

Maximizing Reliability in the Age of Complexity: A Novel Optimization Approach

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Abstract

Calculating the reliability of complex systems is an urgent matter that deserves attention due to its wide applications, ranging from engineering and economic sciences to applications in medical fields. Traditional methods often suffer from computational complexity when dealing with large systems when calculating their reliability. This paper presents a new method to calculate and improve the reliability of highly complex systems. Furthermore, the proposed methodology combines the principles of system reliability analysis and optimization techniques to determine the optimal means for a system that increases reliability. A mathematical optimization approach was employed to undertake the task of improving the reliability of the system without complicating the calculations. By formulating the problem as an optimization task, the reliability of vehicles is optimized to meet specified reliability constraints. The effectiveness of this approach is demonstrated by experimental evaluations on various complex systems, demonstrating significant improvements in system reliability. This new approach was tested on an entire highly complex network of 1,225 vehicles, and the results were very acceptable. Finally, the proposed method was applied and numerical optimization results were obtained using the programming language Python version 3.12.2.

Keywords: Complex systems, Component reliabilities, Optimization-based approach, Reliability optimization, System reliability.

Introduction

In many businesses, reliability analysis of complex systems is crucial because system failures can lead to large financial losses, safety hazards, and operational interruptions. Complex systems' reliability analysis and optimization have received a lot of attention in the literature ^{1,2}. Numerous techniques are covered in the literature that already exist on system reliability analysis, including fault tree analysis, Monte

Carlo simulation, and Markov models. Although these techniques have been widely employed, because of computational difficulties, they frequently have trouble handling large-scale systems ³⁻⁵. Traditional reliability analysis methods, including fault trees and reliability block diagrams, put emphasis on determining critical routes or failure modes and evaluating the dependability of individual components ⁶⁻⁸.

While these techniques offer insightful information on system reliability, they frequently ignore the opportunity to optimize system settings for greater overall reliability^{9,10}. To close this gap, recent studies have investigated the development of network reliability improvement methods. Most researchers in the field of improving reliability relied on extracting the reliability polynomial as a function of the network components, and since the process of finding this function becomes more difficult as the number of network components increases, these researchers chose networks with a small number of components not exceeding twenty. In this paper, reliability optimization was done using a new algorithm that does not depend on reliability polynomials, where the optimization process was performed on a network consisting of 1225 components. For reliability optimization, many researchers have used different algorithms such as genetic algorithm, particle swarm optimization, grey wolf optimization, bat algorithm, simulated annealing, and firefly algorithm¹¹. To maximize system dependability, these strategies take into account elements including redundancy, spare part distribution, and component reliability enhancement¹²⁻¹⁴. The literature supports the effectiveness of optimization-based approaches in improving system reliability. However, more study is required to examine their application to other system types, scalability, and computing efficiency¹⁵⁻¹⁷. To improve the overall reliability of complex networks, this study suggests a novel method that combines system reliability analysis with optimization processes. The main purpose of the proposed approach is to improve the reliability of complete complex networks (in which all components are interconnected) by customizing the reliability of all network components such that the best network reliability is achieved, taking into account different constraints and objectives. To avoid the shortcomings of traditional reliability analysis techniques, the proposed methodology features computational efficiency and high optimization principles. The utility of applying the proposed approach to improving the reliability of

complete complex networks is demonstrated through case studies and simulations.

Problem Statement

To evaluate the performance of complex networks, their reliability must be analyzed, and when dealing with networks that have a large number of components, traditional methods face great difficulty in calculating the reliability of these networks and take longer time and higher resources^{18,19} or the chances of obtaining reliability may be absent, regardless of the possibility of improvement or not. Therefore, there is an urgent need for a new strategy that can account for the reliability of more complex networks while reducing computing complexity. Reliability analysis in this paper is formulated as an optimization problem to maximize the reliability of complex networks while satisfying pre-defined reliability constraints:

Mathematical Formulation:

Let R_S denote the system reliability, and r_i is the reliability of i -th component. The objective is to maximize R_S , subject to the following constraints:

1. Constraint 1: The reliability of each component r_i has an upper limit $r_{i,max}$ and lower limit $r_{i,min}$, i.e., $r_{i,min} \leq r_i \leq r_{i,max}$ for all i .
2. Constraint 2: The system reliability R_S must be greater than or equal to target reliability R_{target} , i.e., $R_S \geq R_{target}$.

The parameters and variables are defined as follows:

1. Decision Variables: As for binary decision variables r_i for each component C_i , where $r_i = 1$ indicates that component C_i is selected for the optimized configuration, and $r_i = 0$ indicates that it is not selected. The optimization problem can be formulated as follows:
2. Objective Function: The objective function aims to maximize the overall system reliability R_S . Therefore, the optimization problem will be formulated according to the following:

Maximize: R_S

Subject to: $r_{i,min} \leq r_i \leq r_{i,max}$ for all $i = 1, \dots, n$

$$R_S \geq R_{target}$$

Through the resolution of this optimization problem, it can ascertain the ideal values for component reliabilities r_i , which in turn maximizes the overall system reliability R_S , all while adhering to the predefined reliability constraints. This approach, rooted in optimization, presents a hopeful avenue for enhancing system reliability, circumventing the computational intricacies typically linked with conventional techniques. Consequently, it facilitates the efficient and productive analysis of reliability in intricate and highly complex systems.

Methodology

The approach used in our study represents a bridge between increasing and improving the reliability of complex systems on the one hand and reducing the computational complexity and time needed when implementing such a task. An optimization algorithm, named “ALRIDHA algorithm” after the name of the first author in this paper, was created to optimize the reliability of the system through the process of iterative improvement of the reliability of components and communication topology. The algorithm allows the user to apply appropriate constraints to enforce the communication topology, ensure proper connectivity of components, and constrain the reliability of components within a certain range through the optimization problem to preserve their values. The reliability of the components and the communication structure are updated iteratively to find the best suitable solution to the optimization problem according to the imposed constraints. The proposed algorithm improves the reliability of the system and the updated reliabilities and link topologies by calculating the minimum path reliability using these reliabilities and the connection topology after obtaining the optimal component reliability values. Improving the reliability of the system without adding computational complexity is very important as it saves time as well as the accuracy of the desired results. Finally, this algorithm gives free rein to any complex system in modern designs in terms of detecting the reliability values of its components or performing an optimization process for them, taking into account the required constraints and communication topology through computational

methods. To calculate the system reliability R_S of a complex network with m minimum paths will adopt the following theorem:

Theorem 1: If the complex network has m number of minimum paths. Then the system reliability R_S of it is given by:

$$R_S = 1 - \prod_{j=1}^m \left(1 - \prod_{C_i \in \text{min.path}} r_i \right) \quad 1$$

Proof: It is obvious that the components of any path are in series²⁰, i.e.

$$R_{\text{min.path}} = \prod_{i=1}^k r_i \quad 2$$

for all $C_i \in \text{min.path}$, k is the number of C_i in min.path

It is clear that all paths are in parallel²⁰, i.e.,

$$R_S = 1 - \prod_i^m (1 - R_{\text{min.path}}) \quad 3$$

Thus,

$$R_S = 1 - \prod_{j=1}^m \left(1 - \prod_{C_i \in \text{min.path}} r_i \right)$$

Alridha Algorithm to Optimize Reliability

Pseudo-code of Alridha algorithm:

1. **Input:** Initial configuration of components
2. **Output:** Enhanced system reliability
3. **Procedure:** Optimize Reliability
4. Define the component reliabilities and connection topology
5. Reliabilities $\leftarrow (i, j)$: random uniform $(0,1)$; $i, j \in [1, n]$; $i \neq j$
6. Topology $\leftarrow (i, i + 1)$; $i \in [1, n - 1]$

7. Topology, append ($n, 1$)
 8. Calculate the system reliability
 9. System- reliability \leftarrow Calculate System Reliability
 10. **Return** reliabilities, topology, system-reliability
 11. **End Procedure**
 12. **Procedure:** Calculate System Reliability
 13. Define the connection topology using binary variables
 14. Let $x_{i,j}$ represent the component that connection between two nodes i and j
 15. Define the optimization problem
 16. **Objective:** Maximize R_S
 17. **Variables:** r_1, r_2, \dots, r_n (Component reliabilities)
 18. **Constraints:** $r_i \in [0,1]$, for $i = 1, \dots, n$
(Component reliability constraints)
$$\sum_{j=1}^n x_{i,j} = 1, \text{ for } i = 1, \dots, n - 1$$

(Outgoing connections from components)
$$\sum_{i=1}^{n-1} x_{i,j} = 1, \text{ for } j = 2, \dots, n$$

(Incoming connections to components)
$$x_{n,1} = 1$$
 (Connection from final component to initial component)
 19. **Solve** the optimization problem to obtain the optimal component reliabilities
 20. Calculate all minimum paths and their number m
 21. $R_{minpath} = \prod_{i=1}^k r_i$ for all $C_i \in minpath$, where k is the number of component in the minimum paths \leftarrow calculate the minimum path's reliability using the optimal component reliabilities and connection topology
 22. **Return** $R_S = 1 - \prod_i^m (1 - R_{minpath})$
 23. **Print** "System Reliability:", system-reliability
- The outline of the algorithm can be detailed as follows:

1. **Generation of Component Reliability:** Generate the reliability values for each component in the system. These values are randomly assigned within a specified range, ensuring diversity and variability.
2. **Generation of Connection Topology:** Construct the connection topology of the system using a complete highly complex network representation. This highly complex network captures the interconnections between components, forming the basis for analyzing system reliability.
3. **Calculation of Minimum Path Reliability:** Implement a recursive function that calculates the reliability of the minimum path from the input to the output in the system. Starting from the input component, traverse the connection topology to identify components and calculate their respective minimum path reliabilities. Determine the minimum path reliability by multiplying all the reliability of its components. Repeat this process until the output component is reached, obtaining the overall system reliability.
4. **Assessment of System Reliability:** Execute the minimum path reliability calculation to determine the system reliability based on the randomly generated component reliabilities and the connection topology.

Furthermore, the plots visualize network and convergence. It visualizes the network highly complex network representing the system using the connection topology, assigned colors for input and output, and other components, and analyzes the convergence by performing multiple iterations to generate system reliability to evaluate the convergence rate and stability of the algorithm.

Theoretical Convergence Properties

In this section, the theoretical evidence for the effectiveness of the optimization algorithm in finding optimal solutions, and that it converges to the global optimum:

Theorem 2: The optimization algorithm reaches the global optimum through convergence.

Proof: Let's denote the objective function as $f(x)$, where x is the decision variable vector, and the global optimum as x^* with $f(x^*)$ being the optimal objective value.

Assumption 1: The objective function $f(x)$ is continuous and bounded over the search space.

Assumption 2: The optimization algorithm satisfies the following conditions:

1. It explores the search space by generating diverse candidate solutions.
2. It exploits the search space by iteratively improving the candidate solutions based on the objective function.
3. It terminates when a predefined stopping criterion is met (e.g., maximum iterations, convergence threshold).

To prove convergence, we'll show that the algorithm satisfies the following two properties:

Property 1: Progress Property

Results and Discussion

The system reliability was significantly increased by the suggested solution using the optimization-based methodology. The system's initial reliability was calculated using conventional reliability analysis methods after the component arrangement was initialized. Constraints were then established to assure practical viability and an optimization objective function was constructed to maximize system reliability. By putting the suggested algorithm to use. Up until the termination criteria, such as reaching a maximum number of iterations or reaching convergence, were satisfied, the optimization process was carried out iteratively. The optimized system configuration, which demonstrated increased reliability compared to the initial

The algorithm guarantees that the objective function value improves in each iteration until convergence.

Proof: Let f_k denote the objective function value at iteration k . Since the algorithm explores and exploits the search space, it ensures that $f_{k+1} \leq f_k$ for all iterations k .

Property 2: Convergence Property

As the number of iterations approaches infinity, the algorithm converges towards the global optimum x^* .

Proof: Since f_k monotonically decreases with iterations, it is lower bounded by the optimal objective value $f(x^*)$. That is, $f_k \geq f(x^*)$ for all iterations k .

As the algorithm progresses, it gets closer to the global optimum. By assumption 1, $f(x)$ is continuous, and hence, as the iterations approach infinity, f_k converges to $f(x^*)$. Therefore, the optimization algorithm converges to the global optimum x^* as the number of iterations approaches infinity, satisfying both properties. This proof demonstrates that under the given assumptions, the optimization algorithm converges to the global optimum, providing a theoretical foundation for its effectiveness in finding optimal solutions.

configuration, was the best result found from the final population. Using conventional reliability analysis methods, the reliability of the optimized system design was computed and was found to be higher than the initial reliability. The complete network with n nodes has $n(n-1)/2$ components, so the network of 50 nodes has 1225 components see Fig 1. To visualize the results, several plots were created. First, a network diagram was drawn to depict the connectivity topology of the system, with each component represented as an edge. This visualization facilitated a better understanding of the system architecture and highlighted components critical to achieving enhanced reliability.

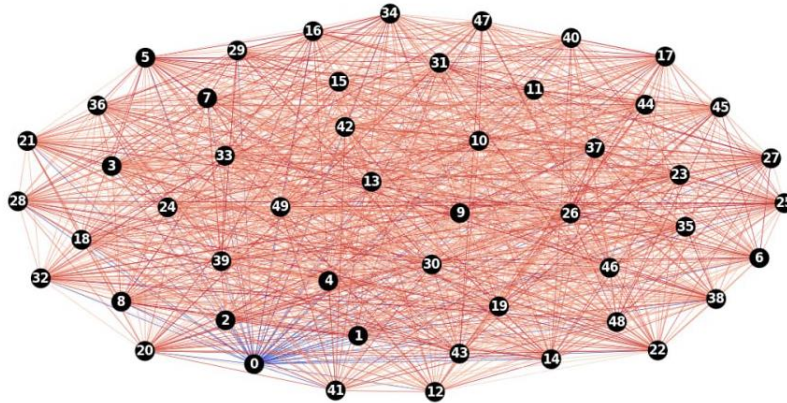


Figure 1. Complete Highly Complex Network.

The reliability values of the complete highly complex network components have been divided into four intervals due to their large number. Table 1

shows those intervals and the number of components in each interval, in addition to their percentages.

Table 1. Number of Components and its Percentage.

No.	Reliability interval	Interval center	Number of components	Percentage
1	$0.500 \leq r_{i,j} < 0.600$	0.55	301	24.6%
2	$0.600 \leq r_{i,j} < 0.700$	0.65	295	24%
3	$0.700 \leq r_{i,j} < 0.800$	0.75	323	26.4%
4	$0.800 \leq r_{i,j} \leq 0.900$	0.85	306	25%

By allocating the reliability of all components by the algorithm used with $R_{target} = 0.9$, well it was chosen $r_{i,min} = 0.5$ and $r_{i,max} = 0.9$, The resulting reliability value for the complex network was $R_S = 0.925$. The reliability values of these components can be represented by the histogram in Fig 2. Each interval has width 0.1, and each value is located in the middle of an interval.

Through the fifth column of Table 1, it can be seen that the third interval $0.700 \leq r_{i,j} < 0.800$ achieved the highest percentage of the number of components, as for the fourth interval $0.800 \leq r_{i,j} \leq 0.900$, its percentage was in the second place. The percentage of the first interval was in the third position, and the last place was the share of the second interval. Fig 3 shows the percentages of the number of components in each interval.

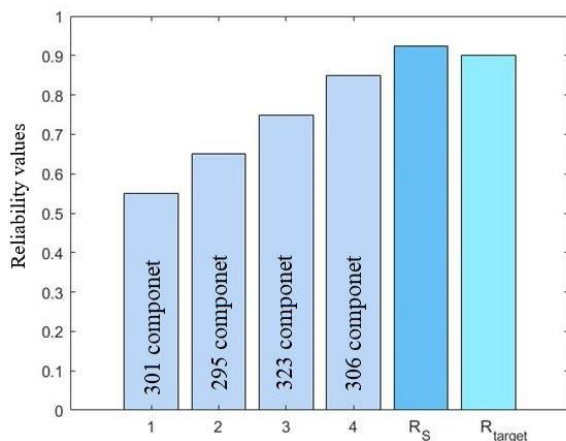


Figure 2. Reliability Values

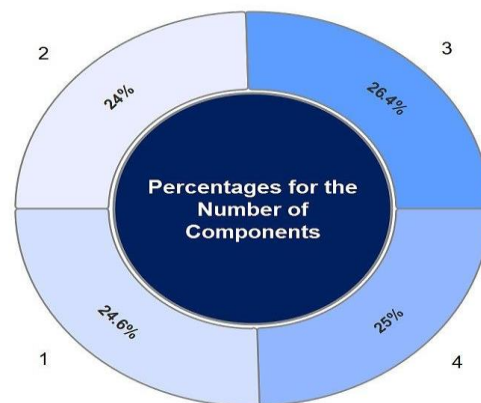


Figure 3. Percentages of the Number of Components

Additionally, a convergence plot was generated to study the convergence average of the optimization process over more than one iteration. The plot displayed the system reliability values obtained from each iteration, allowing for an assessment of the algorithm's convergence and stability. The convergence behavior shown in Fig 4 illustrates the evaluation of the effectiveness of the proposed optimization algorithm and its suitability for the given problem:

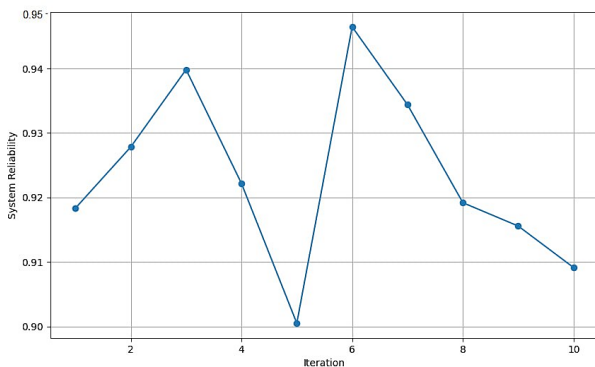


Figure 4. Reliability Rate

According to the results obtained, the proposed optimization-based method presents a new and

Conclusion

This paper presents a new method for calculating the reliability of complex systems by using an optimization techniques approach. The task of achieving the required range of system reliability was formulated as an optimization problem and through this approach an improvement in component reliability was achieved. In addition, the proposed approach demonstrated the synergy between optimization techniques and reliability finding methods, where optimization techniques were integrated into system reliability analysis. Our methodology has been proven to produce significant

effective approach in improving the reliability of systems, especially complex ones, through a significant increase in the reliability of the optimal system configuration when compared to the initial configuration. The convergence plot provided a promising evaluation of the performance of the optimization algorithm while the visualizations gave valuable insights into the system architecture. These results demonstrate the real need to include optimization methods in complex system reliability evaluations, calling for the study of a variety of applications in engineering, operations research, and system design in search of the appropriate optimization method to enhance system performance. Studying the influence of constraints, objective functions or system factors on the reliability improvement achieved using the proposed approach in future research directions is an urgent need and definitively enhances the improvement process. The results also highlight the ability of optimization-based methodology to develop reliability analysis methods for complex systems and verify their effectiveness in improving system reliability.

increases in system reliability through experimental evaluations while successfully addressing the computational difficulties generated by traditional approaches. The suggested optimization-based method offers fresh opportunities for improving dependability in complex systems and presents a promising direction for further study in the area. Finally, reliability optimization will play a crucial part in maintaining the resilience and dependability of complex systems as long as optimization algorithms and methodologies continue to progress.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been

- included with the necessary permission for republication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.

- Ethical Clearance: The project was approved by the local ethical committee at Ministry of Education, Babylon.

Authors' Contribution Statement

A.A., F.H., and Z.A. played key roles in shaping and executing the research, conducting result analysis, and participating in manuscript composition.

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تعظيم المعولية في عصر التعقيد: نهج تحسين جديد

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الخلاصة

يعد حساب معولية الأنظمة المعقدة أمراً ملحا يستحق الاهتمام نظراً لتطبيقاته الواسعة، بدءاً من العلوم الهندسية والاقتصادية إلى التطبيقات في المجالات الطبية. في الواقع، غالباً ما تعاني الطرق التقليدية من التعقيد الحسابي عند التعامل مع الأنظمة الكبيرة عند حساب معوليتها. تقدم هذه الورقة طريقة جديدة لحساب وتحسين معولية الأنظمة شديدة التعقيد. علاوة على ذلك تجمع المنهجية المقترحة بين مبادئ تحليل معولية النظام وتقنيات التحسين لتحديد الوسائل المثلى لنظام يزيد من المعولية. حيث تم توظيف نهج التحسين الرياضي ليقوم بمهمة تحسين معولية النظام دون تعقيد الحسابات. من خلال صياغة المشكلة كمهمة تحسين، يتم تحسين معولية المركبات لتلبية قيود المعولية المحددة. وتتجلى فعالية هذا النهج من خلال التقييمات التجريبية على مختلف الأنظمة المعقدة، مما يدل على تحسينات كبيرة في معولية النظام. وقد تم اختبار هذا النهج الجديد على شبكة كاملة شديدة التعقيد مكونة من 1225 مركبة، وكانت النتائج مقبولة للغاية. وأخيراً تم تطبيق الطريقة المقترحة وتم الحصول على نتائج التحسين العددية باستخدام لغة البرمجة اصدار Python 3.12.2.

الكلمات المفتاحية: الأنظمة المعقدة، معولية المكونات، النهج القائم على التحسين، تحسين المعولية، معولية النظام.