Effect of Different Parameters on Powell-Eyring Fluid Peristaltic Flow with the Influence of a Rotation and Heat Transform in an Inclined Asymmetric Channel

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Abstract

In this article, the effect of the rotation variable and other variables on the peristaltic flow of Powell-Eyring fluid in an inclined asymmetric channel with an inclining magnetic field through a porous medium with heat transfer is examined. Long wavelength and low Reynolds number are assumed, where the perturbation approach is used to solve the nonlinear governing equations in the Cartesian coordinate system to produce series solutions. Distributions of velocity and pressure gradients are expressed mathematically. Through the collection of figures, the impact of various criteria is explained and graphically represented. These numerical results were attained using the mathematical application MATHEMATICA.

Keywords: Heat transfer, Inclined channel, Magnetic field, Peristaltic flow, Porous medium, Powell-Eyring fluid, Rotation.

Introduction

Peristaltic pumping is a specific sort of pumping when a wide range of intricate rheological fluids can be moved readily from between two locations. This pumping principle is referred to as peristaltic. The ducts through which the fluid passes undergo intermittent involuntary constriction and then expand. As a result, the pressure gradient rises, causing the fluid to move forward. After Latham's groundbreaking work and due to the fact that it is utilized in biological, engineering, and physiological systems academics have become increasingly interested in the different applications of peristalsis. Due to the fact that it is utilized in biological, engineering, and physiological systems, peristaltic transport has received significant attention in recent years. Generally, the peristaltic wave’s circular contractions and the successive longitudinal contractions that occur during peristalsis are generated by the sinuses which propagate along the fluid-containing duct. This technique is the basis for several muscular tubes, including the gastrointestinal tract, fallopian tubes, bile ducts, ureters, esophageal tubes, and others. Moreover, non-Newtonian fluids are better than numerous industrial and physiological processes that use Newtonian fluids. Among the models of non-Newtonian fluids (which can exhibit various rheological effects), that can be accessed is Paul-Erying fluid. Although this model is more difficult mathematically than models of non-Newtonian fluids, it deserves more attention because of its distinct benefits. Numerous researchers have been interested in the Powell-Eiryng fluid's peristaltic flow mechanism since it was studied by Hina and Mustafa and Hayat and Alsaedi, Hayat and Naseema and Rafiq and Fuad, Hayat and Ahmed, Hussain and Alvi and Latif and Asghar, and Ali and Liqaa. The static magnetohydrodynamic flow and heat transfer of an Eyring-Powell fluid on an expansion plate with viscous dissipation were studied and numerically explained. The exchange of thermal energy between different system components is referred to as heat transfer. However, the medium's physical characteristics and the separate compartments' temperatures affect the
speed. In recent years, research has been conducted about studying the effect of heat transport on non-Newtonian fluids. In a tapered asymmetric channel, the issue of peristaltic transport of an incompressible non-Newtonian fluid is examined. Peristalsis is used as the basis for the creation of devices such as peristaltic pumps, roller pumps, hose pumps, tube pumps, finger pumps, heart-lung machines, blood pump machines, and dialysis machines. These applications include the transportation of aggressive chemicals, high solid slurries, toxic (nuclear industries), and other materials. With regard to well-established problems of the stir of semi-conductive physiological fluids, such as blood and blood pump machines, magnetic drug forcing, and pertinent methods of human digestion, the advantage of applied magnetic field (MHD) on peristaltic efficacy is crucial. It is also helpful in treating gastroparesis, chronic constipation, and morbid obesity as well as magnetic resonance imaging (MRI), which is used to identify brain, vascular diseases, and tumors. A substance that has several tiny holes scattered throughout it is referred to as a porous medium. In riverbeds, fluid infiltration and seepage are sustained by flows over porous media. Important examples of flows through a porous material are those through the ground, water, and oil. Oil is trapped in rock formations like limestone and sandstone, which make up the majority of an oil reservoir. Natural porous media can be found in many different forms, such as sand, rye bread, wood, filters, bread loaves, human lungs, and the gallbladder. Food processing, oxygenation, hemodialysis, tissue condition, heat convection for blood flow from tissues' pores, and radiation between the environment and its surface all depend on the action of heat transfer in the peristaltic repositioning of fluid. The aforementioned processes all benefit from mass transfer; in particular, the mass transfer that occurs as nutrients diffuse from the blood into nearby tissues cannot be understated. Greater mass transfer participation is typical in the distillation, diffusion of chemical contaminants, membrane separation, and combustion processes. It should be observed that when mass and heat transmission happens at the same time, there is a connection between driving potentials and fluxes. However, the temperature gradient is what causes the gradients in mass flux and composition (termed soret action). The study of fluid peristaltic transport in the presence of an external magnetic field and rotation is necessary for many issues involving the flow of conductive physiological fluids, such as blood and saline water. A variety of values are used for the rotational parameters, the porous medium, density, amplitude wave, and taper of the channel, as well as a variety of values for the Hartman number and Darcy number, to study the effects of varying the velocity and pressure gradient. This article's objective is to look into the rotational effects of the peristaltic transport of a Powell-Eyring fluid through a porous media under the combined influence of inclined MHD.

**Problem Mathematical Description**

Consider the peristaltic motion of an incompressible Powell-Eyring fluid in a two-dimensional, asymmetric conduit with a width of (d’+d). An endless sinusoidal wave traveling along the channel walls at a constant forward speed (c) is what generates flow. The geometry of the wall structure is described as:

\[ h_1(x, t) = d - a_1 \sin \left[ \frac{2\pi}{\lambda} (x - ct) \right] \]

\[ h_2(x, t) = -d' - a_2 \sin \left[ \frac{2\pi}{\lambda} (x - ct) + \Phi \right] \]

In which \( h_1(x, t) \), \( h_2(x, t) \) are the lower and upper walls respectively, \((d, d')\) denote the channel width, \((a_1, a_2)\) are the amplitudes of the wave, \(\lambda\) is the wavelength, \((c)\) is wave the wave speed, \((\Phi)\) varies in the range \((0 \leq \Phi \leq \pi)\), when \(\Phi = 0\) is a symmetric channel with out-of-phase waves and \(\Phi = \pi\) waves are in phase, the rectangular coordinate system is chosen so that the \(X - axis\) is in the direction of the wave’s motion, and the \(\bar{Y} - axis\) perpendicular to \(X\), where \(\bar{t}\) is the time as shown in Error! Reference source not found.. Further \(a_1, a_2, d, d'\) and \(\Phi\) fulfill the following condition:

\[ a_1^2 + a_2^2 + 2a_1a_2 \cos \Phi \leq (d + d')^2 \]

The Cauchy stress tensor \(\vec{\tau}\) for a fluid that obeys the Powell- Eyring model is given as follows:-

\[ \vec{\tau} = -\Pi + \vec{S} \]

\[ \vec{S} = \left[ \mu + \frac{1}{\beta \gamma} \sinh^{-1} \left( \frac{\bar{Y}}{\bar{Y}_0} \right) \right] A_{11} \]
\[
\rho \left( \frac{\partial V}{\partial t} + \vec{U} \frac{\partial V}{\partial x} + \nabla \frac{\partial V}{\partial y} \right) - \rho \Omega \left( \nabla \vec{U} + 2 \frac{\partial V}{\partial y} \right) = \\
- \frac{\partial P}{\partial x} + \frac{\partial S_{xx}}{\partial y} + \frac{\partial S_{xy}}{\partial y} - \sigma \beta_0^2 \cos \beta (\nabla \cos \beta - \nabla \sin \beta) \\
- \frac{\mu}{k} \vec{U} + pg \sin \alpha^* \tag{11}
\]

Where \( S \) is the extra stress tensor, \( I \) is the identity tensor, \( \nabla = (\partial \vec{X}, \partial \vec{Y}) \) is the gradient vector, \((\beta, c_1)\) are the material parameters of Powell-Eyring fluid, \( P \) is the fluid pressure, and \( \mu \) is the dynamic viscosity. The term \( \sinh^{-1} \) is approximately equivalent to

\[
\sinh^{-1} \left( \frac{\vec{Y}}{c_1} \right) = \frac{\vec{Y}}{c_1} - \frac{\vec{Y}^3}{6c_1^3}, \quad \left| \frac{\vec{Y}^3}{6c_1^3} \right| \ll 1 \tag{12}
\]

The flow is governed by three coupled nonlinear partial differentials of continuity, momentum, and energy, which are expressed in frame \((\vec{X}, \vec{Y})\) as

\[
\frac{\partial \vec{U}}{\partial x} + \frac{\partial \vec{V}}{\partial y} = 0 \tag{13}
\]

Natural peristaltic motion is an erratic occurrence, but it applying the transformation from a laboratory frame, stability can be assumed (fixed frame) \((\vec{X}, \vec{Y})\) to wave frame (move frame) \((\vec{x}, \vec{y})\). The subsequent transformations determine the relationship between coordinates, velocities, and pressure in the laboratory frame \((\vec{X}, \vec{Y})\) and wave frame \((\vec{x}, \vec{y})\)

\[
\vec{x} = \vec{X} + ct, \quad \vec{y} = \vec{Y}, \quad \vec{u} = \vec{U} - c, \quad \vec{v} = \vec{V}, \quad p(x, y) = p(x, y, t) \tag{14}
\]

Where \( \vec{u} \) and \( \vec{v} \) represent the velocity factors and \( p \) represents the pressure in the wave frame. Now that Eq.15 has been substituted into Eqs.1, 2, and 9–14, the resulting equation has been normalized using the non-dimensional variables shown below:

\[
\begin{align*}
\lambda &= \frac{1}{\alpha}, \quad \mu = \frac{1}{\alpha c_1}, \\
\beta &= \frac{1}{c_1}, \quad \lambda &= \frac{1}{\alpha}, \\
Re &= \frac{\rho c \mu}{\lambda}, \quad \alpha &= \frac{\rho \lambda}{\mu}, \\
\xi &= \frac{d}{\delta}, \quad \gamma &= \frac{d}{\delta}, \\
\beta_0 &= \frac{\beta}{\alpha}, \quad \bar{S}_{xx} = \frac{1}{\mu \beta_0} S_{xx}, \quad \bar{S}_{xy} = \frac{1}{\mu \beta_0} S_{xy}, \quad d' = \frac{d}{\beta_0}, \\
a &= \frac{a_1}{d'}, \quad b &= \frac{a_2}{d'}, \quad \bar{S}_{yy} = \frac{1}{\mu \beta_0} \bar{S}_{yy} 
\end{align*} \tag{15}
\]
Where, \( \delta \) is the wave number, \( h_1 \) and \( h_2 \) are non-dimensional lower and upper wall surfaces respectively, \( Re \) is the Reynolds number, \( Ha \) is the Hartman number, \( \Phi \) is the amplitude ratio, \( w \) is the non-dimensional permeability of the porous medium parameter, \( Da \) is the Darcy number, \( A \) is the Powell-Eyring fluid parameter, \( T_0 \) and \( T_1 \) are the temperatures at the upper and lower walls, \( Fr \) is the Froude number, and \( \alpha^* \) the inclination angle of the channel to the horizontal axis.

Following that is

\[
\begin{align*}
{h_1(x)} &= 1 - a \sin(2\pi x) \\
{h_2(x)} &= -d^* - b \sin(2\pi x + \Phi)
\end{align*}
\]

Where \( a, b, d^* \), and satisfy Eq.3, then

\[
a^2 + b^2 + 2ab \cos \Phi \leq (1 + d^*)^2
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
Re \delta \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) - \frac{\rho d^2 \Omega}{\mu} \left( \Omega u + 2 \frac{\delta c}{d} \frac{\partial v}{\partial t} \right) - \frac{\rho d^2 \Omega}{\mu} \left( \Omega u + 2 \frac{\delta c}{d} \frac{\partial v}{\partial t} \right) = \frac{\partial^2 \Omega}{\partial t^2} + \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} + E_c \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial u}{\partial x} \right) + 2 \delta^2 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \delta^2 \left( \frac{\partial v}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \delta^2 \left( \frac{\partial v}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \delta^2 \left( \frac{\partial v}{\partial x} \right)^2
\]

\[
S_{xx} = 2(1 + w) \frac{\partial u}{\partial x} - 2A \left[ 2 \delta^2 \left( \frac{\partial u}{\partial x} \right) + \frac{\partial u}{\partial y} \right] - 2 \delta^2 \left( \frac{\partial u}{\partial x} \right) + 2 \delta^2 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \delta^2 \left( \frac{\partial v}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \delta^2 \left( \frac{\partial v}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial v}{\partial y} \right)^2
\]

\[
S_{yy} = 2 \left( \delta^2 \left( \frac{\partial u}{\partial y} \right) + 2A \delta \left( \frac{\partial u}{\partial y} \right) + 2 \delta^2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial u}{\partial y} \right)^2 + 2 \delta^2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial u}{\partial y} \right)^2 \right)
\]

In previous equations, \( Pr \) is the Prandtl number, \( Ec \) is the Eckert number and \( \theta \) is the dimensionless temperature.

Following are the relations between the stream function \( \psi \) and velocity components:

\[
u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}
\]

Substituting Eq.28 into Eqs. 21 to 27, noting that the mass balance displayed by Eq.21 is similarly satisfied, produces the consequence that Eq.28 is satisfied.

\[
Re \delta \left( \frac{\partial^2 \psi}{\partial t^2} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} \right) - \frac{\rho d^2 \Omega}{\mu} \left( \Omega \frac{\partial \psi}{\partial y} - 2 \frac{\delta c}{d} \frac{\partial \psi}{\partial t} \right) = -2 \frac{\delta c}{d} \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial x} S_{xx} + \frac{\partial \psi}{\partial y} S_{yy}
\]

\[
Ha^2 \cos \beta \left( \frac{\partial \psi}{\partial y} \cos \beta + \frac{\partial \psi}{\partial x} \sin \beta \right) - \frac{1}{\delta a} \frac{\partial \psi}{\partial y} + \frac{Re}{Fr} \sin \alpha^*
\]

\[
Re \delta \left( \frac{\partial^2 \psi}{\partial t^2} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} \right) - \frac{\rho d^2 \Omega}{\mu} \left( \Omega \frac{\partial \psi}{\partial y} - 2 \frac{\delta c}{d} \frac{\partial \psi}{\partial t} \right) = -2 \frac{\delta c}{d} \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial x} S_{xx} + \frac{\partial \psi}{\partial y} S_{yy}
\]

\[
Ha^2 \sin \beta \left( \frac{\partial \psi}{\partial y} \cos \beta + \frac{\partial \psi}{\partial x} \sin \beta \right) + \frac{1}{\delta a} \frac{\partial \psi}{\partial y} + \frac{Re}{Fr} \cos \alpha^*
\]

\[
Re \delta \left( \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial y} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial x} \right) = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial t^2} + \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)
\]

\[
+ Ec \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + 2 \delta^2 \left( \frac{\partial \psi}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial \psi}{\partial y} \right)^2 + 2 \delta^2 \left( \frac{\partial \psi}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial \psi}{\partial y} \right)^2 \right)
\]

\[
2 \delta^2 \left( \frac{\partial \psi}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial \psi}{\partial y} \right)^2 + 2 \delta^2 \left( \frac{\partial \psi}{\partial x} \right)^2 + 2 \delta^2 \left( \frac{\partial \psi}{\partial y} \right)^2
\]
\[
S_{xx} = 2(1 + w) \frac{\partial^2 \psi}{\partial x \partial y} - 2A \left[ 2\delta^2 \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 \psi}{\partial x^2} - \delta^2 \frac{\partial^2 \psi}{\partial y^2} \right)^2 \right] \]  
\[
S_{xy} = (1 + w) \left( -\delta^2 \frac{\partial^2 \psi}{\partial x^2} + \delta^2 \frac{\partial^2 \psi}{\partial y^2} \right) - A \left[ 2\delta^2 \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2 - \left( -\delta^2 \frac{\partial^2 \psi}{\partial x^2} + \delta^2 \frac{\partial^2 \psi}{\partial y^2} \right)^2 \right] + 2\delta^2 \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2 \right] \left( -\frac{\delta^2 \frac{\partial^2 \psi}{\partial x^2} + \delta^2 \frac{\partial^2 \psi}{\partial y^2}}{\delta \frac{\partial^2 \psi}{\partial y^2}} \right) \left( -\frac{\delta^2 \frac{\partial^2 \psi}{\partial x^2} + \delta^2 \frac{\partial^2 \psi}{\partial y^2}}{\delta \frac{\partial^2 \psi}{\partial x^2}} \right) \]  
\[
S_{yy} = -2(1 + w)\delta \frac{\partial^2 \psi}{\partial x \partial y} - 2A\delta \left[ 2\delta^2 \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right)^2 \right] \left( -\frac{\delta^2 \frac{\partial^2 \psi}{\partial x^2} + \delta^2 \frac{\partial^2 \psi}{\partial y^2}}{\delta \frac{\partial^2 \psi}{\partial y^2}} \right) \left( -\frac{\delta^2 \frac{\partial^2 \psi}{\partial x^2} + \delta^2 \frac{\partial^2 \psi}{\partial y^2}}{\delta \frac{\partial^2 \psi}{\partial x^2}} \right) \]  

Now, Eqs. 29-34 become the form when \((Re \text{ and } \delta < 1)\) are present:
While the component of the extra stress tensor becomes the form of

Also, if Eq.39 is entered into Eq.35 as well as the derivative with regard to y and by (w+1) is taken, then the following equation is obtained:

\[
-\frac{\partial^2 \omega^2}{\mu \frac{\partial^2 \psi}{\partial y^2}} = -\frac{\partial^2 \omega^2}{\partial y^2} + \frac{\partial^2 \psi}{\partial y^2} S_{xy} - \left( H a^2 \cos^2 \beta + \frac{1}{\partial a} \right) \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial y^2} \sin \alpha^* \]  
\[
-\frac{\partial^2 \omega^2}{\partial a^2} = 0 \]  
\[
-\frac{\partial^2 \chi}{\partial y^2} = -Ec. Pr \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \]  
\[
S_{xx} = 2(1 + w) \frac{\partial^2 \psi}{\partial x \partial y} - 2A \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \frac{\partial^2 \psi}{\partial x \partial y} \]  
\[
S_{xy} = (1 + w) \left( \frac{\partial^2 \psi}{\partial y^2} \right) - A \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 \]  
\[
S_{yy} = 0 \]  

Where

\[
\zeta = \frac{Ha^2 \cos^2 \beta + \frac{1}{\partial a} \frac{\partial^2 \omega^2}{\mu}}{w + 1}, \quad \eta = \frac{1}{w + 1} \]  

In the wave frame, the dimensionless volume flow rate and boundary condition are as follows:
\(F\) represents the dimensionless temporal average flow in the wave frame.

\[
\Psi = \frac{F}{2} \frac{\partial^2 \psi}{\partial y^2} = -1, \theta = 0 \quad \text{at} \quad y = h_1 \]  
\[
\Psi = -\frac{F}{2} \frac{\partial^2 \psi}{\partial y^2} = -1, \theta = 0 \quad \text{at} \quad y = h_2 \]  

Problem’s Resolution

A non-linear system of partial differential equations is solved using the perturbation method by increasing flow amounts in a power series of \(A\).

\[
\Psi = \Psi_0 + A\Psi_1 + O(A^2) \]  
\[
P = P_0 + AP_1 + O(A^2) \]  

Now, by substituting Eqs. 44 – 45 into Eqs. 35 – 40 and boundary conditions (42), (43) and comparing the coefficients of the same \(A\) power up to the first order yields the two system solutions listed below:

1. Zeroth Order System

When the terms of order \((A)\) in a zeroth-order system are negligible, then

\[
\Psi_{0yyyy} - \zeta \Psi_{0yy} = 0 \]  

Such is the case

\[
\Psi_0 = \frac{F_0}{2}, \frac{\partial^2 \psi_0}{\partial y^2} = -1 \quad \text{at} \quad y = h_1 \]  
and

\[
\Psi_0 = -\frac{F_0}{2}, \frac{\partial^2 \psi_0}{\partial y^2} = -1 \quad \text{at} \quad y = h_2 \]  

2. First Order System

\[
\Psi_{1yyyy} - \eta \frac{\partial^2 \psi_0}{\partial y^2} (\Psi_{0yy})^3 - \zeta \Psi_{1yy} = 0 \]  
\[
\Psi_{1yyyy} - \zeta \Psi_{1yy} = \eta \frac{\partial^2 \psi_0}{\partial y^2} (\Psi_{0yy})^3 \]  
\[
\Psi_1 = \frac{F_1}{2}, \frac{\partial^2 \psi_1}{\partial y^2} = 0 \quad \text{at} \quad y = h_1 \]  
and

\[
\Psi_1 = -\frac{F_1}{2}, \frac{\partial^2 \psi_1}{\partial y^2} = 0 \quad \text{at} \quad y = h_2 \]  

Solving associated zeroth and first-order systems yields the final equation for the stream function.

\[
\Psi = \Psi_0 + A\Psi_1 \]  
\[
\Psi = e^{-\sqrt{\eta} \left( e^{2\sqrt{\zeta}\eta} c_1 + c_2 \right)} + c_3 + yc4 + \]  
\[
A \left[ e^{-3\sqrt{\eta} \left( e^{3(h+1)\sqrt{\eta}} (F_0 + h_1 - h_2)^3 \zeta^3 \eta - e^{6\sqrt{\eta}} (F_0 + h_1 - h_2)^3 \zeta^3 \eta + \right. \right] \]
Results and Discussion

This section consists of two subsections. In the first, the pressure gradient is discussed, while in the second, the temperature distribution is illustrated using the MATHEMATICA software.

Pressure Gradient $\frac{dp}{dx}$:

Case variation of $\frac{dp}{dx}$ indicates the variance in the axial pressure gradient across the channel. The influence of various values ($\text{Ha, } \beta, \text{ Da, } \Omega, w, \phi, A, \text{ Fr, } Re, \alpha^*$) on the axial pressure gradient $\frac{dp}{dx}$ is illustrated in Figs. 2 - 11.

Figs. 2 and 8 demonstrate that increases in the values of the Hartman number (Ha) and the material fluid parameter (A) cause the axial pressure gradient to decrease as the curve's vertex, but have no effect on the axial pressure gradient near the right or left channel wall.

In Figs. 3 and 4, the increases in the values of Darcy number (Da) and the inclination of the magnetic field ($\beta$) lead to the axial pressure gradient increasing as the vertex of the curve is twisted to the right but the axial pressure gradient close to the right or left walls of the channel is unaffected. While in Fig.6 the increases in the values of the porous medium parameter ($w$) lead to the axial pressure gradient increases as the vertex of the curve only but the axial pressure gradient close to the right or left walls of the channel is unaffected.

With the fixed frame, the axial velocity component is expressed as

$$u(x, y, t) = \Psi_y$$

It is possible to rewrite Eq.35 as

$$\frac{\partial p}{\partial x} = \Psi_{0yy} - \zeta \Psi_{0y} + A \Psi_{1yy} - \eta A \frac{\partial}{\partial y} (\Psi_{0yy})^2 - A \zeta \Psi_{1yy}$$

Energy Equation Solution

The long wavelength and low Reynolds approximation are used to get Eq. 37.

$$\frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial y^2} \right) + Ec \left( \frac{\partial \Psi_y}{\partial y^2} \right)^2 = 0$$

The solution of Eq.57 with boundary conditions

$$\theta = 0 \ at \ y = h_1 \ and \ \theta = 1 \ at \ y = h_2$$

It is possible to prove that $r_1$ and $r_2$ are constants using the boundary conditions, and can be stated $\theta$ as in the index.

Temperature Distribution $\theta$:

The effect of relevant parameters on the temperature distribution $\theta$ is graphically illustrated in Figs. 12 - 21 whereas depicted in the following figures, the behavior of temperature distribution is parabolic.

Figs. 12, 17, 18, 19, and 20 illustrate that increases in the values of the Hartman number (Ha), the amplitude ratio ($\phi$), the material fluid parameter (A), the Eckert number (Ec), and the
Prandtl number (Pr) have no effect on the temperature field in the channel's central region, whereas the temperature field decreases near the channel wall.

- Figs. 13, 14, 15, and 16 display the increases in the values of Darcy number (Da), the inclination of the magnetic field (β), the rotation (Ω), and porous medium parameter (w) there is no effect on the temperature field in the central region of the channel, whereas the temperature field is rising close to the channel wall's margin.

- The temperature field does not change as the Reynolds number (Re), the Froude number (Fr), and the inclination angle of the channel to the horizontal axis (α*) values increase, as shown in Figs. 21, 22, and 23.

**Figure 2.** Pressure gradient variation for different (Ha) values when β = 0.1, Da = 0.2, ρ = 0, d = 0.5, Ω = 0.2, μ = 3, w = 0.3, φ = 0.2, α = 0.2, b = 0.2, d₁ = 0.5, F₀ = 0.4, F₁ = 0, A = 0.3, Fr = 0.7, Re = 0.2, α* = 0.5

**Figure 3.** Pressure gradient variation for different (Da) values when Ha = 2.5, β = 0.1, ρ = 0.1, d = 0.5, Ω = 0.2, μ = 3, w = 0.3, φ = 0.2, α = 0.2, b = 0.2, d₁ = 0.5, F₀ = 0.4, F₁ = 0, A = 0.3, Fr = 0.7, Re = 0.2, α* = 0.5

**Figure 4.** Pressure gradient variation for different (β) values when Ha = 2.5, Da = 0.2, ρ = 0.1, d = 0.5, Ω = 0.2, μ = 3, w = 0.3, φ = 0.2, a = 0.2, b = 0.2, d₁ = 0.5, F₀ = 0.4, F₁ = 0, A = 0.3, Fr = 0.7, Re = 0.2, α* = 0.5

**Figure 5.** Pressure gradient variation for different (Ω) values when Ha = 2.5, β = 0.1, Da = 0.2, ρ = 0.1, d = 0.5, μ = 3, w = 0.3, φ = 0.2, a = 0.2, b = 0.2, d₁ = 0.5, F₀ = 0.4, F₁ = 0, A = 0.3, Fr = 0.7, Re = 0.2, α* = 0.5

**Figure 6.** Pressure gradient variation for different (w) values when Ha = 2.5, β = 0.1, Da = 0.2, ρ = 0.1, d = 0.5, Ω = 0.2, μ = 3, φ = 0.2, α = 0.2, b = 0.2, d₁ = 0.5, F₀ = 0.4, F₁ = 0, A = 0.3, Fr = 0.7, Re = 0.2, α* = 0.5
Figure 7. Pressure gradient variation for different (φ) values when Ha = 2.5, β = 0.1, Da = 0.2, ρ = 0.1, d = 0.5, Ω = 0.2, μ = 3, w = 0.3, α = 0.2, b = 0.2, d₁ = 0.5, F0 = 0.4, F1 = 0, A = 0.3, Fr = 0.7, Re = 0.2, α* = 0.5

Figure 8. Pressure gradient variation for different (A) values when Ha = 2.5, β = 0.1, Da = 0.2, ρ = 0.1, d = 0.5, Ω = 0.2, μ = 3, w = 0.3, φ = 0.2, α = 0.2, b = 0.2, d₁ = 0.5, F0 = 0.4, F1 = 0, A = 0.3, Fr = 0.7, α* = 0.5

Figure 9. Pressure gradient variation for different (Fr) values when Ha = 2.5, β = 0.1, Da = 0.2, ρ = 0.1, d = 0.5, Ω = 0.2, μ = 3, w = 0.3, φ = 0.2, α = 0.2, b = 0.2, d₁ = 0.5, F0 = 0.4, F1 = 0, A = 0.3, Fr = 0.7, α* = 0.5

Figure 10. Pressure gradient variation for different (Re) values when Ha = 2.5, β = 0.1, Da = 0.2, ρ = 0.1, d = 0.5, Ω = 0.2, μ = 3, w = 0.3, φ = 0.2, α = 0.2, b = 0.2, d₁ = 0.5, F0 = 0.4, F1 = 0, A = 0.3, Fr = 0.7, α* = 0.5

Figure 11. Temperature variation for various (Ha) values when Ha = 0.5, β = 0.1, Da = 10, ρ = 0.1, d = 0.5, Ω = 0.5, μ = 3, w = 0.3, φ = 1.3, α = 0.2, b = 0.2, d₁ = 0.5, F0 = 0.4, F1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, α* = 1.5

Figure 12. Temperature variation for various (Fr) values when Ha = 0.5, β = 0.1, Da = 10, ρ = 0.1, d = 0.5, Ω = 0.5, μ = 3, w = 0.3, φ = 1.3, α = 0.2, b = 0.2, d₁ = 0.5, F0 = 0.4, F1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, α* = 1.5
Figure 13. Temperature variation for various (Da) values when \( Ha = 0.5, \beta = 0.1, Da = 10, \rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, a = 0.2, b = 0.2, d_1 = 0.5, F0 = 0.4, F1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha^* = 1.5 \)

Figure 14. Temperature variation for various (\( \beta \)) values when \( Ha = 0.5, \beta = 0.1, Da = 10, \rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, a = 0.2, b = 0.2, d_1 = 0.5, F0 = 0.4, F1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha^* = 1.5 \)

Figure 15. Temperature variation for various (\( \Omega \)) values when \( Ha = 0.5, \beta = 0.1, Da = 10, \rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, a = 0.2, b = 0.2, d_1 = 0.5, F0 = 0.4, F1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha^* = 1.5 \)

Figure 16. Temperature variation for various (w) values when \( Ha = 0.5, \beta = 0.1, Da = 10, \rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, a = 0.2, b = 0.2, d_1 = 0.5, F0 = 0.4, F1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha^* = 1.5 \)

Figure 17. Temperature variation for various (\( \phi \)) values when \( Ha = 0.5, \beta = 0.1, Da = 10, \rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, a = 0.2, b = 0.2, d_1 = 0.5, F0 = 0.4, F1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha^* = 1.5 \)

Figure 18. Temperature variation for various (A) values when \( Ha = 0.5, \beta = 0.1, Da = 10, \rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, a = 0.2, b = 0.2, d_1 = 0.5, F0 = 0.4, F1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha^* = 1.5 \)
Figure 19. Temperature variation for various (Ec) values when $Ha = 0.5, \beta = 0.1, Da = 10$, $\rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, \alpha = 0.2, b = 0.2, d_1 = 0.5, F_0 = 0.4, F_1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha' = 1.5$

Figure 20. Temperature variation for various (Pr) values when $Ha = 0.5, \beta = 0.1, Da = 10$, $\rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, \alpha = 0.2, b = 0.2, d_1 = 0.5, F_0 = 0.4, F_1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha' = 1.5$

Figure 21. Temperature variation for various (Re) values when $Ha = 0.5, \beta = 0.1, Da = 10$, $\rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, \alpha = 0.2, b = 0.2, d_1 = 0.5, F_0 = 0.4, F_1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha' = 1.5$

Figure 22. Temperature variation for various (Fr) values when $Ha = 0.5, \beta = 0.1, Da = 10$, $\rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, \alpha = 0.2, b = 0.2, d_1 = 0.5, F_0 = 0.4, F_1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha' = 1.5$

Figure 23. Temperature variation for various ($\alpha'$) values when $Ha = 0.5, \beta = 0.1, Da = 10$, $\rho = 0.1, d = 0.5, \Omega = 0.5, \mu = 3, w = 0.3, \phi = 1.3, \alpha = 0.2, b = 0.2, d_1 = 0.5, F_0 = 0.4, F_1 = 0, A = 0.1, Ec = 0.05, Pr = 0.05, Re = 2, Fr = 1, \alpha' = 1.5$
Conclusion

In this study, the rotational effects of peristaltic transport of a Powell-Eyring fluid in an asymmetric channel through a porous material susceptible to the combined acts of inclined MHD are investigated. The asymmetric channel is formed by selecting peristaltic waves with varying amplitudes and phases on the non-uniform walls and a low Reynolds number. Using the perturbation approach, the formulas for the axial velocity and pressure gradient are produced. Multiple graphs are utilized for parameter analysis:

I) When the values of Hartman number (Ha) and material fluid parameter (A) increase, the axial pressure gradient decreases as the vertex of the curve is twisted to the right. However, the axial pressure gradient close to the right or left walls of the channel is unaffected. However, the opposite occurs when the values of Darcy number (Da), and the inclination of magnetic field (β) increase, while the increases in the values of the porous medium parameter (w) lead to the axial pressure gradient increases as the vertex of the curve only while demonstrating that the axial pressure gradient does not change as the rotation (Ω), the Froude number (Fr), the Reynolds number (Re), and the inclination angle of the channel to the horizontal axis (α*) values increase. Whereas for approximately -1.9<x<0, and -2.8<x<-2.2, the axial velocity increases as the amplitude ratio increases (ϕ), but for approximately -4<x<-2.8, the axial pressure gradient decreases slightly. However, for -2.2<x<-1.9 and 0<x<1, the axial pressure does not change.

II) As the Hartman number (Ha), the amplitude ratio (ϕ), material fluid parameter (A), Eckert number (Ec), and Prandtl number (Pr) increases, the temperature field decreases in the vicinity of the channel's wall but no change in the channel's central region, while the increases in the values of Darcy number (Da), the inclination of magnetic field (β), the rotation (Ω), and porous medium parameter (w) there is no effect on the temperature field in the channel’s central region, the temperature field increases in the vicinity of the channel’s wall and furthermore increasing values of Reynolds number (Re), Froude number (Fr), and inclination angle of the channel to the horizontal axis (α*) have no effect on the temperature field.

Authors’ Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

Authors’ Contribution Statement

This work was carried out in collaboration between both authors. R. Gh. I. and L. Z. H. read and approved the final manuscript.

References


تأثير المعلمات المختلفة على التدفق التمتعجي لسائل باول آرينغ مع تأثير الدوران والانتقال الحراري في قناة غير متماثلة مائلة

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الخلاصة

في هذه المقالة، يتم فحص تأثير متغير الدوران والمتغيرات الأخرى على التدفق التمتعجي لسائل باول آرينغ في قناة غير متماثلة مائلة مع مجال مغناطيسي مائل عبر وسط مسامي مع نقل الحرارة. يفترض الطول الموجي الطويل وعدد رينولدز المنخفض، حيث يتم استخدام نهج الاضطراب لحل المعادلات الحاكمة غير الخطية في نظام الإحداثيات الديكارتية لإنتاج حلول متسلسلة. يتم التعبير عن توزيعات السرعة وتدرجات الضغط رياضيًا. من خلال جمع الأرقام، يتم شرح تأثير المعايير المختلفة وتعميتها بيانياً. تم الحصول على هذه النتائج العددية باستخدام التطبيق الرياضي MATHEMATICA.

الكلمات المفتاحية: انتقال الحرارة، قناة المائلة، مجال مغناطيسي، تدفق تمتعجي، وسط مسامي، سائل باول آرينغ، الدوران.