On CSO-Compact Space

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Abstract:

The aim of this paper is to introduces and study the concept of CSO-compact space via the notation of simply-open sets as well as to investigate their relationship to some well known classes of topological spaces and give some of his properties.

1. Introduction and Preliminaries

Let *(X,T)* be a topological space. For a subset *S* of *X* , the closure, the interior and the complement of *S* with respect to *(X,T)* will be denoted by *clS*, *intS* and(*X-S = S^c*) respectively. And let *w* be denoted the set of all positive integers. The subspace topology on a subset *S* of a space *(X,T)* is denoted by *T/S*.

Semi-open subsets were defined by Levine. Recall that *S* is called semi-open [1] if *S*⊂cl(int *S*). A *semi-closed* set is a set, whose complement is semi-open [2].

Recently there has been some interest in the notion of a *simply-open* subset of topological space *(X,T)*. According to Neubrunnovia [3] a subset *S* of a space *(X, T)* is called simply-open if it is *S* = O∪N, where O is open and N is *nowhere dense* (=nwd, int (cl N) = Φ) subset of a space *(X,T)*. In [4] Ganster, Reilly and Vamanmurthy showed that a subset *S* of a space *(X,T)* is simply-open if and only if it is intersection of semi-open and semi-closed subsets of a space *(X,T)*. In [5] and [6] simply-open sets called as *semi-locally closed* set and *NDB-set*, respectively.

Nour [7] showed that a subset *S* of a space *(X,T)* is called *regular open* (resp. *regular closed*) if *S* = int (cl *S*) (resp. *S* = cl (int *S*)). A set *S* is called *semi-regular* [8] if it is both semi-open and semi-closed. Di Maio and Noiri [8] have pointed out that a subset *S* of a

space *(X,T)* is called semi-regular if and only if there is regular open set U such that U ⊂ *S* ⊂ cl U. Cameron [9] used the term *regular semi-open* for a semi-regular set.

In 1994, Dlaska, Ergun and Ganster [10] introduced the class of *countably S-closed* space. A space *X* is called countably S-closed if every countable cover of regular closed sets has finite subcover. In 1984, Porter and Woods [11] defined the concept of *feebly compact spaces*. A space *X* is called feebly compact if every countable open cover of *X* has finite subfamily such that the closures of whose members cover *X* .

Recall that *X* is *countably rs-compact* [12] (resp. *semi-countably compact* [13], *mildly compact* [14]) if every countable cover of *X* by semi-regular (resp. semi-open, regular open) sets has a finite subcover.

The families of regular open(resp. regular closed) subsets of *(X,T)* is denoted by *RO(X,T)*(resp. *RC(X,T)*).And *RO(X,T)* is a base for a coarser topology T_s on *X*, called the *semi-regularization* topology on *X*.

Grossely and Hildebrand [15] defined a *irresolute* (resp. *pre-semi-open*) function as, a function f: *X* → *Y*, such that *X* and *Y* be a topological spaces, is said to be irresolute(resp. pre-semi-open) if and only if, for any semi-open subset *S* of *Y* we have, f⁻¹(*S*) is semi-open in *X*(resp. for all *S*∈ *SO(X)*, f(*S*)∈*SO(Y)*) and they

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defined a *semi - homeomorphism* function between two topological spaces as, a function f: $X \rightarrow Y$ is semi-homeomorphism if and only if it is one-to-one, onto, irresolute, and pre-semi-open.

2.The relation between CSO-compact space and some
well-known classes of topological spaces

We begin by defining the class of spaces we will study in this paper.

Definition 2.1. A topological space *(X,T)* is called ***CSO-compact*** if every countable cover of *X* by simply-open subset of *X* has a finite subcover.

Lemma2.2. Every semi-open subset of a space *(X, T)* is simply-open set.

Proof. Let *A* be a semi-open subset of a space *(X,T)*. By [1],there exist an open set O such that, $O \subset A \subset clO$, then $A = O \cup (A-O)$. Let B=*A*-O, then $B \subset (clO-O)$. Now only we shall to do is to prove that B is nwd in *X*, so we have, int(cl(clO-O)) = $[int(cl(clO \cap O^c)] \subset [int(clO \cap clO^c)] = [int(clO \cap (intO)^c] \subset [clO \cap int(intO)^c]$ = [clO $\cap$ (clO)^c]=Φ.So clO-O is nwd but $B \subset (clO-O)$, therefore B is nwd. This shows the Lemma■

By upper lemma we can prove the following result

Proposition 2.3. Every CSO-compact space is countably S-closed space .

Proof. Let {**G$_i$: i**$\in$I} be a countable cover of apace *X* by regular closed sets and ,but we know that every regular closed sets is semi open set so {**G$_i$: i**$\in$I} is countable cover of *X* by semi-open set and so by simply-open set by Lemma2.2 but *X* is CSO-compact space so {**G$_i$: i**$\in$I} have finite subcover ■

Lemma2.4. The closure of semi-open set is regularly closed set.

Proof. For let A be a semi-open set of a topological space (X,T). Then A$\subseteq$cl(int(A)).Hence cl(A)$\subseteq$cl(cl(intA))=cl(int(A))Which implies cl(A)$\subseteq$cl(int(A)) So cl(A)=cl(int(A)) . But cl(int($\overline{A}$)) $\subseteq$ $\overline{A}$ and cl(int(A))$\subseteq$cl(int(clA)) so clA=cl(int(clA)) and therefore $\overline{A}$ is regular closed set■

Proposition 2.5. Every CSO-compact space is feebly compact.

Proof. Let {**G$_i$: i**$\in$I} be a countable cover of apace *X* by open sets, but we know that every open set is semi-open and so {**G$_i$: i**$\in$I} is countable semi-open cover of *X* , so by Lemma2.4 {cl**G$_i$: i**$\in$I} is regular closed cover of *X* and so by Proposition 2.3 {cl**G$_i$: i**$\in$I} have finite subcover■

In our next example we show that the converses of upper propositions need not be true.

Example 2.6. Let *X* be the real line *R* with a topology in which the only non-trivial open subsets are {0},{1}and {0,1}clearly *X* has only a finite number of regular open and open sets, and hence *X* is countably S-closed and feebly compact space. Note now that the countable cover {*R-w* }$\cup${{1,n}:n$\in$*w*} by simply-open sets has no finite subcover, hence *X* is not CSO-compact space.

The following proposition is very easy and the proof will not be given.

Proposition 2.7. In a discrete space every feebly compact space is CSO-compact space.

Proposition 2.8. Every semi-regular subset of a space X is simply-open.

Proof. Let A be a semi-regular subset of a space X so A is open and so semi-open then by Lemma2.2 A is simply-open set∎

Proposition 2.9. Every CSO-compact space is countably rs-compact(resp. semi- countably compact, mildly compact).

Proof. The proof coming from the fact say that every cover of X by semi-regular (resp. semi-open, regular open) is simply-open cover, but X is CSO-compact space, so its have subcover∎

The cofinite topology will show that the converse of previous proposition need not be true.

under what condition the converse of above proposition showed be true? The following theorem will answer the question.

Theorem 2.10. Every semi-countably compact space, in which every semi-closed set is semi-open, is CSO-compact space.

Proof. Let A be a simply-open subset of a space X, then A=O∪N where, O is open and N is nwd subsets of a space X , so O is open and N is semi-closed and so N, by theorem's condition, is semi-open. But the union of two semi-open set is semi-open, so A is semi-open. Now let $\bigcup_{i\in I} A_i$, such that I is countable, is countably simply-open cover of X then $\bigcup_{i\in I} A_i$ is

countably semi-open cover of X, but X is semi- countably compact space then $\bigcup_{i\in I} A_i$ have a finite subcover of X ∎

Lemma2.11. [10] Let (X,T) be a space, then if $G \subset X$ is locally dense, i.e. $G \subset \text{int}(\text{cl}G)$, then $RC(G,T/G)=\{F \cap A : F \in RC(X,T)\}$

Theorem 2.12. Let (X,T) be a CSO-compact space, then if $G \in RO(X,T)$, then $(G,T/G)$ countably S-closed space.

Proof. Let $\{A_{n:} n \in w\} \subset RC(G,T/G)$ be a cover of G. By lemma 2.10, for each $n \in w$, $A_{n=} G \cap F_n$ for some $F_n \in RC(X,T)$. Since $\{ F_n : n \in w \} \cup \{ X- G \}$ is a regular closed cover of (X,T) ,but (X,T) is a CSO-compact space and so countably S-closed space by Proposition 2.3, so there exists $m \in w$ such that $X=\{ X- G \} \cup F_1 \cup F_2 \cup .... \cup F_{m.}$ Consequently, $G= A_1 \cup ...... \cup A_m$ and thus $(G,T/G)$ countably S-closed space ∎

3. Characterization and properties of CSO-compact space

Lemma3.1 [15] if f: $(X,T) \rightarrow (Y,F)$ is homeomorphism, then f is semi-homeomorphism

Lemma3.2 [15] if f: $(X,T) \rightarrow (Y,F)$ is semi-homeomorphism and $N \subset X$ is nwd in X, then f(N) is nwd in Y.

Now we are in a position to prove our main result.

Theorem 3.3 If f: $(X,T) \rightarrow (Y,F)$ is homeomorphism and (Y,F) is CSO-compact space, then is(X,T) CSO-compact.

Proof. Let $\{G_{i=}O_i \cup N_{i:}\ i \in I\}$ be a countable simply-open cover of X, such that O_i is open and N_i is nwd subsets of a space X. so by lemma3.1 , that f is semi-homeomorphism, so by lemma3.2 we have f(N_i), $i \in I$, provide that I countable, is nwd subsets of a space Y, and since f is homeomorphism then f(O_i), $i \in I$ is open subsets of a space Y.

Therefore $\{f(O_i) \cup\ f(N_i),\ i \in I\}$ is countable simply-open cover of a space Y. But Y is CSO-compact space, hence there exists a finite subcover of Y, say $\{f(O_{i_1})\ \cup f(N_{i_1}),\ f(O_{i_2})$ $\cup f(N_{i_2}),\dots\ ,\ f(O_{i_n})\ \cup f(N_{i_n})\}$, such that $\bigcup\limits_{j}^{n}\ \{f(O_{i_j})\ \cup f(N_{i_j})\}=Y$, hence $\left(f(O_{i_1})\ \ \cup f(N_{i_1})\right)\cup\ \ \left(f(O_{i_2})\right.$ $\left.\cup f(N_{i_2})\right)\cup\dots\dots\ \cup\left(\ \ f(O_{i_n})\right.$ $\left.\cup f(N_{i_n})\right)=\ Y$, therefore we have, $f(O_{i_1}\cup N_{i_1}\cup\ \ O_{i_2}\cup N_{i_2}\cup\dots\dots\cup$ $O_{i_n}\cup N_{i_n})=\ Y$, but $G_{i=}O_i\cup N_{i,}\ i \in I$, then $f(Gi_1\ \cup\ Gi_2\cup\dots\dots\cup\ Gi_n)=Y$, so $Gi_1\ \cup\ Gi_2\cup\dots\dots\cup\ Gi_n=X$. And this shows the validity of this theorem ∎

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حول الفضاء المرصوص-CSO

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الخلاصة:

الهدف من هذا البحث هو تعريف و دراسة موضوع الفضاء المرصوص- CSO المعرف عن طريق موضوع المجموعات المفتوحة–حقا بالإضافة إلى التحقق من علاقة الفضاء المرصوص- CSO مع بعض الفضاءات التوبولوجية المعروفة وذكر بعض خواصه.