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A Stage Structure Prey Predator Model Using Pentagonal Fuzzy Numbers and Functional Response

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Abstract

In the present study, our work is focused on a prey predator model with a stage structure for the prey. The objective of the study is to find the behavior of the model using parameter values in the presence of Pentagonal fuzzy numbers. The interaction between the species is done by using functional responses, such as the Holling type I reaction for immature prey and the Crowley Martin functional response for mature prey. Prey population categorized as immature and mature prey. The idea of the problem is to construct a fuzzy theoretical method which helps us to create a model to solve this uncertainty problem by using fuzzy parameters and initial values. The mathematical formulation is developed by using a prey predator model in a fuzzy environment. The existence of equilibrium points is carried out. By using the concept of alpha cut the parameters which are used in the prey – predator model is treated as pentagonal fuzzy numbers. The parameters which they have used in the mathematical formulation can be taken as a crisp value by applying the defuzzification method. Here a Robust ranking technique method is utilized. The stability of each equilibrium point is studied by computing the Jacobian matrix and finding the eigenvalues evaluated at each equilibrium point. By utilizing the prey stage structure stability analysis is also discovered. For the dynamical system, numerical simulations are provided using a MATLAB program and also, they show the behavior of the system and determine if it is stable or unstable.

Keywords: Equilibrium Points, Functional Response, Fuzzy Numbers, Immature Prey, Mature Prey, Prey Predator.

Introduction

In mathematical ecology, a Prey-Predator model is essential tool, especially for understanding how populations interact in the real world. A real-world problem is transformed into a mathematical form through the process of mathematical modeling. Modeling[1-4] entails creating circumstances from real life or transforming difficulties from mathematical explanations into situations that are credible or realistic. A prey-predator interaction model among immature prey, mature prey, and predator has been developed. A functional response has been utilized for both mature and immature prey. Holling[5] proposed a functional response model, which is still used by ecologists. This model is sometimes called the "disc equation" because Holling replicated the area that predators would inspect using paper discs.

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The model is also mathematically equivalent to the enzyme kinetics model created by Lenor Michalis and Maude Amenten in 1913. In his analysis[6], Holling divides these impacts into two categories: the functional response and the numerical response. One of the common ways to establish dynamic relationships between predators and their prey is by using a mathematical ecology model. In 1965, L.A. Zadeh[7], proposed the fuzzy set theory. The fuzzy theory[8] can be applied in a variety of fields where the information is ambiguous or imperfect. To eliminate the ambiguity of the current problems, many forms of fuzzy sets are defined. The idea of fuzzy numbers is a broadening of the idea of real numbers. Dubois and Prade[9] define fuzzy numbers as a fuzzy subset of the real line. So far, many researchers have used triangular fuzzy numbers, trapezoidal fuzzy numbers, Pentagonal fuzzy numbers[10-12], and hexagonal, octagonal, and pyramid fuzzy numbers. Also, authors[13] investigated the prey-predator model under the fluctuation rescue effect. In the present paper, they utilize Pentagonal fuzzy numbers for the parameters that will be proposed in the Mathematical formulation in the presence of functional responses. Also, apply the concept of the alpha cut to the parameters that are used in the mathematical model. A defuzzification process can be done by using an alpha-cut notation. For the defuzzification process, they approach a robust ranking technique method[14-17] which helps us to find the stability of the stage structure prey-predator model. Many researchers[18-22] have done their research based on Prey – Predator model. But only a few authors[23,24] can apply the concept of fuzzy. So far, they have discussed the concepts of fuzzy uncertainties, fuzzy parameters[25], fuzzy ratios, and fuzzy initial conditions. But here our work is focused on a new concept of how a pentagonal fuzzy number is applied in the prey-predator model by using alpha cut notation and robust ranking techniques. The benefits of the results are obtained by using the Routh Hurwitz Stability Criteria Condition and the defuzzification method. Without a fuzzifier, parameters are not able to achieve stability. Since they applied a mathematical program for the numerical analysis, the parameters are supposed to be fuzzified. The fuzzy parameter cannot be used as it is because it is an uncertain number.

The following is the structure of the paper: Basic preliminaries are carried out in section 2. The mathematical formulation for the prey-predator model using fuzzy logic is described in section 3. In section 4, equilibrium points are carried out. In section 5, the defuzzification method is provided. In section 6, stability analysis is presented and in section 7 results and discussion are shown. Finally, it ends with a conclusion.

Preliminaries

Some of the basic concepts used in this paper such as fuzzy numbers, the α – level of fuzzy numbers, and Pentagonal fuzzy numbers will be introduced in this section.

Fuzzy set[7] – A collection of elements with a membership function in the range [0,1] that enables its members to have various levels of membership. The degree to which an element belongs to a specific set is indicated by a numerical number between 0 and 1, also known as the membership value.

Normal and Convex Fuzzy Set[8] - The complete ordering of a fuzzy set based on a universe of discourse has a height (maximum membership value of 1), and any elements between two arbitrary boundary elements have membership grades larger than or equal to the smaller membership grades of the two arbitrary boundary elements.

Fuzzy Number[7] - "A fuzzy set defining a fuzzy interval in the real number $\mathfrak{R}$". It may also meet the following characteristics. Fuzzy sets that are normalized and convex. It has a piecewise continuous membership function and is also defined as a real number.

Pentagonal Fuzzy Number[10] – Let be $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ is called a pentagonal fuzzy number if its membership function is as,

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$$\mu_{\tilde{B}}(x)=\begin{cases}0, & if \quad x<b_1, x\geq b_5\\ \frac{x-b_1}{b_2-b_1}, & if \quad b_1\leq x\leq b_2\\ 1, & if \quad x=b_3\\ \frac{x-b_2}{b_3-b_2}, & if \quad b_2\leq x\leq b_3\\ \frac{b_4-x}{b_4-b_3}, & if \quad b_3\leq x\leq b_4\\ \frac{b_5-x}{b_5-b_4}, & if \quad b_5\leq x\leq b_4\end{cases}$$

For the PFN$\tilde{B}=(b_1,b_2,b_3,b_4,b_5)$, $b_3$is the mid value, (b_1,b_2) and (b_4,b_5) are the left and right-side values respectively.

Operation on PFN[9] - If $\tilde{B}=(b_1,b_2,b_3,b_4,b_5)$ and $\tilde{C}=(c_1,c_2,c_3,c_4,c_5)$ be two PFNs then it

$$(i)\tilde{B}\oplus\tilde{C}=(b_1+c_1,b_2+c_2,b_3+c_3,b_4+c_4,b_5+c_5)$$

$$(ii)\tilde{B}\ominus\tilde{C}=(b_1-c_1,b_2-c_2,b_3-c_3,b_4-c_4,b_5-c_5)$$

(iii)Scalar Multiplication: Let $\tilde{B}=(b_1,b_2,b_3,b_4,b_5)$ be a PFN and k be any scalar then

$k\tilde{B}=(kb_1,kb_2,kb_3,kb_4,kb_5)\ if\ k\geq 0$ and $k\tilde{B}=(kb_5,kb_4,kb_3,kb_2,kb_1) if\ k\leq 0$

Alpha cut for Pentagonal Fuzzy Numbers[14]– "Let $\tilde{B}_\alpha\in F(X)$ and $\alpha\in[0,1]$ then the set $\tilde{B}_\alpha=\{x\in X/\mu_{\tilde{B}_\alpha}(x)\geq\alpha\}$ is called the α - cut set". For a PFN, let us take

$\tilde{B}_\alpha=(b_1,b_2,b_3,b_4,b_5)$ then $\tilde{B}_\alpha=\begin{cases}U_1(\alpha),U_2(\alpha)\ for\ \alpha\in[0,0.5)\\ V_1(\alpha),V_2(\alpha)\ for\ \alpha\in(0.5,1]\end{cases}$ where

$U_1(\alpha)=2\alpha(b_2-b_1)+a_1\ ; U_2(\alpha)=-2\alpha(b_5-b_4)+b_5\ ; V_1(\alpha)=2\alpha(b_3-b_2)-b_3+2b_2\ ; V_2(\alpha)=2\alpha(b_4-b_3)+2b_3-b_4$

$$\tilde{B}_\alpha=\begin{cases}[2\alpha(b_2-b_1)+b_1\ ,-2\alpha(b_5-b_4)+b_5]\ for\ \alpha\in[0,0.5)\\ [2\alpha(b_3-b_2)-b_3+2b_2\ ,2\alpha(b_4-b_3)+2b_3-b_4]\ for\ \alpha\in(0.5,1]\end{cases}$$

Model Construction for Prey – Predator Model

The biological system can be framed by the presence of Functional response and stage structure in the dynamics of the fuzzy prey-predator model. Here, the predator feeds on each unique prey by utilizing several functional responses. The k is the carrying capacity where the immature evolves. The functional response describes a typical predator interaction with prey. Here the immature prey grows alone without mature prey due to the logistic growth rate and carrying capacity. Also, the immature prey can be mating with predators alone. Here the biomass conversion from immature prey goes to mature prey due to its growth rate of maturity.

Dian Savitri and Abadi[18] proposed a two-species prey-predator model with Holling type II functional response in crisp form. But here our work is focused on a model for a prey-predator with Crowley martin functional response in fuzzy form. Table 1 shows the parameter representation which is available in Eq 1. Also, by applying a concept of pentagonal fuzzy numbers for the parameters that are present in the fuzzy prey-predator model. The formulation for the immature prey, mature prey, and predator is given by,

$$\frac{d\tilde{x}}{dt}=\tilde{r}x\left(1-\frac{x}{k}\right)-\tilde{\beta}x-\tilde{\alpha}xz$$

$$\frac{d\tilde{y}}{dt}=\tilde{\beta}x-\frac{\tilde{\varepsilon}yz}{(1+\tilde{\sigma}x)(1+\tilde{\gamma}y)}-\tilde{\mu}_1 y$$

$$\frac{d\tilde{z}}{dt}=\frac{\tilde{\gamma}\tilde{\varepsilon}yz}{(1+\tilde{\sigma}x)(1+\tilde{\gamma}y)}-\tilde{\mu}_2 z \qquad 1$$

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Table 1. Parameters Notation

Parameters	Notation
$\boldsymbol{x}$	Immature prey density
$\boldsymbol{y}$	Mature prey density
$\boldsymbol{z}$	Predator density
$\dfrac{\tilde{\varepsilon}yz}{(1+\tilde{\sigma}x)(1+\tilde{\nu}y)}$	The functional response of an adult prey to a predator
$\tilde{\alpha}xz$	The functional response of a juvenile prey to a predator
$\tilde{\beta}$	Rate of maturity
$\tilde{r}$	The rate of intrinsic growth of immature prey
$\tilde{\mu}_1$	Death rates among mature prey
$\tilde{\mu}_2$	Predator death rate
$\tilde{\sigma},\tilde{\nu}$	Handling time
$\tilde{\varepsilon}$	Effect of capture level
$\tilde{\gamma}$	The growth rate of predator
$\boldsymbol{k}$	Carrying capacity

Existence of Equilibrium Points

By using the definition of the equilibrium point, the system of equations Eq 1 is satisfied when,

$$\frac{d\tilde{x}}{dt}=\frac{d\tilde{y}}{dt}=\frac{d\tilde{z}}{dt}=0$$

Below is a list of the equilibrium points:

(i) $E^0(0,0,0)$ exists

(ii) A point $E^1\left(k\left(1-\frac{\tilde{\beta}}{\tilde{r}}\right),0,0\right)$ exists

Consider,

$$\tilde{r}x\left(1-\frac{x}{k}\right)-\tilde{\beta}x-\tilde{\alpha}xz=0$$

$$x\left[\tilde{r}\left(1-\frac{x}{k}\right)-\tilde{\beta}-\tilde{\alpha}z\right]=0$$

Since $z=0; x\neq 0$

$$\tilde{r}\left(1-\frac{x}{k}\right)-\tilde{\beta}=0$$

$$1-\frac{x}{k}=\frac{\tilde{\beta}}{\tilde{r}}$$

$$1-\frac{\tilde{\beta}}{\tilde{r}}=\frac{x}{k}$$

$$x^*=k(1-\frac{\tilde{\beta}}{\tilde{r}})$$

(iii) The equilibrium of immature prey extinction

$$E^2\left(0,\frac{\tilde{\mu}_2(1+\tilde{\nu}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y)}{\tilde{\gamma}\tilde{\varepsilon}},-\frac{\tilde{\mu}_1(1+\tilde{\nu}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y)}{\tilde{\varepsilon}}\right)$$

Consider,

$$\frac{\tilde{\gamma}\tilde{\varepsilon}yz}{(1+\tilde{\sigma}x)(1+\tilde{\nu}y)}-\tilde{\mu}_2z=0$$

$$z\left[\frac{\tilde{\gamma}\tilde{\varepsilon}y}{(1+\tilde{\sigma}x)(1+\tilde{\nu}y)}-\tilde{\mu}_2\right]=0$$

$$z\neq 0;\frac{\tilde{\gamma}\tilde{\varepsilon}y}{(1+\tilde{\sigma}x)(1+\tilde{\nu}y)}=\mu_2$$

$$\tilde{\gamma}\tilde{\varepsilon}y=\tilde{\mu}_2[1+\tilde{\nu}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y]$$

$$y^*=\frac{\tilde{\mu}_2[1+\tilde{\nu}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y]}{\tilde{\gamma}\tilde{\varepsilon}}$$

Again Consider,

$$\tilde{\beta}x-\frac{\tilde{\varepsilon}yz}{(1+\tilde{\sigma}x)(1+\tilde{\nu}y)}-\tilde{\mu}_1y=0$$

$$Since\ x=0$$

$$y\left[\frac{-\tilde{\varepsilon}z}{(1+\tilde{\sigma}x)(1+\tilde{\nu}y)}-\tilde{\mu}_1\right]=0$$

$$y\neq 0;\ \frac{-\tilde{\varepsilon}z}{(1+\tilde{\sigma}x)(1+\tilde{\nu}y)}=\tilde{\mu}_1$$

$$z^*=\frac{-\tilde{\mu}_1(1+\tilde{\sigma}x+\tilde{\nu}y+\tilde{\sigma}x\tilde{\nu}y)}{\tilde{\varepsilon}}$$

The mature prey comes from immature prey. The immature prey cannot survive alone for all time it will come to maturity at some certain stage.

(iv) The equilibrium of predator extinction $E^3\left(k\left(1-\frac{\tilde{\beta}}{\tilde{r}}\right),\frac{\tilde{\beta}x^*}{\tilde{\mu}_1},0\right)$

(v) $E^4(0,y^*,0)\Longrightarrow E^4(0,0,0)$ Since in the absence of immature prey, maturity doesn't happen which implies that there will be no mature prey

(vi) Interior equilibrium point $E^5(x^*,y^*,z^*)$. This point will obtained by solving very difficult. From the biological point of view, this equilibrium plays a major role, and one of the interesting analyses in

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mathematical biology is that in this case all of the three populations will coexist in the ecosystem.

Defuzzification Method[16]

Finding the singleton value (crisp value), which is the average value of the pentagonal fuzzy numbers, is the process of defuzzification. Due to its simplicity and accuracy, Robust's Ranking approach is utilized in this case to fuzzily pentagonal fuzzy numbers.

Robust Ranking Technique Method[15]

If $\tilde{a}$ is a fuzzy number, then the ranking method is given by $R(\tilde{a}) = \int_0^1 0.5(b_\alpha^L, b_\alpha^R)\, d\alpha$ where

$(b_\alpha^L, b_\alpha^R) = \{[2\alpha(b_2 - b_1) + b_1, -2\alpha(b_5 - b_4) + b_5], [2\alpha(b_3 - b_2) - b_3 + 2b_2, 2\alpha(b_4 - b_3) + 2b_3 - b_4]\}$ the $\alpha -$cut off the fuzzy numbers $\tilde{b}$. "For the parameters which are used in the system of Eq 1 using alpha cut notation are represented as",

$$\tilde{r} = \begin{pmatrix} 2r_L(b_2 - b_1) + b_1, -2r_R(b_5 - b_4) + b_5\ \forall r\epsilon[0,0.5) \\ 2r_L(b_3 - b_2) - b_3 + 2b_2, 2r_R(b_4 - b_3) + 2b_3\ \forall r\epsilon(0.5,1] \end{pmatrix}$$

$$\tilde{\beta} = \begin{pmatrix} 2\beta_L(b_2 - b_1) + b_1, -2\beta_R(b_5 - b_4) + b_5\ \forall \beta\epsilon[0,0.5) \\ 2\beta_L(b_3 - b_2) - b_3 + 2b_2, 2\beta_R(b_4 - b_3) + 2b_3\ \forall \beta\epsilon(0.5,1] \end{pmatrix}$$

$$\tilde{\varepsilon} = \begin{pmatrix} 2\varepsilon_L(b_2 - b_1) + b_1, -2\varepsilon_R(b_5 - b_4) + b_5\ \forall \varepsilon\epsilon[0,0.5) \\ 2\varepsilon_L(b_3 - b_2) - b_3 + 2b_2, 2\varepsilon_R(b_4 - b_3) + 2b_3\ \forall \varepsilon\epsilon(0.5,1] \end{pmatrix}$$

$$\tilde{\sigma} = \begin{pmatrix} 2\sigma_L(b_2 - b_1) + b_1, -2\sigma_R(b_5 - b_4) + b_5\ \forall \sigma\epsilon[0,0.5) \\ 2\sigma_L(b_3 - b_2) - b_3 + 2b_2, 2\sigma_R(b_4 - b_3) + 2b_3\ \forall \sigma\epsilon(0.5,1] \end{pmatrix}$$

$$\tilde{\nu} = \begin{pmatrix} 2\nu_L(b_2 - b_1) + b_1, -2\nu_R(b_5 - b_4) + b_5\ \forall \nu\epsilon[0,0.5) \\ 2\nu_L(b_3 - b_2) - b_3 + 2b_2, 2\nu_R(b_4 - b_3) + 2b_3\ \forall \nu\epsilon(0.5,1] \end{pmatrix}$$

$$\tilde{\gamma} = \begin{pmatrix} 2\gamma_L(b_2 - b_1) + b_1, -2\gamma_R(b_5 - b_4) + b_5\ \forall \gamma\epsilon[0,0.5) \\ 2\gamma_L(b_3 - b_2) - b_3 + 2b_2, 2\gamma_R(b_4 - b_3) + 2b_3\ \forall \gamma\epsilon(0.5,1] \end{pmatrix}$$

$$\widetilde{\mu_2} = \begin{pmatrix} 2\mu_{2L}(b_2 - b_1) + b_1, -2\mu_{2R}(b_5 - b_4) + b_5\ \forall \mu_2\epsilon[0,0.5) \\ 2\mu_{2L}(b_3 - b_2) - b_3 + 2b_2, 2\mu_{1R}(b_4 - b_3) + 2b_3\ \forall \mu_2\epsilon(0.5,1] \end{pmatrix}$$

$$\widetilde{\mu_1} = \begin{pmatrix} 2\mu_{1L}(b_2 - b_1) + b_1, -2\mu_{1R}(b_5 - b_4) + b_5\ \forall \mu_1\epsilon[0,0.5) \\ 2\mu_{1L}(b_3 - b_2) - b_3 + 2b_2, 2\mu_{1R}(b_4 - b_3) + 2b_3\ \forall \mu_1\epsilon(0.5,1] \end{pmatrix}$$

Analysis of System Stability

The stability for the nonlinear system of equations Eq 1 can be obtained by linearizing the system Eq 1 with the Jacobian matrix given by

$$J[x,y,z] = \begin{bmatrix} \tilde{r}\left(1 - \frac{2x}{k}\right) - \tilde{\beta} - \tilde{\alpha}z & 0 & -\tilde{\alpha}x \\ \tilde{\beta} + \frac{\tilde{\varepsilon}\tilde{\sigma}yz(1+\tilde{\nu}y)}{(1+\tilde{\nu}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y)^2} & \frac{-\tilde{\varepsilon}z(1+\tilde{\sigma}x)}{(1+\tilde{\nu}x+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y)^2} - \tilde{\mu}_1 & \frac{-\tilde{\varepsilon}y}{1+\tilde{\nu}x+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y} \\ \frac{-\tilde{\gamma}\tilde{\varepsilon}yz\tilde{\sigma}}{(1+\tilde{\nu}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y)^2} & \frac{\tilde{\gamma}\tilde{\varepsilon}z(1+\tilde{\sigma}x)}{(1+\tilde{\nu}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y)^2} & \frac{\tilde{\gamma}\tilde{\varepsilon}y}{(1+\tilde{\nu}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{\nu}y)} - \tilde{\mu}_2 \end{bmatrix}$$

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At the point $E^0(0,0,0)$ in Eq 2, $\Longrightarrow J[E^0] = \begin{pmatrix} \tilde{r}-\tilde{\beta} & 0 & 0 \\ \tilde{\beta} & -\tilde{\mu}_1 & 0 \\ 0 & 0 & -\tilde{\mu}_2 \end{pmatrix}$

The matrix shown above's characteristic equation is given as

$$J-\lambda I = \begin{pmatrix} \tilde{r}-\tilde{\beta}-\lambda & 0 & 0 \\ \tilde{\beta} & -\tilde{\mu}_1-\lambda & 0 \\ 0 & 0 & -\tilde{\mu}_2-\lambda \end{pmatrix}$$

$$(\tilde{r}-\tilde{\beta}-\lambda)(-\tilde{\mu}_1-\lambda)(-\tilde{\mu}_2-\lambda)=0$$

$$\lambda_1 = \tilde{r}-\tilde{\beta};\ \lambda_2 = -\tilde{\mu}_1;\ \lambda_3 = -\tilde{\mu}_2$$

The Eigenvalues are distinct. λ_2 and λ_3 are having negative values. Therefore E^0 is said to be locally stable only if $\tilde{r} < \tilde{\beta}$ and unstable if $\tilde{r} > \tilde{\beta}$

At the point $E^1(x^*,0,0) \Longrightarrow E^1\left(k\left(1-\frac{\tilde{\beta}}{\tilde{r}}\right),0,0\right)$ in Eq.2,

$$\Rightarrow J-\lambda I = \begin{bmatrix} -\tilde{r}+\tilde{\beta}-\lambda & 0 & -\tilde{\alpha}k\left(1-\frac{\tilde{\beta}}{\tilde{r}}\right) \\ \tilde{\beta} & -\tilde{\mu}_1-\lambda & 0 \\ 0 & 0 & -\tilde{\mu}_2-\lambda \end{bmatrix}$$

From the above matrix, the results are, $\lambda_1 = -(\tilde{r}-\tilde{\beta}); \lambda_2 = -\tilde{\mu}_1; \lambda_3 = -\tilde{\mu}_2$. Since the eigenvalues are real distinct and negative then the equilibrium point is a stable node if $\tilde{r} > \tilde{\beta}$ then the system is asymptotically stable.

At the point $E^2(0,y^*,z^*) \Rightarrow E^2\left(0,\frac{\tilde{\mu}_2(1+\tilde{v}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{v}y)}{\tilde{\gamma}\tilde{\varepsilon}},-\frac{\tilde{\mu}_1(1+\tilde{v}y+\tilde{\sigma}x+\tilde{\sigma}x\tilde{v}y)}{\tilde{\varepsilon}}\right)$ in Eq 2,

$$\Longrightarrow J[E^2] = \begin{bmatrix} \tilde{r}-\tilde{\beta}-\tilde{\alpha}z^* & 0 & 0 \\ \tilde{\beta}+\frac{\tilde{\varepsilon}y^*z^*\tilde{\sigma}}{1+\tilde{v}y^*} & -\frac{\tilde{\varepsilon}z^*}{(1+\tilde{v}y^*)^2}-\tilde{\mu}_1 & \frac{-\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)} \\ \frac{-\tilde{\gamma}\tilde{\varepsilon}y^*z^*\tilde{\sigma}}{1+\tilde{v}y^*} & \frac{\tilde{\gamma}\tilde{\varepsilon}z^*}{(1+\tilde{v}y^*)^2} & \frac{\tilde{\gamma}\tilde{\varepsilon}y^*}{1+\tilde{v}y^*}-\tilde{\mu}_2 \end{bmatrix}$$

The matrix shown above's characteristic equation is given as

$$J-\lambda I = \begin{pmatrix} \tilde{r}-\tilde{\beta}-\tilde{\alpha}z^*-\lambda & 0 & 0 \\ \tilde{\beta}+\frac{\tilde{\varepsilon}y^*z^*\tilde{\sigma}}{1+\tilde{v}y^*} & -\frac{\tilde{\varepsilon}z^*}{(1+\tilde{v}y^*)^2}-\tilde{\mu}_1-\lambda & \frac{-\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)} \\ \frac{-\tilde{\gamma}\tilde{\varepsilon}y^*z^*\tilde{\sigma}}{1+\tilde{v}y^*} & \frac{\tilde{\gamma}\tilde{\varepsilon}z^*}{(1+\tilde{v}y^*)^2} & \frac{\tilde{\gamma}\tilde{\varepsilon}y^*}{1+\tilde{v}y^*}-\tilde{\mu}_2-\lambda \end{pmatrix}$$

From the above matrix, the eigenvalues cannot be obtained directly. So, the characteristic equation will arrive in a cubic form.

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$$-\lambda^3+\lambda^2\left\{\tilde{r}-\tilde{\beta}+\frac{\tilde{\varepsilon}z^*}{(1+\tilde{v}y^*)}-\tilde{\mu}_1+\frac{\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)}-\tilde{\mu}_2-\tilde{\alpha}z^*\right\}+\lambda\left\{\frac{\tilde{r}\tilde{\varepsilon}z^*}{(1+\tilde{v}y^*)^2}+\tilde{r}\tilde{\mu}_1-\frac{\tilde{r}\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)}+\tilde{r}\tilde{\mu}_2-\frac{\tilde{\beta}\tilde{\varepsilon}z^*}{(1+\tilde{v}y^*)^2}-\tilde{\beta}\tilde{\mu}_1+\frac{\tilde{\beta}\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)}-\tilde{\beta}\tilde{\mu}_2-\frac{\tilde{\alpha}\tilde{\varepsilon}z^{*2}}{(1+\tilde{v}y^*)^2}-\frac{\tilde{\varepsilon}z^*\tilde{\mu}_2}{(1+\tilde{v}y^*)^2}+\frac{\tilde{\mu}_1\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)}-\tilde{\mu}_1\tilde{\mu}_2-\tilde{\alpha}z^*\tilde{\mu}_1+\frac{\tilde{\alpha}z^*\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)}-\tilde{\alpha}z^*\tilde{\mu}_2+\tilde{\gamma}\tilde{\varepsilon}^2z^*y^*-\frac{\tilde{\gamma}\tilde{\varepsilon}^2y^*z^*}{(1+\tilde{v}y^*)^3}\right\}+\left\{-\tilde{r}\tilde{\gamma}\tilde{\varepsilon}^2z^*y^*+\frac{\tilde{r}\tilde{\varepsilon}z^*\tilde{\mu}_2}{(1+\tilde{v}y^*)^2}-\frac{\tilde{r}\tilde{\mu}_1\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)}+\tilde{r}\tilde{\mu}_1\tilde{\mu}_2+\frac{\tilde{r}\tilde{\gamma}\tilde{\varepsilon}^2y^*z^*}{(1+\tilde{v}y^*)^3}+\tilde{\beta}\tilde{\gamma}\tilde{\varepsilon}^2z^*y^*-\frac{\tilde{\beta}\tilde{\varepsilon}z^*\tilde{\mu}_2}{(1+\tilde{v}y^*)^2}+\frac{\tilde{\beta}\tilde{\mu}_1\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)}-\tilde{\beta}\tilde{\mu}_1\tilde{\mu}_2-\frac{\tilde{\beta}\tilde{\gamma}\tilde{\varepsilon}^2y^*z^*}{(1+\tilde{v}y^*)^3}+\tilde{\alpha}\tilde{\gamma}\tilde{\varepsilon}^2z^{*2}y^*-\frac{\tilde{\varepsilon}\tilde{\alpha}z^{*2}\tilde{\mu}_2}{(1+\tilde{v}y^*)^2}+\frac{\tilde{\alpha}z^*\tilde{\mu}_1\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*)}-\tilde{\alpha}z^*\tilde{\mu}_1\tilde{\mu}_2-\frac{\tilde{\alpha}\tilde{\gamma}\tilde{\varepsilon}^2y^*z^{*2}}{(1+\tilde{v}y^*)^3}\right\}=0$$

The above equation of the form $\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0$. By using Routh – Hurwitz stability Criteria namely $a_1>0, a_3>0$ and $a_1a_2-a_3>0$ are satisfied. Therefore, the solution of the system attains an equilibrium point E^2 as time goes on. Hence the equilibrium point becomes locally asymptotically stable.

At $E^3(x^*,y^*,0)\Rightarrow E^3\left(k\left(1-\frac{\tilde{\beta}}{\tilde{r}}\right),\frac{\tilde{\beta}x^*}{\tilde{\mu}_1},0\right)$ in Eq 2,

$$\Rightarrow J[E^3]=\begin{pmatrix}-\tilde{r}+\tilde{\beta} & 0 & -\tilde{\alpha}k\left(1-\frac{\tilde{\beta}}{\tilde{r}}\right)\\ \tilde{\beta} & -\tilde{\mu}_1 & \frac{-\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)}\\ 0 & 0 & \frac{\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)}-\tilde{\mu}_2\end{pmatrix}$$

The matrix shown above’s characteristic equation is given as

$$J-\lambda I=\begin{pmatrix}-\tilde{r}+\tilde{\beta}-\lambda & 0 & -\tilde{\alpha}k\left(1-\frac{\tilde{\beta}}{\tilde{r}}\right)\\ \tilde{\beta} & -\tilde{\mu}_1-\lambda & \frac{-\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)}\\ 0 & 0 & \frac{\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)}-\tilde{\mu}_2-\lambda\end{pmatrix}$$

From the above matrix,

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$$-\lambda^3 + \lambda^2\left[\frac{\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)} - \tilde{\mu}_2 - \tilde{\mu}_1 + \tilde{\beta} - \tilde{r}\right] + \lambda\left[-r\tilde{\mu}_1 + \frac{\tilde{r}\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)} - \tilde{r}\tilde{\mu}_2 + \tilde{\beta}\tilde{\mu}_1 - \frac{\tilde{\beta}\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)} + \tilde{\beta}\tilde{\mu}_2 + \frac{\tilde{\mu}_1\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)} - \tilde{\mu}_1\tilde{\mu}_2\right] + \left[\frac{\tilde{r}\tilde{\mu}_1\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)} - r\tilde{\mu}_1\tilde{\mu}_2 - \frac{\tilde{\beta}\tilde{\mu}_1\tilde{\gamma}\tilde{\varepsilon}y^*}{(1+\tilde{v}y^*+\tilde{\sigma}x^*+\tilde{\sigma}x^*\tilde{v}y^*)} + \tilde{\beta}\tilde{\mu}_1\tilde{\mu}_2\right] = 0$$

The above equation of the form $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$. By using Routh – Hurwitz Stability Criteria namely $a_1 > 0, a_3 > 0$ and $a_1a_2 - a_3 > 0$ are satisfied. Therefore, the solution of the system attains an equilibrium point E^3 as time goes on. Hence the equilibrium point becomes locally asymptotically stable.

Comparison Result

In this Existing method[18,] they have analyzed a Holling type – II function response in the formulation of the prey-predator model. The stability and eigenvalues are found directly. In the Proposed method they both have adopted a new idea called Pentagonal fuzzy numbers for the parameters that are present in the formulation of the fuzzy prey-predator model. Also, consider a Holling type I reaction for immature prey and the Crowley Martin functional response for mature prey in the fuzzy prey-predator model. The stability is obtained after applying the defuzzification method. Some of the eigenvalues are found by using the Routh Hurwitz stability criteria condition.

Results and Discussion

In this section, I have carried out some of the results mentioned in Table 2 using numerical simulation. Table 2 brings a sense of completion to the analytical results and tracks the impacts on the dynamics of the system Eq.1. The steady-state curves for our numerical investigation were obtained using the MATLAB application. The numerical analysis provided here demonstrates the dynamic of the prey-predator model based on immature prey, mature prey, and predator. By assuming the initial value and parameters the model is simulated. Here they consider some set of parametric values such as, $\tilde{r} = 0.5, \tilde{\beta} = 0.15, \tilde{\gamma} = 0.8, k = 10, \tilde{\alpha} = 0.5, \tilde{\varepsilon} = 0.2, \tilde{\sigma} = 0.8, \tilde{v} = 0.2, \tilde{\mu}_1 = 0.05, \tilde{\mu}_2 = 0.32$. Using the values of the parameters Fig 1 shows the dynamic behavior $\tilde{\varepsilon} < k$. It shows the equilibrium point of Eq.1 is locally stable. Fig. 2 shows the trivial equilibrium point and it is locally stable when $\tilde{r} < \tilde{\beta}$. Fig 3 shows the axial equilibrium point and it is asymptotically stable when $\tilde{r} > \tilde{\beta}$. Fig 4 shows the extinction of three populations. Because there is an interaction between the predator and immature prey. Fig 5 shows the time series of the stability model Eq 1. This figure shows that all three population points ever become stable. E^0 always exists, but it is never stable both environmentally and analytically. Fig 6 shows the dynamic behavior when $\tilde{\varepsilon} > k$.

Table 2. Existence of Stability

Figures	Existence of Result
1	Locally Stable
2	Locally Stable
3	Asymptotically Stable
4	Time series of stability
5	Stable
6	Shows the existence when $\tilde{\varepsilon} > k$

Phase Portrait Analysis of the Model

Let us consider another set of parameter values and also consider different initial values for the phase portrait of Eq 1 are given as, $\tilde{r} = 0.57, \tilde{\beta} = 0.24, \tilde{\gamma} = 0.46, k = 10, \tilde{\alpha} = 0.15, \tilde{\varepsilon} = 0.4, \tilde{\sigma} = 0.08, \tilde{v} = 0.02, \widetilde{\mu_1} = 0.05, \widetilde{\mu_2} = 0.32$ and $N_1 =$

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$[1.02,1.02,1.02], N_2 = [5,10,15], N_3 = [5,8,10], N_4 = [1,2,3]$. Fig 7 shows the dynamic behavior with the initial condition N_1 and E^0. Fig 8 and Fig 11 show the equilibrium point for system Eq 1 with different initial condition N_4, N_2. Fig 9 shows the phase portrait N_4. Fig 10 shows the phase portrait of the system Eq 1 around E^0. Fig 12 shows the phase portrait analysis N_3.

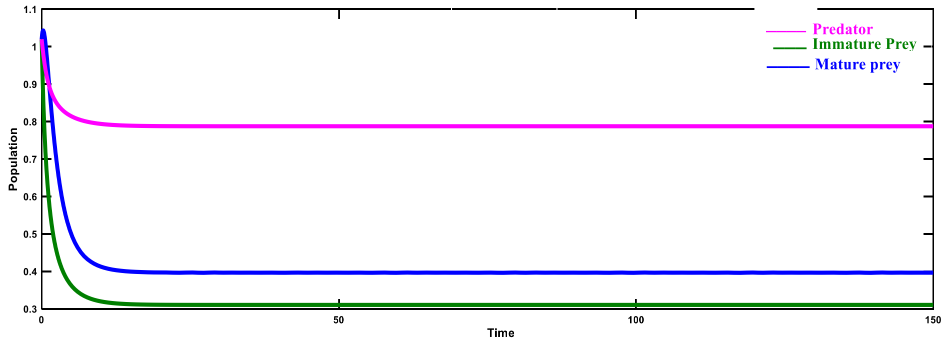

Figure 1. It shows the dynamic behavior when $\tilde{\varepsilon} < k$

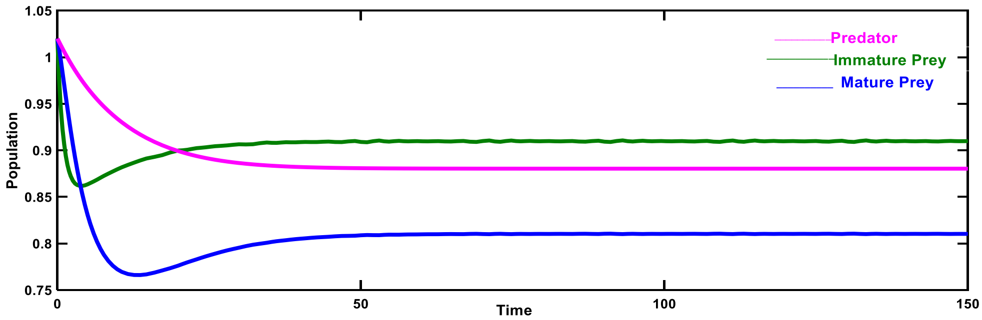

Figure 2. The curve attains a trivial equilibrium when $\tilde{r} < \tilde{\beta}$

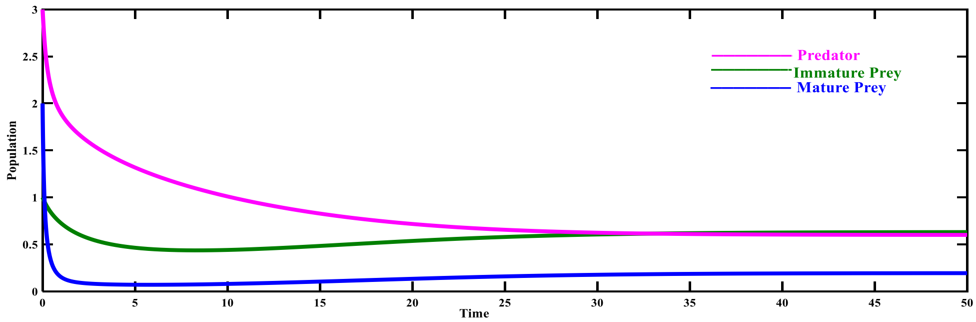

Figure 3. The curves attain an axial equilibrium when $\tilde{r} > \tilde{\beta}$

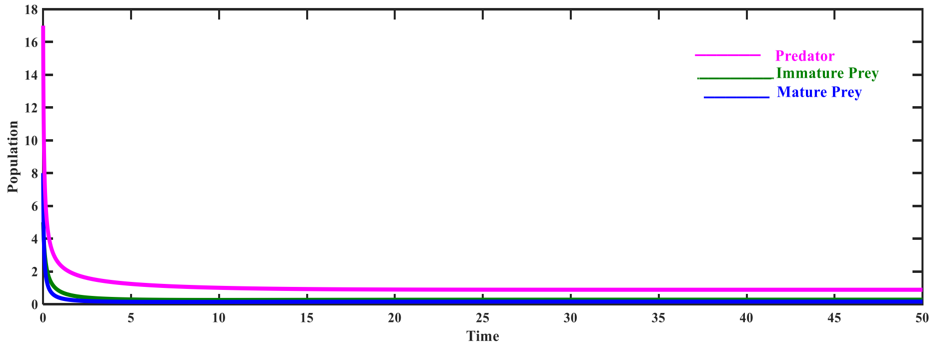

Figure 4. It indicates the extinctions of three populations

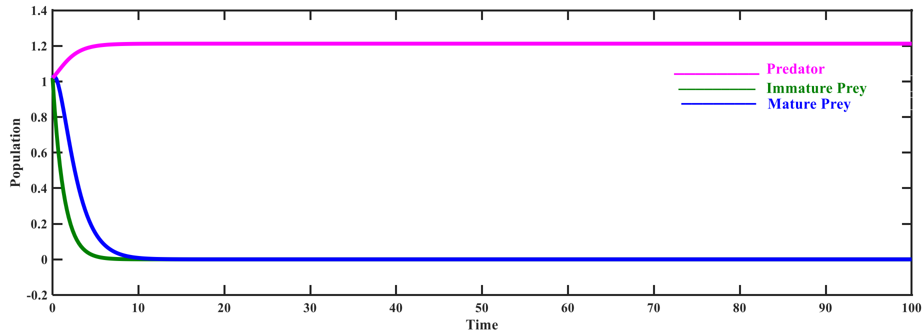

Figure 5. It shows time series stability

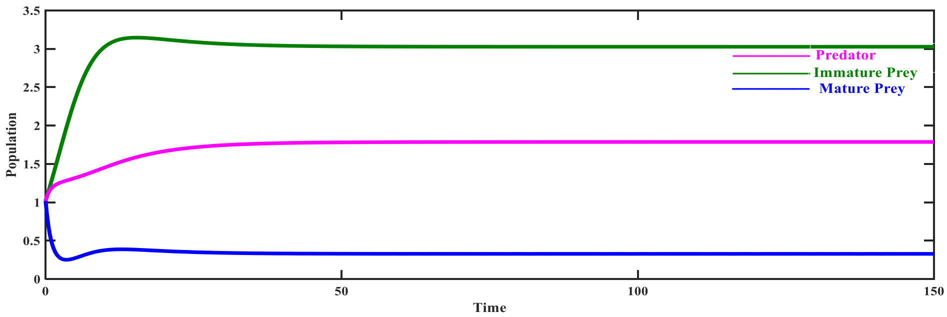

Figure 6. It indicates dynamic behavior when $\tilde{\varepsilon} > k$

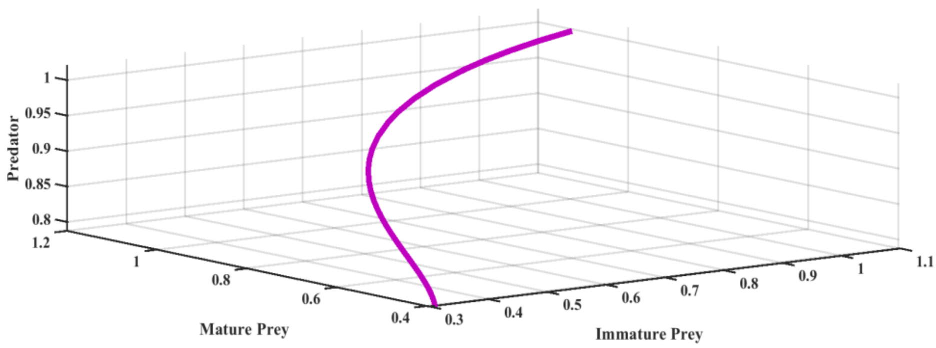

Figure 7. Shows the behavior with N_1 initial condition

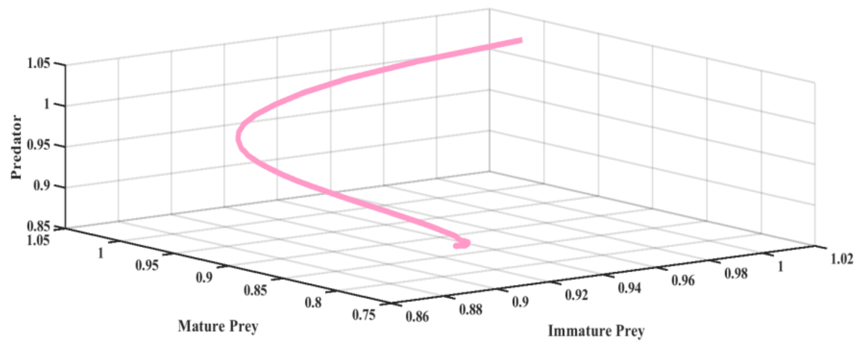

Figure 8. It indicates an equilibrium point with N_4, N_2

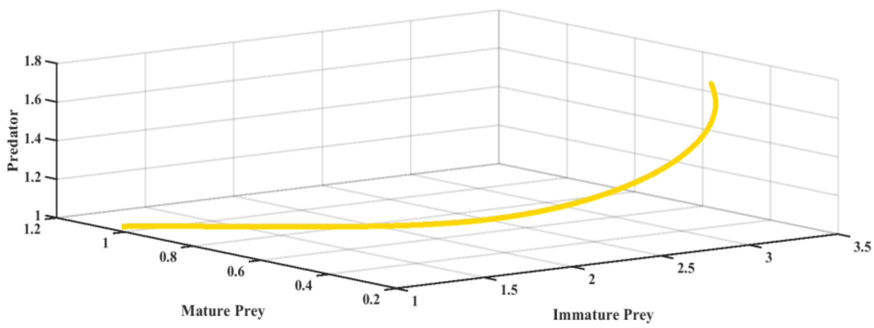

Figure 9. It shows a phase portrait with N_4

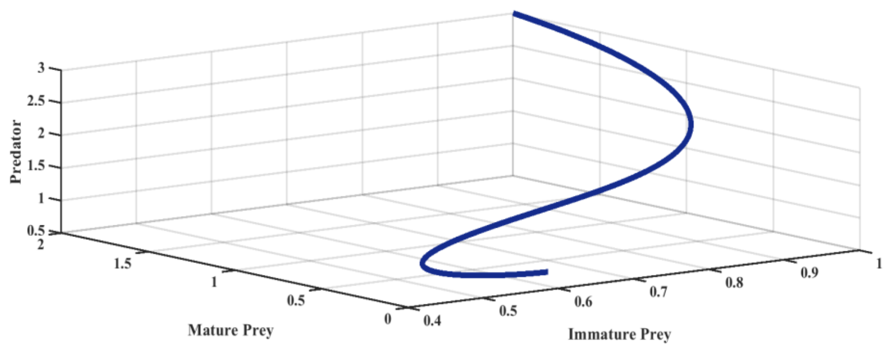

Figure 10. Phase portrait around E^0

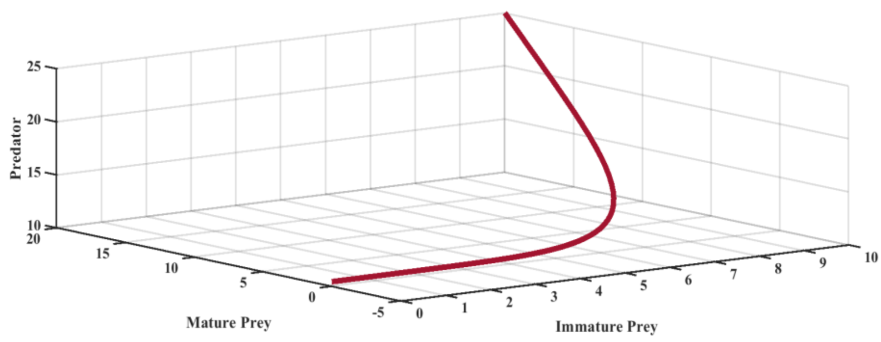

Figure 11. It indicates an equilibrium point with N_4, N_2

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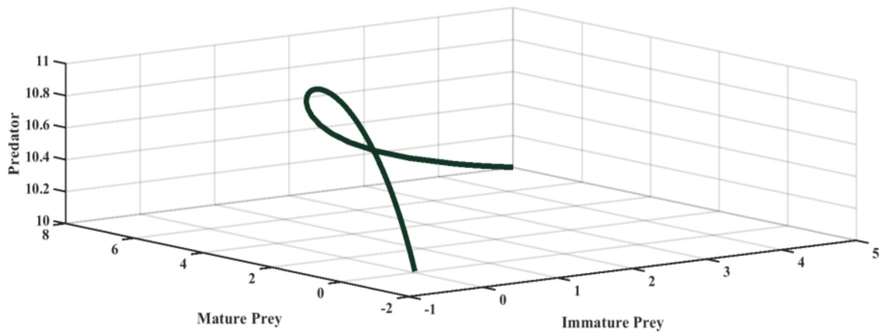

Figure 12. It shows the analysis of phase portrait with N_3

Conclusion

In the present paper, the authors have developed a numerical scheme to find the solution of a three-population model with a stage structure for prey. A Crowley Martin functional response can be utilized for the prey-predator model. Further, they considered a fuzzy parameter as a Pentagonal fuzzy number and fuzzy initial population. The existence of equilibrium points is confirmed. By using an alpha cut definition for pentagonal fuzzy numbers, they can defuzzify the fuzzy number by applying the robust ranking technique method. After the defuzzification process, the stability analysis for the dynamics behavior also exists by using the Community matrix. Through computational methods, they were able to develop several new phase portraits, including the presence of stable or unstable equilibrium points under appropriate parameter values in the prey-predator model. Finally, the authors conclude that the behavior of the model is more realistic which shows the problem of uncertainty through numerical simulation. In the future, this paper can be extended through bifurcation problems and the Allee effect with harvesting for the prey-predator models in a fuzzy environment.

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Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Vellore Institute of Technology, Vellore, India.

Authors' Contribution Statement

This work was carried out in collaboration between P.V and K.K. K.K provided the research paper idea and P.V implemented the idea and generalized the Mathematical formulation. K.K give the idea of fuzzy concept. P.V utilized the concept of fuzzy to find Equilibrium points and Stability analysis. P.V used a software for simulation and K.K supervised the manuscript. K.K edited the manuscript and made some changes in the manuscript. All authors read and approved the final manuscript.

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نموذج هيكل المرحلة للمفترس باستخدام الأرقام الخماسية الغامضة والاستجابة الوظيفية

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الخلاصة

في هذه الدراسة، قمنا بدراسة نموذج مفترس للفريسة مع هيكل مرحلة للفريسة. الهدف من الدراسة هو إيجاد سلوك النموذج باستخدام قيم المعلمات في وجود أرقام ضبابية خماسية.. يتم التفاعل بين الأنواع باستخدام الاستجابات الوظيفية، مثل تفاعل هولينج من النوع الأول للفريسة الناضجة واستجابة كرولي مارتن الوظيفية للفريسة الناضجة . فكرة المشكلة هي بناء نموذج رياضي في بيئة غامضة باستخدام المعلمات الغامضة والقيم الأولية.. يتم تنفيذ وجود نقاط التوازن. وباستخدام مفهوم قطع ألفا للمعلمات المستخدمة في نموذج الفريسة – المفترس تم التعامل مع الأعداد الغامضة الخماسية. يمكن اعتبار المعلمات التي استخدموها في الصياغة الرياضية قيمة واضحة من خلال تطبيق طريقة إزالة الضبابية. هنا يتم استخدام طريقة تقنية التصنيف القوية. تتم دراسة استقرار كل نقطة توازن عن طريق حساب مصفوفة جاكوبي وإيجاد القيم الذاتية التي تم تقييمها عند كل نقطة توازن. من خلال الاستفادة من تحليل استقرار هيكل مرحلة الفريسة يتم اكتشافه أيضًا. بالنسبة للنظام الديناميكي تم توفير عمليات محاكاة عددية باستخدام برنامج MATLAB للكمبيوتر حتى نتمكن من عرض سلوك النظام وتحديد ما إذا كان مستقرًا أم غير مستقر.

الكلمات المفتاحية: نقاط التوازن، الاستجابة الوظيفية، الأرقام الغامضة، الفريسة غير الناضجة، الفريسة الناضجة، الفريسة المفترسة