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Investigation of the Optical Properties of the Achromatic Quadrupole Lens by Using the Rectangular Potential Distribution Model

Oday A. Hussein * *Fatin A. J. Al-Moudarris** *Fatma N. Gaafer**

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Abstract

An optimization calculation is made to find the optimum properties of combined quadrupole lens which consists of electrostatic and magnetic lens. Both chromatic and spherical aberration coefficients are reduced to minimum values and the achromatic aberration is found for many cases.

These calculations are achieved with the aid of transfer matrices method and using rectangular model of field distribution, where the path of charged-particles beam traversing the field has been determined by solving the trajectory equation of motion and then the optical properties for lens have been computed with the aid of the beam trajectory along the lens axis.

The computations have been concentrated on determining the chromatic and spherical aberration coefficients in both convergence and divergence planes and the effects of changing both of excitation and effective length of lens are studied.

Key words: Combined quadrupole lens, Achromatic aberration, Rectangular model.

Introduction

The quadrupole lens systems are formed by aligned cylindrical electrodes which are cut by longitudinal slots into four equal parts to create a quadrupole lens. A combined electrostatic and magnetic quadrupole lens consists of four electrodes and four magnetic poles and.

The quadupole lens is more complex than the axial symmetrical, in construction, in calculations and in operation. As it is known, these lenses have found a wide application in accelerator techniques, such as the high- energy microprobe and the scanning ion microscope, where it is only necessarily to focus ions into a very small spot [1].

The chromatic aberrations are corrected by electric and magnetic qudrupole lenses, but quadrupole – octupole correctors are well suited for correcting chromatic and spherical aberrations [2].

Electric and magnetic lenses can be used for focusing beams of charged particles. Each of the quadrupoles focus the beam of ions in one transverse direction and defocus it in the other. The net effect on the beam after traveling through the system is focusing in all directions. Particle tracing of the ions is used to investigate the focusing effect for the quadrupole.

The symmetry plane of the electric field of such a compound quadrupole lens is placed in

*Department of Physics, College of Science, Al-Nahrain University,Baghdad, Iraq

coincidence with the plane of antisymmetry of the magnetic field. Electrostatic and magnetic quadrupole lenses have found application in the focusing of charge particle, in particular of high energy particles [3].

Electric quadrupole is assembled with the converging principle section of the electric lens coincident with diverging section of the magnetic lens. If the lens excitations are adjusted so that the magnetic force is every where twice the electric force in magnitude, the focal length is independent of particle energy [4].

An achromatic quadrupole lens is formed by the electrostatic and magnetic fields imposed on each other. The compound lens thus obtained possesses first order focusing properties of an ordinary quadrupole lens, however, depending upon the actual electrostatic and magnetic field strengths it may both be achromatic and exhibit negative chromatic aberration [3].

The achromatic quadrupole lenses were found to be an order of magnitude less sensitive to changes in the beam energy away from the operating energy compared to the equivalent magnetic or electrostatic quadrupole lenses [5].

The first order chromatic aberration can be reduced using combined electrostatic and magnetic quadrupole whose excitations are connected by the achromatic condition. This method involves magnetic elements and leads to a complicated construction of the focusing system [6].

The field distribution of a quadrupole lens in the present work may be represented by rectangular model for long narrow quadrupole lens [7]. The function f(z) for rectangular field model of axial width L is represented mathematically as follows:

$$f(z) = (f(z))_{max} = 1 \quad \text{when } -L/2 \le z \le L/2 \qquad (1)$$

At points when $|z| > L/2$ the function $f(z) = 0$.This model is also known as the square-top field distribution.

Trajectory of Charged – Particles Beam

The paraxial ray equation in Cartesian coordinates for the charged-particles beam traversing the field of a quadrupole lens is given as follows [8]:

$$x'' + \beta^2 f(z)\, x = 0 \quad \dots (2)$$

$$y'' - \beta^2 f(z)\, y = 0 \quad \dots (3)$$

where β is the lens excitation.

The general solution of the second-order linear homogeneous differential equations (2) and (3) can always be written in the following matrix form, respectively [8]:

$$\begin{pmatrix} x(z) \\ x'(z) \end{pmatrix} = T_c \begin{pmatrix} x_o(z) \\ x_o'(z) \end{pmatrix} \quad \dots (4)$$

$$\begin{pmatrix} y(z) \\ y'(z) \end{pmatrix} = T_d \begin{pmatrix} y_o(z) \\ y_o'(z) \end{pmatrix} \quad \dots (5)$$

where x_o and y_o are the initial displacements from the optical axis in the x–z and y–z planes respectively, and x_o' and y_o' are the initial gradients of the beam in the corresponding planes. And the parameter T_c and T_d are the transfer matrices in the convergence plane xoz and the divergence plane yoz respectively which are given by [11] and [12].

$$T_c = \begin{pmatrix} \cos(\beta L) & 1/\beta \sin(\beta L) \\ -\beta \sin(\beta L) & \cos(\beta L) \end{pmatrix} \ldots (6)$$

$$T_d = \begin{pmatrix} \cosh(\beta L) & 1/\beta \sinh(\beta L) \\ \beta \sinh(\beta L) & \cosh(\beta L) \end{pmatrix} \ldots (7)$$

In practice, length L is the "effective length" which has been found experimentally to be given by [8].

$$L = \ell + 1.1c \quad \ldots \quad (8)$$

where ℓ is the electrode length and c is the bore radius which is assumed to be very small. Therefore, the effective length L could be equal to the electrode length ℓ by neglecting the second term of equation (8) i.e. $L \approx \ell$ where $\theta = \beta L$ [8].

Figure (1) shows the trajectories of charged particles beam through combined quadrupole lens in both convergence and divergence planes. The charge particles in the convergence plane (x-z) in the figure deflected toward the optical axis and away from the optical axis in the divergence plane (y-z). i.e. the combined quadrupole lens is astigmatic, and the results are coincide with that published by Baranova and Yavor [13].

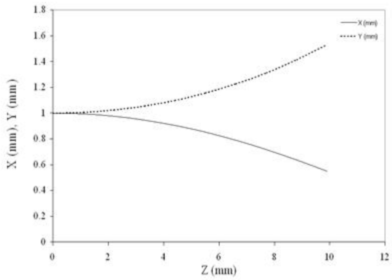


Figure (1): Trajectory of charge particles in combined quadrupole lens for both convergence (x-z) plane and divergence (y-z) plane.

3. Chromatic Aberration

The chromatic aberration coefficients, defined by [8]:

$$\Delta X(z_i) = M_c (Ccx\alpha + Cmx x_o)\frac{\Delta V}{V} \quad \ldots \quad (9)$$

$$\Delta Y(z_i) = M_d (Ccy\gamma + Cmy y_o)\frac{\Delta V}{V} \quad \ldots \quad (10)$$

where x_o and y_o are the initial displacements from optical axis in the x-z plane and y-z plane, respectively; α and γ are the image side semi-aperture angles in the x-z and y-z planes, respectively. M_c and M_d are the magnification in both convergence and divergence planes respectively and are given by:

$$M_c = 1/\cos\theta - u\,\beta\sin\theta \quad \ldots (11)$$

$$M_d = 1/\cosh\theta + u\,\beta\sinh\theta \quad \ldots (12)$$

where u is the object distance.

The coefficients of chromatic aberration in a rectangular model field are given by [8]:

$$\frac{Ccx}{L} = \frac{n-1}{4\sin(\theta)^2}[(1+\frac{\sin(2\theta)}{2\theta})(m_c^2+1) - 2(\cos(\theta) + \frac{\sin(\theta)}{\theta})m_c] \qquad (13)$$

$$\frac{Ccy}{L} = -\frac{n-1}{4\sinh(\theta)^2}[(1+\frac{\sinh(2\theta)}{2\theta})(m_d^2+1) - 2(\cosh(\theta) + \frac{\sinh(\theta)}{\theta})m_d] \quad (14)$$

$$Cmx = -\frac{n-1}{4}\frac{\theta}{\sin(\theta)}[(1+\frac{\sin(2\theta)}{2\theta})m_c - (\cos(\theta) + \frac{\sin(\theta)}{\theta})] \qquad (15)$$

$$Cmy = -\frac{n-1}{4}\frac{\theta}{\sinh(\theta)}[(1+\frac{\sinh(2\theta)}{2\theta})m_d - (\cosh(\theta) + \frac{\sinh(\theta)}{\theta})] \quad (16)$$

where

$$n = \frac{\beta e^2}{\beta^2} = \frac{\beta e^2}{\beta m^2 - \beta e^2} \quad \ldots (17)$$

and βe and βm are the excitation of electric and magnetic lenses respectively.

The optimization calculations for chromatic aberration coefficients in both convergence and divergence plane are made for two values of effective length of lens, (L1 = 0.9mm and L2 = 1mm) to produce achromatic lens.

The plot between the chromatic aberration coefficient and relative excitation parameter (n) is shown for both convergence and divergence planes in the figures (2) to (3), respectively. The relative chromatic aberration coefficient Ccx/L increases as the relative excitation parameter (n) increases and the effective length of lens L1 has values of chromatic aberration coefficient Ccx/L greater than that of L2 as is shown in figure (2).

The same behavior is found for the calculations of chromatic aberration coefficient relative to change of magnification Cmx as shown in figure (3)

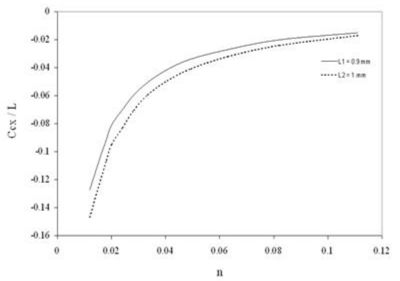


Figure (2): The relative chromatic aberration coefficients as a function of relative excitation parameter (n) for the combined quadrupole lens.

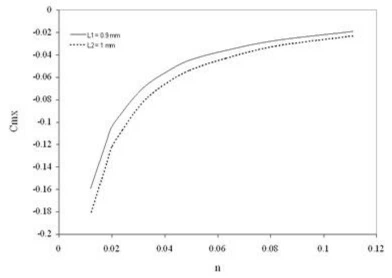


Figure (3): The relative chromatic aberration coefficients change with magnification as a function of relative excitation parameter (n) for the combined quadrupole lens.

In case of divergence plane the results are shown in figures (4) and (5). Figure (4) shows the results of chromatic aberration coefficient Ccy/L which decrease as the relative excitation parameter (n) is increasing and the values of the effective length of lens L1(0.9mm) are lower than that of L2(1mm). The same behavior of chromatic aberration coefficient change with magnification Cmy is found as shown in figure (5).

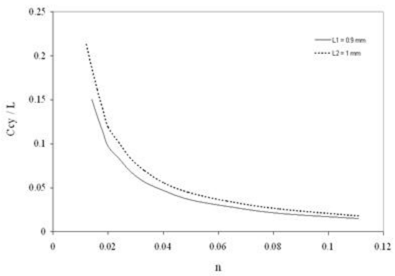


Figure (4): The relative chromatic aberration coefficients as a function of relative excitation parameter (n) for the combined quadrupole lens.

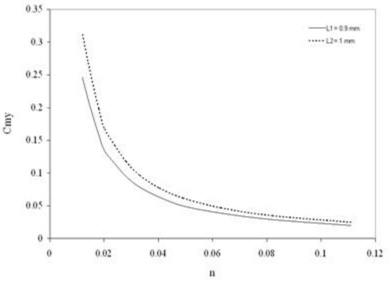


Figure (5): The relative chromatic aberration coefficients change with magnification as a function of relative excitation parameter (n) for the combined quadrupole lens.

Spherical Aberration

The spherical aberration coefficients, defined by [8]:

$$\Delta X(z_i) = M_c (C_{30}\,\alpha^3 + C_{12}\,\alpha\gamma^2) \ldots (18)$$

$$\Delta Y(z_i) = M_d (D_{03}\,\gamma^3 + D_{21}\,\alpha^2\gamma) \ldots (19)$$

The coefficients C characterize the aberration in convergence plane, and D in the divergence plane.

The coefficients of spherical aberration in a rectangular model field are given by [14]:

$$\frac{C_{30}}{L} = \frac{1}{16}[\eta^2 - (1-6\mu^2+\mu^4)(\frac{\sin 4\theta}{4\theta}) + (\frac{u}{L})\xi(1-\cos 4\theta) + \frac{1}{3}(2-2n+3n^2)[3\eta^2 - 4(1-\mu^4)(\frac{\sin 2\theta}{2\theta}) + (1-6\mu^2+\mu^4)(\frac{\sin 4\theta}{4\theta}) + 4(\frac{u}{L})\eta(1-\cos 2\theta) - (\frac{u}{L})\xi(1-\cos 4\theta)]] \ldots (20)$$

$$\frac{C_{12}}{L} = \frac{1}{16}[6\eta\xi + 8(\frac{u}{L}) - [2\xi^2 + (5\xi\eta - 4\mu^2)\cosh 2\theta](\frac{\sin 2\theta}{2\theta}) + [2\eta^2 - (\xi\eta + 20\mu^2)\cos 2\theta](\frac{\sinh 2\theta}{2\theta}) - 2\frac{u}{L}(\xi\cos 2\theta - \eta\cosh 2\theta) + \frac{u}{L}(\eta - 5\xi)\sin 2\theta\sinh 2\theta - \frac{u}{L}(5\eta + \xi)\cos 2\theta\cosh 2\theta + (2+2n-n^2)[-2\eta\xi - 2\frac{u}{L} + [2\xi^2 - (\eta\xi - 4\mu^2)\cosh 2\theta](\frac{\sin 2\theta}{2\theta}) + [2\eta^2 - (\eta\xi + 4\mu^2)\cos 2\theta](\frac{\sinh 2\theta}{2\theta}) + 2\frac{u}{L}(\xi\cos 2\theta + \eta\cosh 2\theta) + \frac{u}{L}(\eta - \xi)\sin 2\theta\sinh 2\theta - \frac{u}{L}(\eta + \xi)\cos 2\theta\cosh 2\theta]] \ldots (21)$$

where $\mu = \beta u$, $\eta = 1 + \mu^2$, and $\xi = 1 - \mu^2$

The coefficients D_{03} and D_{21} are derived from equations (19) and (20) by replacing β by iβ.

The optimization calculations for spherical aberration coefficients in both convergence and divergence plane are made to show the effect of changing the excitation parameter and the effective lengths of lens on the spherical aberration coefficients.

The relation between spherical aberration coefficients and relative excitation parameter (n) is shown for both convergence and divergence planes in figures (6) to (9). The figures (6) and (7), are explaining the spherical aberration coefficients as the function of relative excitation parameter (n) for the case convergence plane. In figure (6) the spherical aberration coefficient C_{30}/L decrease as n is increasing and the effective length L1(0.9mm) has the values of spherical aberration coefficient lower than that of L2(1mm).

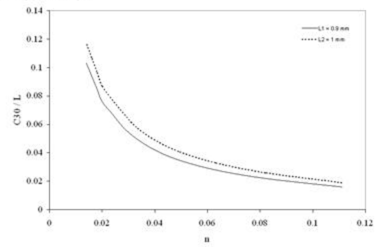


Figure (6): The relative spherical aberration coefficients C_{30}/L as a function of relative excitation parameter (n) for the combined quadrupole lens.

From figure (7) the spherical aberration coefficient C_{12}/L has inverse relation to the relative excitation parameter (n) and the calculations show the slightly difference between two effective lengths of lens and nearly the n = 0.063 both effective lengths have minimum spherical aberration coefficient.

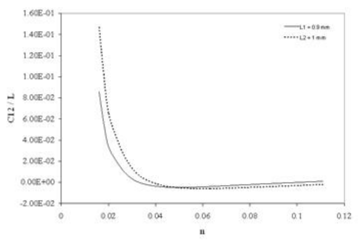
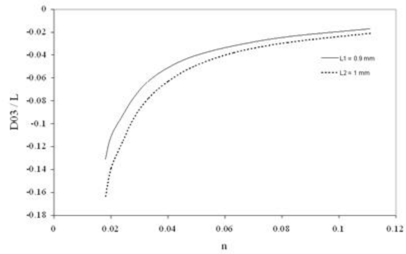


Figure (7): The relative spherical aberration coefficients C_{12}/L as a function of relative excitation parameter (n) for the combined quadrupole lens.

The calculations of divergence case are shown in figures (8) and (9). The figure (8) the relation between spherical aberration coefficient D_{30}/L and relative excitation parameter (n), this coefficient increases as n increases and the effective length L2(1mm) give us the best result with respect to another effective length L1(0.9mm).

The relation between spherical aberration coefficient D_{21}/L and relative excitation parameter (n) is shown in figure (9), this relation is found to be inversely and the effective length of lens L1 give us the optimum results with respect to another effective length of lens L2.



Figure(8): The relative spherical aberration coefficients D_{30}/L as a function of relative excitation parameter (n) for the combined quadrupole lens.

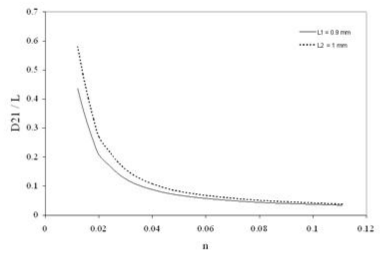


Figure (9): The relative spherical aberration coefficients D_{21}/L as a function of relative excitation parameter (n) for the combined quadrupole lens.

Conclusions

1. The chromatic aberration coefficients of the rectangular model for the effective length of lens L1(0.9mm) is lower than that of the effective length of lens L2(1mm). In the case of spherical aberration opposite behavior is found.
2. From the calculations it is found that the chromatic aberration coefficients can be reduced to zero value but the problems are in the values of the focal lengths are very large and at the same time the effective length and excitation of the lens are very small and not satisfy the optimum design.

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دراسة الخصائص البصرية لعدسة رباعية الاقطاب عديمة الزيغ اللوني باستخدام الأنموذج المستطيلي لتوزيع الجهد

*عدي علي حسين * فاتن عبد الجليل المدرس* فاطمة نافع جعفر*

*قسم الفيزياء، كلية العلوم، جامعة النهرين، بغداد، العراق

كلمات مفتاحية: عدسة رباعية مركبة، الزيغ اللالوني، الانموذج المستطيلي

الخلاصة:

حسابات الامثلية أجريت لايجاد افضل الخواص لعدسة رباعية مركبة مكونة من عدسة كهروسكونية ومغناطيسية. كل من الزيغين اللوني والكروي أختزلا لاقل قيم والزيغ اللالوني وجد لحالات عدة.
الحسابات أنجزت بالاستعانة بطريقة المصفوفات الانتقالية وباستخدام الانموذج المستطيلي لشكل توزيع المجال، حيث حسب مسار الجسيمات المشحونة التي تعبر المجال بحل معادلة مسار الحركة ثم حسبت الخواص البصرية للعدسة بمساعدة مسار الحزمة على امتداد محور العدسة.
الحسابات ركزت على ايجاد البعد البؤري والتكبير والزيغين اللوني والكروي لكلا المستويين التقاربي والتباعدي، وكذلك تم داسة تأثيرات تغير التهييج والطول الفعال للعدسة.