \in (0,1)$ such that $\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_{n+1}, \omega) \to r$ as $n \to \infty$. Now letting $n \to \infty$ in the inequality (11), thus obtain the following:

$$\widetilde{\Psi}(r) \leq \widetilde{\Psi}(r) - \widetilde{\Phi}(r)$$

$$\Rightarrow \widetilde{\Phi}(r) \leq 0$$

and $\widetilde{\Phi}(r) \geq 0$ therefore one get $\widetilde{\Phi}(r) = 0$. Then by property of $\widetilde{\Phi}$, conclude that $r = 1$, that is mean,

$$\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_n, \mathscr{b}_{n+1}, \omega) \to 1 \quad \text{as} \quad n \to \infty \quad 14$$

Next, to prove that $\{\mathscr{b}_n\}$ is a Cauchy. Conversely, if $\{\mathscr{b}_n\}$ is not Cauchy then for any $\epsilon > 0$ it is possible to find a subsequence $\{\mathscr{b}_{n_k}\}, \{\mathscr{b}_{m_k}\}$ of $\{\mathscr{b}_n\}$ with $n_k > m_k \geq k$ such that

$$\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k}, \mathscr{b}_{m_k}, \omega) \leq 1 - \epsilon \quad 15$$

Now select n_k such that it is the smallest integer with $n_k > m_k$ and satisfying Eq 15. Then,

$$\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k-1}, \mathscr{b}_{m_k}, \omega) > 1 - \epsilon \quad \text{and} \quad \mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k-1}, \mathscr{b}_{m_k-1}, \omega) > 1 - \epsilon \quad 16$$

Now

$$\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k}, \mathscr{b}_{m_k}, \omega) \geq \mathfrak{F}_{\mathcal{M}}\left(\mathscr{b}_{n_k}, \mathscr{b}_{n_k-1}, \frac{\omega}{2}\right) \odot \mathfrak{F}_{\mathcal{M}}\left(\mathscr{b}_{n_k-1}, \mathscr{b}_{m_k}, \frac{\omega}{2}\right)$$

$$> \mathfrak{F}_{\mathcal{M}}\left(\mathscr{b}_{n_k}, \mathscr{b}_{n_k-1}, \frac{\omega}{2}\right) \odot 1 - \epsilon$$

and from Eq 14 obtain $\mathfrak{F}_{\mathcal{M}}\left(\mathscr{b}_{n_k}, \mathscr{b}_{n_k-1}, \frac{\omega}{2}\right) = 1 \ as\ k \to \infty$, then

$$\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k}, \mathscr{b}_{m_k}, \omega) > 1 \odot 1 - \epsilon$$

Thus $\lim_{k\to\infty} \mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k}, \mathscr{b}_{m_k}, \omega) > 1 - \epsilon$ 17

From Eqs 15 and 17 conclude that:

$$\lim_{k\to\infty} \mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k}, \mathscr{b}_{m_k}, \omega) = 1 - \epsilon.$$

Now consider,

$$\widetilde{\Psi}\left(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k}, \mathscr{b}_{m_k}, \omega)\right) = \widetilde{\Psi}\left(\mathfrak{F}_{\mathcal{M}}(\Gamma\mathscr{b}_{n_k-1}, \Gamma\mathscr{b}_{m_k-1}, \omega)\right)$$

$$\leq \widetilde{\Psi}\left(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k-1}, \mathscr{b}_{m_k-1}, \omega)\right) - \widetilde{\Phi}\left(\mathfrak{F}_{\mathcal{M}}(\mathscr{b}_{n_k-1}, \mathscr{b}_{m_k-1}, \omega)\right) + \mathfrak{D}\left(1 - \mathcal{A}(\mathscr{b}_{n_k-1}, \mathscr{b}_{m_k-1})\right) \quad 18$$

where

$$\mathcal{A}(\mathscr{b}_{n_k-1}, \mathscr{b}_{m_k-1}) = \max\{\mathfrak{F}_{\mathcal{M}}(\Gamma\mathscr{b}_{n_k-1}, \mathscr{b}_{n_k-1}, \omega),$$

2024, 21(10): 3205-3213
https://doi.org/10.21123/bsj.2024.9288
P-ISSN: 2078-8665 - E-ISSN: 2411-7986

$$\mathfrak{F}_{\mathcal{M}}(\Gamma b_{n_k-1}, b_{m_k-1}, \omega),$$

$$\sqrt{\mathfrak{F}_{\mathcal{M}}(\Gamma b_{m_k-1}, b_{n_k-1}, \omega)\mathfrak{F}_{\mathcal{M}}(\Gamma b_{m_k-1}, b_{m_k-1}, \omega)}\}$$

$= \max\{\mathfrak{F}_{\mathcal{M}}(b_{n_k}, b_{n_k-1}, \omega),$

$$\mathfrak{F}_{\mathcal{M}}(b_{n_k}, b_{m_k-1}, \omega),$$

$$\sqrt{\mathfrak{F}_{\mathcal{M}}(b_{m_k}, b_{n_k-1}, \omega)\mathfrak{F}_{\mathcal{M}}(b_{m_k}, b_{m_k-1}, \omega)}\}$$

Taking $k \to \infty$ in the above inequality,

$$\mathcal{A}(b_{n_k-1}, b_{m_k-1}) = \max\left\{\begin{matrix} 1, \mathfrak{F}_{\mathcal{M}}(b_{n_k}, b_{m_k-1}, \omega), \\ \sqrt{\mathfrak{F}_{\mathcal{M}}(b_{m_k}, b_{n_k-1}, \omega).1} \end{matrix}\right\}$$

$$= \max\left\{\begin{matrix} 1, \mathfrak{F}_{\mathcal{M}}(b_{n_k}, b_{m_k-1}, \omega), \\ \sqrt{\mathfrak{F}_{\mathcal{M}}(b_{m_k}, b_{n_k-1}, \omega)} \end{matrix}\right\} = 1$$

Therefore $\mathcal{A}(b_{n_k-1}, b_{m_k-1}) = 1$ as $k \to \infty$.

So,

$$\tilde{\Psi}\left(\mathfrak{F}_{\mathcal{M}}(b_{n_k}, b_{m_k}, \omega)\right) \le \tilde{\Psi}(1-\epsilon) - \tilde{\Phi}(1-\epsilon) + \mathfrak{D}(1 - \mathcal{A}(b_{n_k-1}, b_{m_k-1}))$$

$\tilde{\Psi}(1-\epsilon) \le \tilde{\Psi}(1-\epsilon) - \tilde{\Phi}(1-\epsilon) + \mathfrak{D}(1-1)$ as $k \to \infty$.

$$\Longrightarrow \tilde{\Psi}(1-\epsilon) \le \tilde{\Psi}(1-\epsilon) - \tilde{\Phi}(1-\epsilon)$$

$$\Longrightarrow \tilde{\Phi}(1-\epsilon) \le 0$$

as well as by the definition of $\tilde{\Phi}$ acquire $\tilde{\Phi}(1-\epsilon) \ge 0$, then $\tilde{\Phi}(1-\epsilon) = 0$. Agaian by property of a function $\tilde{\Phi}$ conclude that $1 - \epsilon = 1 \Longrightarrow \epsilon = 0$, but this a contradiction, then $\{b_n\}$ is a Cauchy. Because Ω is complete, thus there is $d \in \Omega$ such that $b_n \to d$ as $n \to \infty$.

Next to prove that d is Fp of Γ in Ω.

Now consider

$$\tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(b_n, \Gamma d, \omega)) = \tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(\Gamma b_{n-1}, \Gamma d, \omega)) \le \tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(b_{n-1}, d, \omega)) - \tilde{\Phi}(\mathfrak{F}_{\mathcal{M}}(b_{n-1}, d, \omega)) + \mathfrak{D}(1 - \mathcal{A}(b_{n-1}, d))$$

Let $n \to \infty$ in the above inequality one get,

$$\tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(d, \Gamma d, \omega)) \le \tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(d, d, \omega)) - \tilde{\Phi}(\mathfrak{F}_{\mathcal{M}}(d, d, \omega)) + \mathfrak{D}(1 - \mathcal{A}(d, d))$$

where $\mathcal{A}(d, d) = \max\{\mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega), \mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega),$

$$\sqrt{\mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega)\mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega)}\}$$

$= \max\{\mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega), \mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega),$

$$\mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega)\}$$

$$= \mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega)$$

So,

$$\tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(d, \Gamma d, \omega)) \le \tilde{\Psi}(1) - \tilde{\Phi}(1) + \mathfrak{D}(1 - \mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega))$$

$$\le 0 + \mathfrak{D}(1 - \mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega))$$

$$\Longrightarrow \tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(d, \Gamma d, \omega)) \le \mathfrak{D}(1 - \mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega))$$

which is possible if and only if $\mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega) = 1 \Longrightarrow \Gamma d = d$. Thus d is Fp of Γ in Ω.

The last stage is to demonstrate the Fp's uniqueness. Take into account that d and ϑ are two Fps of Γ in Ω where $d \ne \vartheta$.

Then,

$$(\mathfrak{F}_{\mathcal{M}}(d, \vartheta, \omega)) = \tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(\Gamma d, \Gamma\vartheta, \omega))$$

$$\le \tilde{\Psi}(\mathfrak{F}_{\mathcal{M}}(d, \vartheta, \omega)) - \tilde{\Phi}(\mathfrak{F}_{\mathcal{M}}(d, \vartheta, \omega)) + \mathfrak{D}(1 - \mathcal{A}(d, \vartheta))$$

where $\mathcal{A}(d, \vartheta) = \max\{\mathfrak{F}_{\mathcal{M}}(\Gamma d, d, \omega), \mathfrak{F}_{\mathcal{M}}(\Gamma d, \vartheta, \omega),$

$$\sqrt{\mathfrak{F}_{\mathcal{M}}(\Gamma\vartheta, d, \omega)\mathfrak{F}_{\mathcal{M}}(\Gamma\vartheta, \vartheta, \omega)}\}$$

$= \max\{\mathfrak{F}_{\mathcal{M}}(d, d, \omega), \mathfrak{F}_{\mathcal{M}}(d, \vartheta, \omega),$

$$\sqrt{\mathfrak{F}_{\mathcal{M}}(\vartheta, d, \omega)\mathfrak{F}_{\mathcal{M}}(\vartheta, \vartheta, \omega)}\}$$

$$= \max\{1, \mathfrak{F}_{\mathcal{M}}(d, \vartheta, \omega), \sqrt{\mathfrak{F}_{\mathcal{M}}(\vartheta, d, \omega)}\} = 1.$$

So,

$$\left(\mathfrak{F}_{\mathcal{M}}(d,\vartheta,\omega)\right) \leq \tilde{\Psi}\left(\mathfrak{F}_{\mathcal{M}}(d,\vartheta,\omega)\right) - \tilde{\Phi}\left(\mathfrak{F}_{\mathcal{M}}(d,\vartheta,\omega)\right) + \mathfrak{D}(1-1)$$

$$\tilde{\Psi}\left(\mathfrak{F}_{\mathcal{M}}(d,\vartheta,\omega)\right) \leq \tilde{\Psi}\left(\mathfrak{F}_{\mathcal{M}}(d,\vartheta,\omega)\right) - \tilde{\Phi}\left(\mathfrak{F}_{\mathcal{M}}(d,\vartheta,\omega)\right)$$

$\Longrightarrow$ $\tilde{\Phi}\left(\mathfrak{F}_{\mathcal{M}}(d,\vartheta,\omega)\right) \leq 0$, and hence $\tilde{\Phi}\left(\mathfrak{F}_{\mathcal{M}}(d,\vartheta,\omega)\right) = 0$. Again by property of $\tilde{\Phi}$ one get $\mathfrak{F}_{\mathcal{M}}(d,\vartheta,\omega) = 1$. Hence $d = \vartheta$, which means the Fp is unique.

Example 1:

Consider $\Omega = \mathrm{N}$ with $\mathfrak{F}_{\mathcal{M}}: \Omega \times \Omega \times \mathbb{R} \to [0,1]$ specified by:

$$\mathfrak{F}_{\mathcal{M}}(b,q,\omega) = \begin{cases} \frac{b}{q} & if \quad b \leq q \\ \frac{q}{b} & if \quad q < b \end{cases}$$

$\forall b, q \in \Omega$ and $\omega > 0$. Let $\Gamma: \Omega \to \Omega$ be self-mapping defined by

$$\Gamma b = b^2$$

Let $\tilde{\Phi}, \tilde{\Psi}: [0,1] \to [0,\infty)$ be mappings defined by $\tilde{\Psi}(\delta) = 1-\delta^2$ and $\tilde{\Phi}(\delta) = 1-\sqrt{\delta}$. Then Γ satisfies inequality (9) with $\mathfrak{D} \geq 0$. Thus Γ is $(\tilde{\Psi}, \tilde{\Phi})$-almost weakly contraction mapping on Ω. Hence Γ satisfies the hypothesis of Theorem 2 and so, possesses a unique Fp. In this example $b^{\star} = 1$ is the Fp of Γ.

Example 2:

Let $\Omega = [0,1]$ and $\mathfrak{F}_{\mathcal{M}}: \Omega \times \Omega \times \mathbb{R} \to [0,1]$ defined by:

$$\mathfrak{F}_{\mathcal{M}}(b,q,\omega) = \begin{cases} 1 & if \quad b = 0 \text{ or } q = 0 \\ \max\{b,q\} & if \quad b \neq 0 \text{ and } q \neq 0 \end{cases}$$

$\forall b, q \in \Omega$ and $\omega > 0$. Let $\Gamma: \Omega \to \Omega$ be self-mapping defined by

$$\Gamma b = \begin{cases} \frac{1}{4} & if \quad b = 0 \\ 4b & if \quad 0 < b < \frac{1}{2} \\ 1 & if \quad \frac{1}{2} \leq b \leq 1 \end{cases}$$

Let $\tilde{\Phi}, \tilde{\Psi}: [0,1] \to [0,\infty)$ be mappings defined by $\tilde{\Psi}(\delta) = 1-\delta^2$ and $\tilde{\Phi}(\delta) = 1-\delta$. then Γ satisfies inequality (9) with $\mathfrak{D} \geq 0$. Thus Γ is $(\tilde{\Psi}, \tilde{\Phi})$-almost weakly contraction mapping on Ω. On the other hand, it is clear that $(\Omega, \mathfrak{F}_{\mathcal{M}}, \odot)$ is complete $F\mathcal{M}$-space , therefore Γ satisfies the hypothesis of Theorem 2. In this example $b^{\star} = 1$ and $b^{\star} = \frac{1}{4}$ are Fp of Γ.

Conclusion

In this work, the idea of almost $\hat{\boldsymbol{Z}}$-contraction mapping and $(\tilde{\Psi}, \tilde{\Phi})$-almost weakly contraction mapping is presented in a $F\mathcal{M}$-space. The fixed point theorem for these various contraction mappings is then verified. Several examples are offered to demonstrate the usefulness of the obtained results. This work provides the framework for future research on additional new forms of almost weakly contraction mappings and their applications in fuzzy metric spaces.

Acknowledgment

Our deepest gratitude goes out to the people who reviewed our work and the editors of the "Baghdad Science Journal" for giving us this incredible chance.

Authors' Declaration

- Conflicts of Interest: None.
- Ethical Clearance: The project was approved by the local ethical committee at University of Technology.

Authors' Contribution Statement

R. I. S. made a valuable contribution to the analysis, implementation, and design of the research findings. B. A. A. made a contribution to the process of revising and proofreading the findings. Through a collaborative analysis of the findings, the two authors enhanced the final research.

References

1. Abed S, Hasan MZ. Weak Convergence of Two Iteration Schemes in Banach Spaces. Eng Technol J. 2019; 37(2B): 32–40. https://doi.org/10.30684/etj.37.2B.1.
2. Sabri RI. Fuzzy Convergence Sequence and Fuzzy Compact Operators on Standard Fuzzy Normed Spaces. Baghdad Sci J. 2021; 18(4): 1204. https://doi.org/10.21123/bsj.2021.18.4.1204.
3. Al-Jawari NJ, Ahmed IN. Controllability of Nonlinear Boundary Value Control Systems in Banach Spaces Using Schauder Fixed Point Theorem. Eng Technol J. 2015; 33(4B): 640-653. https://doi.org/10.30684/etj.33.4B.8.
4. Gheeab MN, Kider JR. Properties of the Adjoint Operator of a General Fuzzy Bounded Operator. Baghdad Sci. J. 2021; 18(1): 0790. https://doi.org/10.21123/bsj.2021.18.1(Suppl.).0790.
5. Ciric LB. A Generalization of Banach's Contraction Principle. Proc Am Math Soc. 1974; 45(2): 267-273.
6. Khudhair ZA, Kider JR. The Hausdorff Algebra Fuzzy Distance and its Basic Properties. Eng Technol J. 2021; 39(7): 1185-1194. https://doi.org/10.30684/etj.v39i7.2001.
7. 7. Kadhm AE. Schauder Fixed Point Theorems in Intuitionistic Fuzzy Metric Space. Iraqi J Sci. 2022; 58(1C): 490–496. https://doi.org/10.24996.ijs.2017.58.1C.12
   Awasthi T, Dean SB. An Analysis on Fixed Point Theorem and its Application in Fuzzy Metric Space. J Adv Sch Res Allied Educ. 2018; 15(5): 65–69. https://doi.org/10.29070/JASRAE.
8. Sabri RI. Compactness Property of Fuzzy Soft Metric Space and Fuzzy Soft Continuous Function. Iraqi J Sci. 2021; 62(9): 3031–3038. https://doi.org/10.24996/ijs.2021.62.9.18.
9. Sihag V, Dinesh V. Fixed Point Theorem in Fuzzy Metric Space Via α-Series Contraction. In International Conference on Recent Advances in Pure and Applied Mathematics. Springer, Singapore. 2018; 151-159. https://doi.org/10.1007/978-981-15-1153-0_13.
10. Chauhan S, Khan MA, Kumar S. Unified Fixed Point Theorems in Fuzzy Metric Spaces via Common Limit Range Property. J Inequal Appl. 2013; 182(2013). https://doi.org/10.1186/1029-242X-2013-182.
11. Mathuraiveeran J, Selvi RP, Poovaragavan D. Common Fixed Point Theorems In Anti Fuzzy Metric Spaces. J Math Anal Model. 2023; 4(1): 106-114. https://doi.org/10.48185/jmam.v4i1.664.
12. Hardan B, Patil J, Abdo MS, Chaudhari A. A Fixed Point Theorem for Hardy-Rogers Type on Generalized Fractional Differential Equations. Adv. Theory Nonlinear Anal. Appl. 2020; 4(4): 407-420. https://doi.org/10.31197/atnaa.767331.
13. Gupta V, Singh B, Kumar S, Tripathi AK. On Variants of Compatible Mappings in Fuzzy Metric Spaces and Related Fixed Point Theorems. J Anal. 2019; 27: 197–208. https://doi.org/10.1007/s41478-018-0117-2.
14. Schweizer B, Sklar A. Statistical Metric Spaces. Pacific J Math. 1960; 10(1): 313-34. http://dx.doi.org/10.2140/pjm.1960.10.313 .
15. George A, Veeramani P. On Some Results in Fuzzy Metric Spaces. Fuzzy Sets Syst. 1994; 64(3): 395-9. http://dx.doi.org/10.1016/0165-0114(94)90162-7.
16. Isik H, Gungor NB, Park C, Jang SY. Fixed Point Theorems for Almost Z-Contractions with an Application. Math. 2018; 6(7): 117. https://doi.org/10.3390/math6030037.

2024, 21(10): 3205-3213
https://doi.org/10.21123/bsj.2024.9288
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

نتائج النقطة الصامدة لدوال الانكماش القريبة في الفضاء المتري الضبابي

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الخلاصة

في بعض المسائل الرياضية والحاسوبية والاقتصادية والنمذجة، يكون وجود حل لمشكلة نظرية أو مشكلة في العالم الحقيقي مرادفًا لوجود نقطة صامدة (Fp) لدالة مناسبة. وبالتالي، فأن نظرية النقطة الصامدة Fp تلعب دورًا أساسيًا في مجموعة واسعة من السياقات الرياضية والعلمية. تعتبر نظرية النقطة الصامدة Fp في حد ذاتها مزيجًا مذهلاً مكون من التحليل (التحليل الصرف والتحليل التطبيقي)، والهندسة، والطوبولوجيا. لقد أظهرت السنوات الأخيرة أن نظرية النقطة الصامدة Fp هي أداة قوية ومفيدة للغاية في دراسة الحالات غير الخطية. تهتم نظريات النقطة الصامدة Fp بدالة f من المجموعة X الى المجموعة X نفسها والتي في ظل ظروف معينة، فأنها تسمح بوجود نقطة صامدة Fp ، بعبارة اخرى انه بمعنى لكل نقطة x موجودة في المجموعة X ($x\in X$)بحيث أن $f(x)=x$. يقدم هذا العمل ويثبت نظرية النقطة الصامدة Fp لأنواع مختلفة من دوال الانكماش في الفضاء المتري الضبابي (FM-space)والتي تسمى بدالة الانكماش $\widehat{Z}$ القريبة ودالة الانكماش الضعيفة القريبة ($\tilde{\Phi}, \tilde{\Psi}$). في البداية، تم التذكير بمفهوم الفضاء المتري الضبابي(FM-space) وبعض المصطلحات المستخدمة في الإطار الضبابي. ثم بعد ذلك تم اعطاء مفهوم دالة المحاكاة. يتم استخدام مفهوم دالة المحاكاة هذا لتقديم تعريف دالة الانكماش $\tilde{Z}$ القريبة في إطار الفضاء المتري الضبابي. بالإضافة إلى ذلك، لقد تم استخدام هذا المفهوم(دالة الانكماش $\tilde{Z}$ القريبة) لبرهان وجود النقطة الصامدة ووحدانية النقطة الصامدة لهذا النوع من الدوال.ثم بعد ذلك تم تقديم فكرة دالة الانكماش الضعيفة القريبة ($\tilde{\Phi}, \tilde{\Psi}$) في إطار الفضاء المتري الضبابي(FM-space) ، بالإضافة إلى تقديم نظرية النقطة الصامدة Fp لهذا النوع من من الدوال. وفي نهاية البحث تم تقديم بعض الأمثلة لدعم النتائج.

الكلمات المفتاحية: الدوال الانكماشية القريبة، دالة الانكماش $\tilde{Z}$ القريبة، دالة الانكماش الضعيفة القريبة ($\tilde{\Phi}, \tilde{\Psi}$)، النقطة الصامدة، الفضاء المتري الضبابي.