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Coupled of Semi Analytic Approach Associated with Laplace Transform First Step for Solving Matrix Differential Equations Quadratic Form when the Time-Delay in Noise Term

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Abstract

A novel technique and an efficient modification based on Adomian decomposition method and homotopy approach for finding accurately analytic solutions to non-linear (noise term) quadratic matrix retarded delay equations connected with the method of steps to make the problem more easily is discussed. These approaches more efficiently, effectively and accurately. Wholly integration for homotopy analysis method use in state the wholly integration for Adomian approach. Main advantage of this technique is to get more an accurate and efficient results with more extended of the convergence region of iterative approximate solutions obtained with bigger and whole time interval and to know the accurate solution with long interval under delay influence until $t = 8$ and can more. Term of delay is disappeared after apply the method of steps. Absolute residual error is conducted. To reduce the time and more complicated calculations, Laplace transform for each components is applied. Finally, the results which obtained by this technique is an effective and rapidly converge for exact solution for whole time interval with more extended of the convergence region. This technique can used to different nonlinear problem. The Adomian decomposition method is a semi analytical technique for solving different type of differential equations ordinary, partial, fractional, delay differential equations and many type. This method was developed by George Adomian. It is rapidly converge to exact solution and used for linear , nonlinear, homogeneous and nonhomogeneous equations. Adomian polynomial allow the solution converge to exact solution without simply linearizing the problem under consideration. The same for homotopy.

Keywords: Adomian method, Adomian- Homotopy technique, Laplace transform, Method of steps, Quadratic matrix retarded delay differential equation.

Introduction

Currently, Adomian-Homotopy technique applies for quadratic matrix retarded delay differential equation (QMRDDE):

$$\dot{G}(t) + G(t-\tau)D + D^T G(t) - G(t)PG(t) + S(t) = 0, \quad t \in [c,T], \qquad 1$$

where $\tau > 0$ constant, $c \in R$.

and G, P, S and D are $m \times m$ matrices; $P = P^T$, $S = S^T$, $\tau = 1$,

and the initial matrix function

$$G(t) = Z_0(t), \qquad c-\tau \le t \le c, \quad c \text{ a positive} . \qquad 2$$

Adomian decomposition approach (ADM) a semi analytic method and may apply for many kinds include partial differential equation [1].

This solution of this approach is infinitely series converge with closed form easily [2,3]. Application of ADM with Laplace transform for third-order dispersive fractional partial differential equations [4]. Homotopy analysis method using Jumaries approach

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for nonlinear wave-like equations of fractional order is presented [5]. Analytic solutions for matrix and delay matrix differential equations is discussed [6,7]. Approximate and accurate solution for solving higher order initial value problems is discussed [8]. Approximate analytic solution for bright optical soliton to nonlinear Schrödinger Equation is presented [9]. A modification of ADM for fractional diffusion equations with initial conditions is discussed [10].

Previously, Liao applied the basic idea for homotopy based on topological for suggesting approximate analytics method to nonlinear equation, which name Homotopy Analysis Method (HAM) [11]. This method the series solution for many types of nonlinear problem [12]. It's strongly method use for finding solutions of nonlinear form [13]. The application of HAM to solve nonlinear equation systems with integrated genetic algorithm is considered [14]. The quotient HAM for solving nonlinear equations is studied [15]. Application of HAM for solving fractional barrier PDEs is presented [16]. Application of Lagrange Polynomials to find a numerically solutions fractional-Volterra Fredholm-integro type is studied [17]. Implementation of HAM with time-fractional black-scholes equations is discussed [18].

For this study, a QMRDDE is solved analytically by Adomian-Homotopy (ADM-HAM) technique. A motivation of this technique is to provide us a solution of quadratic matrix retarded delay differential equations in infinite series associated with the method of steps and to get more an accurate with efficient results for this type of questions with more extended of the convergence region of iterative approximate solutions for whole time interval obtained under delay influence whenever the iteration is increased. Absolute residual error is conducted. Furthermore, that is capable of to provide us a continuous representation of the approximate solution, which gives a better information of the results with whole time interval.

Analysis of the Adomian-Homotopy Technique for Solving QMRDDE

First, apply HAM to QMDDE discussed. Hence non-linear $m \times m$ of QMRDDE, Eq 1:

$$\dot{G}(t) + G(t-\tau)D + D^T G(t) - G(t)PG(t) + S(t) = 0, \quad t \in [c, T], \qquad 3$$

where $\tau > 0$ constant, c qualitative nonnegative in R ,

G matrix $m \times m$, $P \& D$ constant matrices ; $P = P^T$, $S = S^T$, $\tau = 1$, and $G(t)$ is a matrix, based on historic:

$$G(t) = Z_0(t), \; t \in [c-\tau, c]. \qquad 4$$

With apply method of steps on delay differential equations [7], general for each time steps

$$[c + i\tau, c + (i+1)\tau]:], \quad i = 1,2, \dots, n; n \in N$$

$$\dot{G}(t) + D^T G(t) - G(t)PG(t) + (S(t) + Z_i(t-\tau)D) = 0, \qquad 5$$

$$\dot{G}(t) + D^T G(t) - G(t)PG(t) + S^*(t) = 0, \qquad 6$$

where $S^*(t) = S(t) + Z_i(t-\tau)D$. Suppose $\mu(t,s)$ be a matrix function homotopy, so:

$$V_m(\emptyset) = \frac{1}{m!}\frac{\partial^m \mu}{\partial s^m}\bigg|_{s=0},$$

Zero order is:

$$(1-s)\mathcal{L}[\mu(t,s) - G_0(t)] = s\hbar N[\mu(t,s)], \qquad 7$$

for Eq.3 become:

$$\mathcal{L}[G_m(t) - \chi_m G_{m-1}(t)] = \hbar V_{m-1}(N[G]), \; m \geq 1, \qquad 8$$

$$N[G] = \dot{G}(t) + D^T G(t) - G(t)PG(t) + S^*(t), \qquad 9$$

where $\mathcal{L}[G] = \dot{G}(t)$, $\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases}$,

$$G(t) = \sum_{i=0}^{\infty} G_i(t)\, s^i, \qquad 10$$

$$R_m G_{m-1} = \frac{\partial G_{m-1}}{\partial t} + D^T G_{m-1} - \sum_{i=0}^{m-1} G_i P G_{m-1-i} + (1-\chi_m)(S^*). \qquad 11$$

Then, Eq 11 after apply inverse operator with initial condition, for $m \geq 1$ becomes:

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$$G_m(t) = \chi_m G_{m-1} + \hbar \int_0^t (G_{m-1}(u) + D^T \dot{G}_{m-1}(u) - \sum_{i-0}^{m-1} G_i(u)P\, G_{m-1-i}(u) + (1 - \chi_m)(S^*))du$$

$$= \chi_m G_{m-1} + \hbar[(G_{m-1}(t) - (1-\chi_m)G_{m-1}(0)] + \hbar \int_0^t (D^T G_{m-1}(u) - \sum_{i-0}^{m-1} G_i(u)P\, G_{m-1-i}(u) + (1 - \chi_m)(S^*))du,$$

$$= (\chi_m + \hbar)G_{m-1}(t) - \hbar(1-\chi_m)G_{m-1}(0) + \hbar \int_0^t (D^T G_{m-1}(u) - \sum_{i-0}^{m-1} G_i(u)P\, G_{m-1-i}(u) + (1-\chi_m)(S^*))du. \quad 12$$

For ADM to solve Eq 1, consider nonlinear $m \times m$ QMRDDE Eq 1:

$$\dot{G}(t) + G(t-\tau)D + D^T G(t) - G(t)PG(t) + S(t) = 0, \quad t \in [c,T], \quad 13$$

with initial function: $G(t) = Z_0(t), \quad t \in [c-\tau, c]$.

Now, apply method of steps on delay differential equations, general for each time steps

$$[c + i\tau, c + (i+1)\tau], \quad i = 1,2,\dots,n; n \in N$$

$$\dot{G}(t) + D^T G(t) - G(t)PG(t) + (S(t) + Z_i(t-\tau)D) = 0,$$

$$\dot{G}(t) + D^T G(t) - G(t)PG(t) + S^*(t) = 0, \quad 14$$

where $S^*(t) = S(t) + Z_i(t-\tau)D$.

$$G(c) = Q \quad 15$$

where Q is $m \times m$ constant matrix, and G assumed to be bounded matrix, for $t \in [c,T]$, that is $|g_{ij}(t)| \le M, c \le t \le T,\ G(t) = (g_{ij}(t))_{n\times n}$, the noise term $N(G) = GPG$, has polynomial matrices:

$$N(G) = GPG = \sum_{n=0}^{\infty} F_n,$$

and F_n can be express:

$$F_0(t) = G_0(t)PG_0(t)$$

$$F_1(t) = G_0(t)PG_1(t) + G_1(t)PG_0(t)$$

$$\vdots \quad 16$$

$$F_n(t) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\left(\sum_{i=0}^{\infty} \lambda^i G_i(t)\right) P \left(\sum_{i=0}^{\infty} \lambda^i G_i(t)\right)\right]\Bigg|_{\lambda=0},$$

$F_n(t)$ is polynomial Adomian matrices. Where:

$$W_n = \sum_{i=0}^{n} G_i, \quad 17$$

Application of ADM on Eq 13 is:

$$G(t) = \sum_{i=0}^{\infty} G_i(t),$$

and $\quad G_0(t) = Q + L^{-1}\big(-S^*(t)\big),$

$$G_i(t) = L^{-1}(-F^T G_{i-1}) + L^{-1}F_{i-1}, \quad i \ge 1, \quad 8$$

Now, whole integration for ADM in Eq 18 is replaced by the integration of Eq 12 for HAM. In this case, accurately and efficiently solution with more extended of the convergence region until $t = 8$ and can be more is obtained under delay influence compare with the obtained by Eq 18, to reduce the time and more complicated calculations, Laplace transform will be used for each term, as:

$$G_i(t) = L^{-1}\left(D^T G_{m-1}(u) - \sum_{i-0}^{m-1} G_i(u)P\, G_{m-1-i}(u) + (1-\chi_m)(S^*)\right), \quad i \ge 1, \quad 19$$

Numerical Simulation

Now, ADM-HAM technique which presented above for solving QMRDDE implemented. Terms of delay in non-linear parts will disappeared. One may see that this technique strongly, effectively, reliable with rapidly converge to exact solution with extended of the convergence region for whole time interval whenever the iteration is increased.

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Example 1:

Consider the non-linear 2 × 2 QMRDDE of the form:

$$\dot{G}(t) + G(t)D + D^T G(t) - G(t)PG(t-\tau) + S(t) = 0, \quad \tau = 1,\ t_0 = 0, \qquad 20$$

$D = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix},\ P = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},\ S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ with stationary history condition

$$G_0(t) = \begin{pmatrix} t & 0 \\ 0 & 1 \end{pmatrix}, \quad -1 \le t \le 0.$$

Now, by using method of first step:

$$\dot{G}(t) + G(t)D + D^T G(t) - G(t)PG_0(t-1) + S(t) = 0, \qquad 21$$

$$\begin{pmatrix} \dot{x}(t) & \dot{y}(t) \\ \dot{z}(t) & \dot{w}(t) \end{pmatrix} + \begin{pmatrix} x(t) & y(t) \\ z(t) & w(t) \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x(t) & y(t) \\ z(t) & w(t) \end{pmatrix} +$$

$$\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} x(t) & y(t) \\ z(t) & w(t) \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} t-1 & 0 \\ 0 & 1 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad 0 \le t \le 1$$

$$\rightarrow \begin{pmatrix} 4x(t) + \dot{x}(t) + y(t) + z(t) + x(t)(t-1) + 1 & w(t) + x(t) + 3y(t) + \dot{y}(t) \\ w(t) + x(t) + 4z(t) + \dot{z}(t) + z(t)(t-1) & 3w(t) + \dot{w}(t) + y(t) + z(t) + 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \qquad 22$$

$$\rightarrow \begin{pmatrix} \dot{x}(t) + (3+t)x + y(t) + z(t) + 1 & \dot{y}(t) + 3y(t) + x(t) + w(t) \\ \dot{z}(t) + (3+t)z + x(t) + w(t) & \dot{w}(t) + 3w(t) + y(t) + z(t) + 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In this example, the term of delay in nonlinear part Eq 21 will disappears after apply method of steps Eq. 22, and may considered as quadratic matrix differential equation. Then apply the Adomian-Homotopy to Eq 22 to every equation and initially approximation $G_0(t) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, as explained to every term. To reduce the time and more complicated calculations, Laplace transform will be used for each term. With Mathematica Software, the iteration can be obtained. Absolute residual error for each component to show the high an accurate solution is conducted in Table 1, Figs 1 and 2.

Results and Discussion

Table 1. Numerical results of ADM-HAM QMRDDE

t	***Absolute residual error of ADM-HAM for the first component $x_{11}(t)$***	***Absolute residual error of ADM-HAM for the second component $x_{12}(t)$***	***Absolute residual error of ADM-HAM for the third component $x_{21}(t)$***	***Absolute residual error of ADM-HAM for the fourth component $x_{22}(t)$***
0.	0	0	0	0
0.5	2.2204×10^{-16}	0	0	0
1.	4.4408×10^{-16}	8.8817×10^{-16}	2.2204×10^{-16}	1.7763×10^{-15}
1.5	9.5479×10^{-15}	1.0214×10^{-14}	1.6653×10^{-15}	1.1990×10^{-14}
2.	4.1522×10^{-14}	7.8381×10^{-14}	5.7620×10^{-14}	7.1054×10^{-14}
2.5	2.8510×10^{-13}	1.1474×10^{-12}	6.0196×10^{-13}	1.8296×10^{-12}
3.	3.3819×10^{-12}	2.2738×10^{-12}	3.5264×10^{-12}	1.5688×10^{-11}
3.5	2.4843×10^{-11}	4.0017×10^{-11}	3.4411×10^{-11}	1.0550×10^{-10}
4.	1.8563×10^{-10}	7.5669×10^{-10}	7.5178×10^{-11}	7.5669×10^{-10}
4.5	8.1595×10^{-10}	5.1220×10^{-9}	1.0087×10^{-9}	5.5877×10^{-9}
5.	3.7321×10^{-8}	3.7398×10^{-9}	2.0348×10^{-8}	5.2150×10^{-8}
5.5	1.0333×10^{-7}	3.5825×10^{-7}	7.9658×10^{-8}	3.0912×10^{-8}
6.	3.8974×10^{-7}	2.7591×10^{-6}	1.3451×10^{-6}	2.6060×10^{-6}
6.5	6.6614×10^{-6}	2.0799×10^{-6}	7.4046×10^{-6}	2.6779×10^{-5}
7.	9.8977×10^{-5}	2.6244×10^{-4}	1.3548×10^{-4}	4.9654×10^{-7}
7.5	5.9631×10^{-4}	9.2208×10^{-4}	7.7945×10^{-5}	1.7489×10^{-4}
8.	1.9054×10^{-3}	5.9478×10^{-3}	4.7471×10^{-3}	6.1852×10^{-3}

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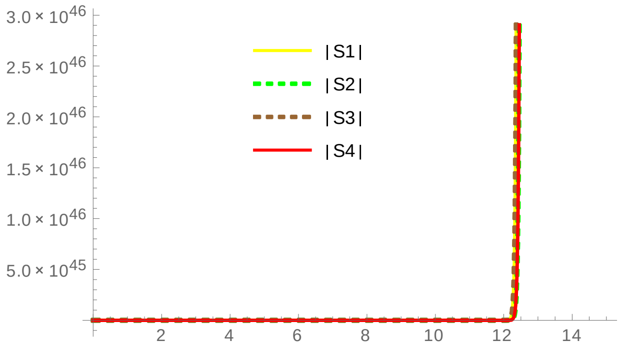

Figure 1. Absolute residual error of ADM-HAM for all components

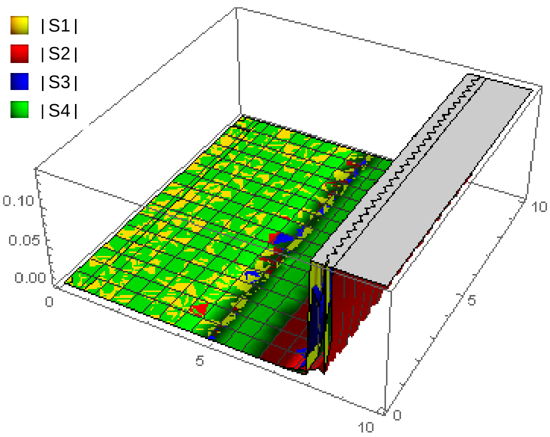

Figure 2. Absolute residual error of ADM-HAM for all components

Example 2:

Consider the non-linear 2×2 QMRDDE of the form

$$\dot{G}(t) + G(t)D + D^T G(t) - G(t-\tau)PG(t) + S(t) = 0,$$

$$\tau = 1, \ \ t_0 = 0, \qquad 23$$

$$D = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \ \ P = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \ \ S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

with stationary history condition

$$G_0(t) = \begin{pmatrix} t & 0 \\ 0 & 1 \end{pmatrix}, \quad -1 \le t \le 0.$$

Hence, connection with the method of first step:

$$\dot{G}(t) + G(t)D + D^T G(t) - G_0(t-1)PG(t) + S(t) = 0, \qquad 24$$

$$\begin{pmatrix} \dot{x}(t) & \dot{y}(t) \\ \dot{z}(t) & \dot{w}(t) \end{pmatrix} + \begin{pmatrix} x(t) & y(t) \\ z(t) & w(t) \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x(t) & y(t) \\ z(t) & w(t) \end{pmatrix} - \begin{pmatrix} t-1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x(t) & y(t) \\ z(t) & w(t) \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, 0 \le t \le 1$$

$$\rightarrow \begin{pmatrix} 4x(t) + \dot{x}(t) + y(t) + z(t) + x(t)(t-1) + 1 & w(t) + x(t) + 4y(t) + \dot{y}(t) + y(t)(t-1) \\ w(t) + x(t) + 3z(t) + \dot{z}(t) & 3w(t) + \dot{w}(t) + y(t) + z(t) + 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

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$$\rightarrow \begin{pmatrix} \dot{x}(t) + (3+t)x + y(t) + z(t) + 1 & \dot{y}(t) + (3+t)y + x(t) + w(t) \\ \dot{z}(t) + 3z(t) + x(t) + w(t) & \dot{w}(t) + 3w(t) + y(t) + z(t) + 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In this example, use delay terms with nose portion Eq 24 then it disappear when apply a steps technique Eq 25 and implement Adomian with initially approximation condition: $G_0(t) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, to reduce the time and more complicated calculations, Laplace transform will be used for each term. Using Mathematica software, to get the iterative. Absolute residual error for each component to show the high an accurate solution is conducted in Table 2, Figs. 3 and 4.

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Table 2. Numerical results of ADM-HAM QMRDDE

t	*Absolute residual error of ADM-HAM for the first component* $x_{11}(t)$	*Absolute residual error of ADM-HAM for the second component* $x_{12}(t)$	*Absolute residual error of ADM-HAM for the third component* $x_{21}(t)$	*Absolute residual error of ADM-HAM for the fourth component* $x_{22}(t)$
0.	0	0	0	0
0.5	2.2204×10^{-16}	0	0	0
1.	6.6613×10^{-16}	0	4.4408×10^{-16}	6.6613×10^{-16}
1.5	2.4424×10^{-15}	1.4432×10^{-15}	3.1036×10^{-15}	1.2434×10^{-14}
2.	6.8167×10^{-14}	4.6185×10^{-14}	4.5963×10^{-14}	8.5265×10^{-14}
2.5	1.0480×10^{-13}	1.7057×10^{-13}	3.1563×10^{-13}	1.5909×10^{-12}
3.	4.1959×10^{-12}	7.2724×10^{-12}	9.7311×10^{-13}	1.2782×10^{-11}
3.5	5.3486×10^{-11}	2.7512×10^{-11}	2.7470×10^{-11}	8.2309×10^{-11}
4.	5.5382×10^{-11}	7.4942×10^{-10}	1.4962×10^{-10}	6.8394×10^{-10}
4.5	1.0533×10^{-10}	4.1909×10^{-9}	2.0419×10^{-9}	6.5192×10^{-9}
5.	3.3622×10^{-8}	3.7253×10^{-9}	1.1694×10^{-8}	6.7055×10^{-8}
5.5	5.5614×10^{-8}	3.5762×10^{-8}	2.5821×10^{-8}	5.9602×10^{-8}
6.	6.5373×10^{-7}	2.3845×10^{-6}	1.9670×10^{-6}	3.5764×10^{-6}
6.5	5.3836×10^{-6}	5.7146×10^{-6}	1.0373×10^{-5}	3.2439×10^{-5}
7.	8.1918×10^{-5}	2.4342×10^{-4}	1.8125×10^{-4}	8.1062×10^{-6}
7.5	2.8742×10^{-4}	1.1501×10^{-3}	5.3249×10^{-4}	4.3484×10^{-4}
8.	8.1455×10^{-3}	2.9249×10^{-3}	2.6362×10^{-3}	2.3155×10^{-3}

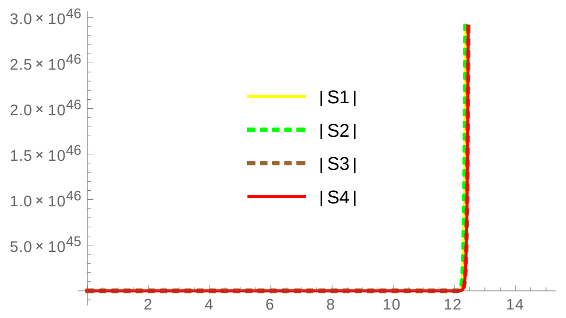

Figure 3. Absolute residual error of ADM-HAM for all components

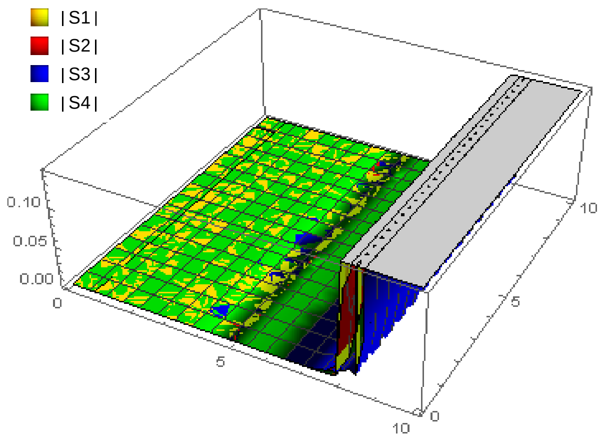

Figure 4. Absolute residual error of ADM-HAM for all components

Conclusion

Adomian-Homotopy technique is discussed to obtain a QMRDDE. Numerical results indicate that the suggested technique is gives highly an accurate solution in more extended of the convergence region for whole time interval under delay influence whenever the iteration is increased until $t = 8$ and can be more. Method of steps is applied for making the problem more easily. Term of delay is disappeared after apply the method of steps. Absolute residual error is obtained. To reduce the time and more complicated calculations, Laplace transform for each component is applied. All results indicate that this technique is capable of to provide us a continuous representation of the approximate solution, which gives a better information of the results for whole time interval as well as the neighborhood of the initial condition.

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Authors' Declaration

- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Furthermore, any Figures and images, that are not mine, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Mustansiriyah University.

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الطريقة شبه التحليلية المدمجة مع تحويلات لابلاس للخطوة الاولى لحل معادلة المصفوفات التفاضلية التباطؤية المربعة عندما يكون التباطؤ في الجزء المزعج

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الخلاصة

في هذا البحث تناولنا طريقة فعالة وجديدة وهي الدمج بين طريقتي الادوميان والهوموتوبي مع إستخدام طريقة الخطوات لتسهيل المسألة والتي تخص المعادلات التفاضلية الاعتيادية التباطئية لحل معادلة المصفوفات التباطئية التربيعية الغير خطية . كلتا الطريقتين على درجة عالية من التأثير والفعالية. جزء التكلمل الكلي لطريقة الهوموتوبي سيستخدم بدلا من جزء التكامل الخاص ب الادوميان . الميزة الرئيسية لهذه التقنية هي الحصول على نتائج اكثر دقة و لفترة و منطقة اوسع واطول ولمعرفة دقة هذه النتائج تحت تأثير التأخير. الجزء الخاص بالتأخير يختفي بعد استخدام طريقة الخطوات. تم حساب الخطأ المتبقي . لتقليل الوقت والعمليات الحسابية المعقدة تم أستخدام تحويلات لابلاس. أخيرا النتائج التي تم الحصول عليها بينت ان التقنية فعالة وسريعة التقارب للحل المظبوط ولفترة ومنطقة اوسع . يمكن استخدام هذه التقنية لحل مسائل غير خطية مختلفة. طريقة الادوميان هي تقنية شبه تحليلية لحل معادلات تفاضلية مختلفة أعتيادية ؛ جزئية ؛ كسرية و تباطؤية . هذه الطريقة تطورت بواسطة جورج ادوميان. هي سريعة التقارب للحل المظبوط وتستخدم للخطية و غير الخطية و المتجانسة و غيلر المتجانسة. متعددة ادوميان تسمح للحل التقارب للحل المظبوط للمسألة قيد الدراسة دون الحاجة الى أي تحوير. طريقة الهوموتوبي هي تقنية شبه تحليلية لحل معادلات تفاضلية مختلفة أعتيادية؛ جزئية ؛ كسرية و تباطؤية وانواع مختلفة. هذه الطريقة تطورت عن طريق العالم ليو . هي سريعة التقارب للحل المظبوط وتستخدم للخطية وغير الخطية و المتجانسة وغير المتجانسة. جاء مفهوم الهوموتوبي من مفهوم التبلوجي في توليد متسلسلة متقاربة للحل المظبوط. هذه الطريقة تم ايجادها من قبل ليو خلال اطروحته. تحتوي الطريقة على متقير او معلمة خلاله تمكننا التقارب.

الكلمات المفتاحية: طريقة الادوميان، طريقة الادوميان مع الهوموتوبي، طريقة الهوموتوبي، طريقة الخطوات، المصفوفة التفاضلية التباطئية الغير خطية.