Um-Salama Science Journal **Vol.6(2)2009**

Limits between the Cosmological Parameters from Strong Lensing Observations for Generalized Isothermal Models

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Date of acceptance 12 /11/ 2008

Abstract

This paper including a gravitational lens time delays study for a general family of lensing potentials, the popular singular isothermal elliptical potential (**SIEP**), and singular isothermal elliptical density distribution (**SIED**) but allows general angular structure. At first section there is an introduction for the selected observations from the gravitationally lensed systems. Then section two shows that the time delays for singular isothermal elliptical potential (**SIEP**) and singular isothermal elliptical density distributions (**SIED**) have a remarkably simple and elegant form, and that the result for Hubble constant estimations actually holds for a general family of potentials by combining the analytic results with data for the time delay and by using the models of distances.

Key words: distance scale -galaxies: distances and redshifts - gravitational lensing – quasars: Strong Lensing (***B 0218+357 B-A*, *PG1115+080 A-B)***

Introduction

Refsdal (1964) first proposed that time delays between images in multiply imaged gravitational lenses can be used to measure the Hubble constant H_0. This method is attractive because it is a single-step process and is based on the well-established theory of General Relativity. The time delays for many gravitational lenses have been measured, which yield (H_0 about 65 ±15 km S^{-1} Mpc^{-1}). The lensing measurement is an important way of confirming and extending local determinations of H_0, at present, the error budget in the lensing measurement is dominated by systematic uncertainties in the lens modeling. Ultimately the accuracy may be limited by the uncertainties induced by the large-scale structure along the line of sight at the few percent level, although if random these effects should shrink as $N^{-1/2}$ as the number N of lenses with measured time delays increases.

Lensing measurements of H_0 are typically derived from models based on isothermal galaxies, because such models are consistent with individual lenses, lens statistics, stellar dynamics, and X-ray galaxies.

Modelers usually adopt a parameterized form, either an isothermal elliptical potential or an isothermal elliptical density, and adjust the parameters to fit the data [1]. The purpose of this work is to gain insights into the time delays in a more general family of lens models that includes these commonly used isothermal models and their variants.

The Selected Observations

In this section, we present some selected observations from six gravitationally lensed systems. The selection depends on the grade for the likelihood that the observed object is a

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lens given by CASTLES [1]. It also due to the availability of the observed data for these systems. The observational data for six time delay lenses is given as in the Table (1) and Figure (1) [2].

Table (1) The observational data for six time delay lenses

Lens / Components	z_l	z_s	$\Delta t_{(i,j)}$ days	r_j (")	r_i (")	$\lvert\theta_i - \theta_j\rvert^\circ$
B 0218+357 / B-A	0.96	0.68	10.5±0.2	0.24±0.06	0.1±0.06	176.4
Q 0957+561 / B-A	1.41	0.36	417±3	5.2275±0.0035	1.034±0.0035	154.2
PG 1115+080 / A-B	1.72	0.31	11.7±1.2	1.147±0.025	0.95±0.004	115.5
PG 1115+080 / C-B	1.72	0.31	25±1.6	1.397±0.004	0.95±0.004	114.6
B 1600+434 / B-A	1.59	0.42	47±6	1.14±0.05	0.25±0.05	179.4
PKS 1830-211 / B-A	2.51	0.89	26±5	0.67±0.08	0.32±0.08	160.5

Where z_l, z_s are the redshifts of the lens, source. Δt is the time delay in days between the two observed images. (r_i", r_J") represents the angular position of the two images. (θ_i-θ_j) are the separation angle between the two images A and B.

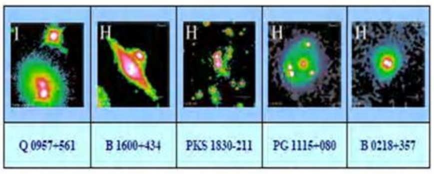

Figure (1) The Images for the six time delay lenses

Time Delay for Generalized Isothermal Models

The Hubble parameter H_0 is one of the most important parameters to characterize the averaged dynamics of universe, and thus its determination is a central issue in observational cosmology, Although there has been tremendous progress regarding its determination recently, it is still worthwhile to determine it by using the relative time delay between images in gravitational lens systems, the method based on the fact that the relative time delay between images is proportional to the scale of the universe and thus is inversely proportional to the Hubble parameter as it given by[3]:

$$\Delta t(x,y)=\frac{D}{2c}(1+z_l)\left[(x-\xi)^2+(y-\eta)^2-2\phi(x,y)\right],$$

$$D\equiv\frac{D_l D_s}{D_{ls}} \qquad 1$$

Where z_l is the redshift of the lens, (ξ, η) are the angular source position, (x, y) is the angular image position, D_l and D_s are angular diameter distances from the observer to the lens and source, respectively, and D_{ls} is the angular diameter distance from the lens to the source. The dimensionless potential ϕ satisfies the two-dimensional Poisson equation [2]:

$$\nabla^2\phi(x,y)=2k \quad , \quad k=\frac{\sum(x,y)}{\sum_{cr}} \qquad 2$$

Where Σ is the projected surface mass distribution of the lens and Σ_{cr} is the critical surface density for lensing. To proceed further it is useful to adopt a specific potential or density form, Individual lenses and lens statistics are usually consistent with isothermal models, Note that the work have chosen a coordinate system that is centered on the lens galaxy and aligned the X-xis along the lens galaxy's major axis. For simplicity, also assumed that the isothermal potential and density distributions are singular at the origin. This step of work including a study for more general family of lens models. The potential is assumed to obey the relation [2]:

$$\phi = x\,\phi_x + y\,\phi_y \qquad 3$$

Where ϕ_x and ϕ_y are the first derivatives of the potential, which are just them components of the deflection angle in the x and y directions. Thus Equation (3) describes a family of scale-free models that includes both the **(SIEP)** and **(SIED)** models but allows for general angular structure; in other words, these are generalized isothermal models. It is important now to show that this family is extremely useful in deriving the time delays between images. The lens equation is given by [2]:

$$\xi = x - \phi_x \quad , \quad \eta = y - \phi_y \qquad 4$$

Substituting Equations (3) and (4) into the time delay expression in equation (1), it's easy to obtain:

$$\Delta t\,(x, y) = \frac{D}{2c}\left(1 + z_l\right) \begin{bmatrix} (x-\xi)^2 + (y-\eta)^2 - \\ 2\,x\,(x-\xi) - 2\,y\,(y-\eta) \end{bmatrix} \qquad 5$$

This can be further simplified into:

$$\Delta t\,(x, y) = \frac{D}{2c}\left(1 + z_l\right)\left[\, \xi^2 + \eta^2 - x^2 - y^2 \,\right] \qquad 6$$

Since only the relative time delay is observable, therefore it is necessary to compute the time delay between two images *i* and *j*, which is given by:

$$\Delta t_{i,j} = \frac{D}{2c}\left(1 + z_l\right)\left[r_j^2 - r_i^2\right] \qquad 7$$

Where is simply the distance of image *i* from the center of the galaxy. This surprisingly simple expression is valid for all lens potentials satisfying Equation (3), including both the (**SIEP**) and (**SIED**) models as well as more general angular structures. Since r_i^2 and r_j^2 are rotationally invariant, Equation (7) is valid even after an arbitrary rotation. In other words, provided it is possible to know the center of the lens galaxy, the predicted time delay can be computed by using Equation (7) irrespective of the lens orientation and without any need to search for the best fit model parameters. Since the predicted time delay scales as $\sim H_0^{-1}$, it can be compared with a measured time delay to determine H_0.

Limits between H_0 and q_0

The aim of this step of the present work is to define the limitations of the values that the Hubble constant may be taken for different adopted cosmological models by using the selected observations from CASTLES [1].

Equation (7) can be written in new form, as:

$$D_{(H_o, q_o)} = \frac{D_l\, D_s}{D_{ls}} = \frac{2c\,\Delta t_{i,j}}{\left(1 + z_l\right)\left\{r_j^2 - r_i^2\right\}} \qquad 8$$

The right hand side of Equation (8) could be easily calculated for each system, but the calculations in the left side is more complicated because it is depending on the adopted cosmological model that used to measure the angular diameter distances between the components of the gravitationally lensed system.

For the simplest model of distance (which depends on H_0, q_0 only), and by assuming that all the matter contains in the observed system or around it are ordinary and smoothly distributed with out any dark matter, and by using the formulas in equations (9), (10) and (11)[4, 5], we get:

$$D_A = \frac{c}{H_o(1+z)^2}\frac{1}{q_o^2}\left[z q_o + (q_o - 1)\left\{(1 + 2 z q_o)^{1/2} - 1\right\}\right] \qquad 9$$

$$D_{z_1, z_2} = \frac{c}{H_o}\left\{\frac{\left(1 - 2q_o - G_1 G_2\right)\left(G_1 - G_2\right)}{2 q_o^2 \left(1 + z_1\right)\left(1 + z_2\right)^2}\right\} \qquad 10$$

$$G_{1,2} = \left(1 + 2\,q_o\,z_{1,2}\right)^{1/2} \qquad 11$$

By substituting the last formulae in the left hand side of Equation (8), we get:

$$D_{(H_o, q_o)} = \frac{D_l D_s}{D_{ls}} = \frac{2c\left\{q_o z_s + (q_o - 1)(G_s - 1)\right\}\left\{q_o z_l + (q_o - 1)(G_l - 1)\right\}}{(1 + z_l) H_o \cdot q_o^2} \qquad 12$$

And by comparing between the last equations with Equation (8) then:

$$H_o = \frac{\left\{q_o z_s + (q_o - 1)(G_s - 1)\right\}\left\{q_o z_l + (q_o - 1)(G_l - 1)\right\}\left\{r_j^2 - r_i^2\right\}}{\Delta t_{i,j} \cdot q_o^2} \qquad 13$$

H_0 can now calculated for each one of the the six observed systems with different adopted cosmological models that represented by the values of the decelerating parameter q_0. The next six figures, represent plots for H_0 against q_0 for each system, it's expected to have three curves for each system because of the minus and plus error bars in the values of the observationally measured values for some factors ($\Delta t_{i,j}$, r_i , r_j) in Table (1). The middle curve represents the average value for H_0 with out any errors, the upper and lower curves represents the maximum and minimum values for H_0 with the effects of the error bars.

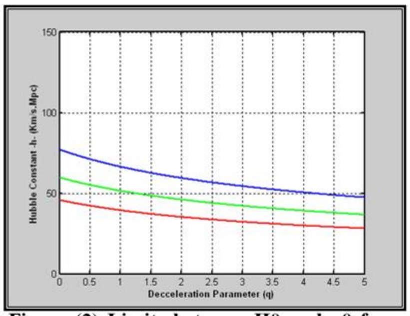

Figure (2) Limits between H0 and q0 from the strong lensing in the ravitationally lensed system PG1115+080 A-B

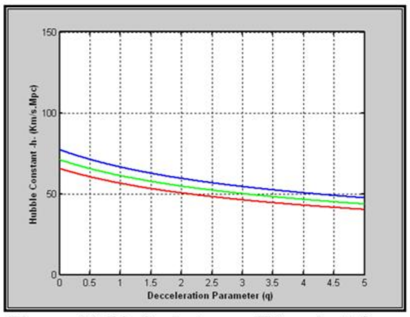

Figure (3) Limits between H0 and q0 from the strong lensing in the gravitationally lensed system PG 1115+080 C-B

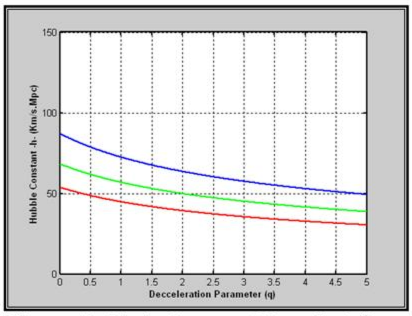

Figure (4) Limits between H0 and q0 from the strong lensing in the gravitationally lensed system B 1600+434 B-A

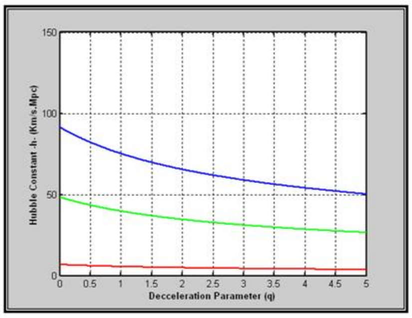

Figure (5) Limits between H0 and q0 from the strong lensing in the gravitationally lensed system B 0218+357 B-A

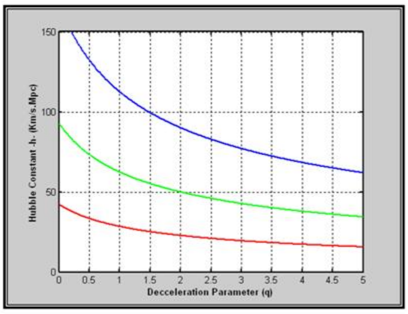

Figure (6) Limits between H0 and q0 from the strong lensing in the gravitationally lensed system PKS 1830-211 B-A

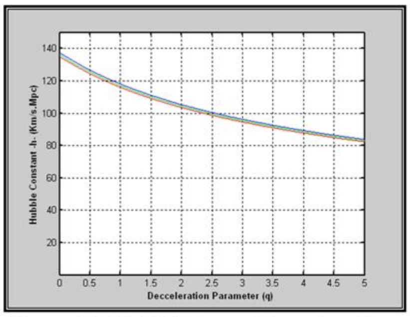

Figure (7) Limits between H_0 and q_0 from the strong lensing in the gravitationally lensed system Q0957+561 B-A

Results

Table (2) H0 values for different cosmological models

	H_0	$q_0 = 0.2$	$q_0 = 0.5$	$q_0 = 1$	$q_0 = 4$
PG 1115+080 A-B	Max	74.55	71.12	66.38	50.46
	Mid	57.74	55.09	51.42	39.08
	Min	44.21	42.18	39.37	29.93
PG 1115+080 C-B	Max	74.63	71.20	66.45	50.51
	Mid	68.62	65.47	61.11	46.45
	Min	63.34	60.43	56.40	42.87
B 1600+434 B-A	Max	83.44	78.72	72.43	52.89
	Mid	65.43	61.74	56.80	41.50
	Min	51.51	48.60	44.71	32.65
B 0218+357 B-A	Max	87.42	82.14	75.14	54.01
	Mid	46.81	43.38	39.69	28.53
	Min	6.47	6.08	5.56	4.00
PKS 1830-211 B-A	Max	150.68	132.41	112.58	68.33
	Mid	83.53	73.39	62.40	37.87
	Min	38.03	33.42	28.41	17.25
Q0957+561 B-A	Max	132.91	126.7	118.09	89.33
	Mid	131.73	125.58	117.04	88.54
	Min	130.57	124.47	116.01	87.76

Table (2) represents four arbitrary values for the deceleration parameter q_0 that represent the three cases of freedman models ($q_0 = 0.2$ for the closed, $q_0 = 0.5$ for the flat and $q_0 = 1$ for the open model) and the fourth value is $q_0 = 4$ for the very open universe model, and give the corresponding minimum, middle and maximum Hubble constant for each selected system for these values. We note from the last six figures and Table (2) that the values of the Hubble constant are acceptable for the first five systems. Only the complicated system Q0957+561B-A is out off the range. Therefore we plot in Figure (8) below the final result for all these five gravitationally lensed systems and exclude Q0957+561B-A by mixing the maximum, middle and minimum values for Hubble constant for all systems in a unique relation between H_0 against q_0.

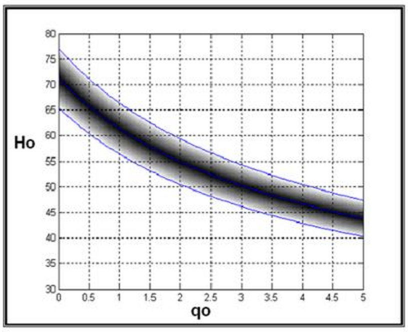

Figure (8) Limits between H0 and q0 from the strong lensing in the gravitationally lensed systems PG1115+080 A-B, PG1115+080 C-B, B1600+434 B-A, B0218+357 B-A and PKS1830-211 B-A

Figure (8) has been calculated by taking in to account the minimum values which represented by the lower curves for all the first five systems, And also calculate the lowest values that the Hubble constant H_0 taken it among its maximum values which represented by the upper curves for these five systems. And the average values which represented by the middle curves for these five systems. This method enables us to mix the relation between H_0 against q_0 in one figure that have more reality with minimum error bars, and it is true and could be applied for all these systems.

Discussion

Lensing time delays are interesting because they offer a measurement of the Hubble constant by a method dependent of the distance ladder and the lens model only. The analytic time delays now offer the ability to estimate H_0 for the time delay lenses with a method that is not only extremely simple but also uniform, in other word we using a uniform set of modeling assumptions and using the same type of data (image positions) for all lenses. We use the data for six of the time delay lenses {summarized in Table (1)} to determine H_0. Figures (2), (3), (4), (5), (6) and (7) shows the H_0 estimates for the generalized isothermal models discussed in Equation (13) for the all gravitationally lensed systems, assuming no external shear. Equation (13) makes it essentially trivial to compute H_0 from easily measured quantities without any modeling.

Moreover, the estimates depend only on the assumption of an isothermal profile for the lensing potential (and not on its angular structure). Finally, the H_0 estimates depend only weakly on the adopted cosmology. With the exception of Q0957+561, the naive H_0 estimates are consistent with each other and with values from other methods. Q0957+561 stand out because the lens is complicated: the lens galaxy appears not to have a simple isothermal profile, two features that invalidate the class of potentials assumed in Equation (4) still the agreement among the remaining five systems is surprising given that we have done no modeling.

On the one hand, the agreement is somewhat misleading because two of the lenses have large systematic uncertainties in the lens galaxy position that lead to enormous H_0 error bars (B0218+357 and PKS1830-211, which are discussed in detail by Leh´ar et al. 1999) [6]. But, the agreement between (PG1115+080 and B1600+434) is intriguing because one has an elliptical lens galaxy and the other has an edge-on spiral lens galaxy. Moreover, the rough agreement between the H_0 estimates from the two time delays in PG1115+080 provides a useful consistency check on the model assumptions.

The last six figures show this relationship clearly. The middle curves represent the average value for H_0 for different adopted cosmological models that represented by the value of the decelerating parameter q_0. The upper curves represent the maximum values for H_0 by taking the effects of the error bars in care. Which calculated by substituting the error bars for the three parameters ($\Delta t_{i,j}$, r_i, r_j) as ($\Delta t_{i,j}$-Δt_{ijerr} , r_i-r_{ierr}, r_j+r_{jerr}). The lower curves represent the minimum values for H_0 by taking the effects of the error bars in care, which calculated by substituting the error bars for the three parameters ($\Delta t_{i,j}$, r_i, r_j) as ($\Delta t_{i,j}$+Δt_{ijerr}, r_i+r_{ierr}, r_j-r_{jerr}) where that's give the minimum value for the over & maximum value of Equation (13).

The general behavior for the entire upper, middle and lower curves for the relation between (H_0 against q_0) in all figures is expected and logical. The value of H_0 should be decrease when q_0 increase. According to the big bang, the increasing in the deceleration parameter q_0 leads to the decreasing in the Hubble constant due to the inverse relation between them as given in [4]:

$$\frac{\ddot{R}(t)}{R(t)} = -q(t)\left[H(t)\right]^2 \qquad 14$$

That's because the speed of the expansion of any part over all the entire universe become very slow if its deceleration increase, and the distances between it's parts to have been very long, then the Hubble constant taken lowest values according to its cosmological definition to be (speed/distance). While, when the deceleration of the universe increase, the speed of the expansion of any part over all the entire universe become higher, and the distances between it's parts to have been shorter, then the Hubble constant taken the highest values.

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الحدود بين المعاملات الكونية من خلال إرصادات التعدس القوي للموديلات الآيزوثرمية المعمّمة

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الخلاصة

هذا البحث يتضمن دراسة ظاهرة التأخر الزمني للتعدس الجذبي لصنف معروف من أصناف الجهد التعدسي، جهد الجمهرة الايزوثرمية المفردة الأهليليجية (**SIEP**)، و توزيع الكثافة للجمهرة الايزوثرمية المفردة الأهليليجية(**SIED**) لكن مع بقاء خصوصية التركيب الزاوي العام. في البداية مقدمة عن الارصادات المختارة من منظومات التعدس الجذبي. ثم في القسم الثاني نبين أنه التأخر الزمني لجهد الجمهرة الايزوثرمية المفردة الأهليليجية (**SIEP**)، و توزيع الكثافة للجمهرة الايزوثرمية المفردة الأهليليجية(**SIED**) صيغة بسيطة ورائعة، و كذلك فان نتيجة التقديرات لثابت هابل- في الحقيقة- هي مقيدة لاي صنف معروف من أصناف الجهد التعدسي باتحاد النتائج التحليلية مع بيانات التأخر الزمني وباستخدام نماذج المسافات.