Semi-Analytical Solutions for Time-Fractional Fisher Equations via New Iterative Method

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Abstract

An effective method for resolving non-linear partial differential equations with fractional derivatives is the New Sumudu Transform Iterative Method (NSTIM). It excels at solving difficult mathematical puzzles and offers insightful information about the behaviour of time-fractional Fisher equations. The method, which makes use of Caputo's sense derivatives and Wolfram in Mathematica, is reliable, simple to use, and gives a visual depiction of the solution. The analytical findings demonstrate that the proposed approach is effective and simple in generating precise solutions for the time-fractional Fisher equations. The results are made more reliable and applicable by including Caputo's sense derivatives. Mathematical modelling relies on the effectiveness and simplicity of the NSTIM approach to solve time-fractional Fisher equations since it enables precise solutions without the use of a lot of processing power. The NSTIM approach is a useful tool for researchers in a variety of domains since it also offers a flexible framework that is easily adaptable to other fractional differential equations. It now becomes possible to examine the dynamics and behaviour of complex systems governed by time-fractional Fisher equations with efficiency and reliability, opening up new research avenues. The ability to solve time-fractional Fisher equations efficiently and reliably using the NSTIM approach has significant implications for various fields such as population dynamics, mathematical biology, and epidemiology. Researchers can now analyze the spread of diseases or study the population dynamics of species with higher accuracy and less computational effort. This advancement in solving fractional differential equations paves the way for deeper insights into the behavior and patterns of complex systems, ultimately advancing scientific understanding and offering new possibilities for practical applications.

Keywords: Caputo fractional derivative, Fisher equations, Fractional Calculus, Iterative method, Sumudu Transform.

Introduction

In recent decades, fractional differential equations have fascinated mathematicians, physicists and engineering researchers1–3. A fractional theory, which includes fractional derivatives and fractional integration, can be used to model a wide range of problems4–7. Various methods have been developed to solve both linear and nonlinear fractional differential equations8–10, including the Cauchy reaction-diffusion method, the Adomian decomposition method(A.D.M.)11,12, the homotopy method (H.A.M.)13, the variational method of iteration (V.I.M.)14,15, and the perturbation method of homotopy (H.P.M.)16. These methods have been applied to the Cauchy-diffusion of time-fractional equations, which are used to model nonlinear and linear systems in fields such as engineering, biology, ecology, chemistry, and physics17–19.
The study of investigates solutions for nonlinear generalized proportional, fractional functional integro-differential Langevin equations using fixed point theorems and Ulam-Hyers stability. It creates a mathematical model to analyse Wolbachia dispersal among Aedes aegypti mosquitoes, analysing symmetrical characteristics. The model is physically meaningful and assessed for equilibrium points in the presence and absence of disease. Eight equilibrium points are determined, and the basic reproduction number is calculated using the next-generation matrix method. Numerical simulations are conducted to evaluate the basic reproduction number and identify the optimal CI value for reducing disease spread. The study also examines the interaction between prey and predator populations, focusing on the additive Allee effect and intraspecific competition. The study highlights the importance of considering precautionary measures in controlling disease spread, with the rate of precautionary measures playing a crucial role in reducing the chance of infection by the Chickenpox virus.

The authors of an article obtained both numerical and analytical solutions to the time-fractional Fisher equations using the New Sumudu transform iterative method (NSTIM). The benefit of this new method is that it makes the calculations easy and gives the most accurate estimate of the exact answer. There are many problems in fractional derivatives, hydrodynamics, chemical diffusion, and option pricing. Partial differential equations can be used to model computational fluid dynamics and control theory. Nonlinear P.D.E.s and processes for finding numerical solutions to nonlinear problems have gotten much attention recently. The main theme of this research is focused on the solution and analysis of a nonlinear time fractional Fisher's equation with specific boundary conditions.

The analytical method focuses on finding an exact solution to PDEs, but solving the time fractional Fisher equation is challenging due to the fractional derivative. The iterative method, using techniques like the New Sumudu transform iterative method, discretizes the equation in space and time, and updates it iteratively until it converges to the desired accuracy. The analytical method approximates the fractional derivative term, while the iterative method transforms the equation into linear or nonlinear algebraic equations that can be solved iteratively.

The software Mathematica provides powerful tools for creating visualizations and graphical representations of nonlinear time fractional Fisher's equations. It supports 2D and 3D plotting, graphing, and interactive visualizations, which aids in better understanding solutions of Fisher's equations.

The new Sumudu transform iterative method offers several advantages, including faster convergence, improved accuracy and precision, wide applicability, robustness, memory and computational efficiency, parallelization, adaptability to problem structure, trade-offs, comparative analysis, and experimental results.

Fisher's equation is a mathematical model that describes the spread of a mutant gene through a population. It is a partial differential equation with constant coefficients.

\[ y_\omega (\xi, \omega) = y_\xi \xi (\xi, \omega) + y (\xi, \omega) (1 - y (\xi, \omega)) \]

This model shows the population density by \( y (\xi, \omega) \), and the logistic form is indicated by \( y (y - 1) \). In chemical kinetics and population dynamics, this equation solves problems like the nonlinear growth of a population in a habitat of 1-dimensional and the number of neutrons in a nuclear reaction. Also, one of the same equations is used in models growth of a logistic population, the spread of a flame, neurophysiology, chemical reactions that happen on their own and branching Brownian motion. In this article, Fisher's equation of time-fractional can be written as follows:

\[ D^\beta_\omega y (\xi, \omega) = y_\xi \xi (\xi, \omega) + \lambda y (\xi, \omega) (1 - y (\xi, \omega)) \]

where \( D^\beta_\omega y (\xi, \omega) \) denotes the caputo fractional derivative (C.F.D.) of order \( \beta \) and \( \lambda \) is a parameter (accurate).

**Background:**

**Definition 1:** The \( \beta \in \mathbb{C}, \text{Re}(\beta) > 0 \) R-L fractional integral \( I^\beta_p f \) of order is defined by

\[
\left( p D^{-\beta}_q f \right)(q) = \left( I^\beta_1 f \right)(q) = \frac{1}{\Gamma(\beta)} \int_p^q \frac{f(c)}{(q-c)^{1-\beta}} \, dc, \quad (q > p, \text{Re}(\beta) > 0).
\]
Definition 2: The R-L fractional derivatives
\((pD^{\beta}_{t}z)(x)\) of order \(\beta \in \mathbb{C}, \text{Re}(\beta) > 0\) is defined by
\[
(pD^{\beta}_{t}z)(x) = \frac{d^{j}}{d\xi^{j}} \left( t^{\beta-j} z(t) \right)(x)
\]
where
\[
\int_{0}^{t} (t-c)^{j-1} c^{\beta-j} dc = \Gamma(j-\beta)
\]
\(c > p, j = \text{Re}(\beta) + 1\).

Definition 3: Function of Mittag-Leffler and generalization
\[
E_{\delta}(z) = \sum_{m=0}^{\infty} \frac{z^{m}}{\Gamma(\delta m + 1)} (\delta \in \mathbb{C}, \text{Re}(\delta) > 0),
\]
\(E_{\delta,\omega}\) is Mittag-Leffler function in two parameters.
\[
E_{\delta,\omega}(z) = \sum_{m=0}^{\infty} \frac{y^{m}}{\Gamma(\delta m + \omega)} \omega, \delta \in \mathbb{C}, \text{Re}(\delta) > 0,
\]
\(\text{Re}(\omega) > 0\).

Definition 4: A caputo fractional derivative of function \(y(\xi, t)\) is defined as
\[
D^{\beta}_{\xi}y(\xi, t) = \frac{1}{\Gamma(\beta-j)} \int_{0}^{\xi} (\xi-c)^{j-1} \frac{d^{j}y(c, t)}{d c^{j}} dc,
\]
\(j-1 < \beta \leq j, j \in \mathbb{N}\).

Definition 5: The Sumudu transform of a function \(f(p), p > 0\) is defined as
\[
S[f(p)] = \int_{0}^{\infty} e^{-pt} f(v) dp, v \in (-P_{1}, P_{2})
\]
and \(f(p) \in W\), where
\[
W = \left\{ f(p), \exists M, P_{1}, P_{2} > 0, |f(p)| \leq M |p|^{\frac{p}{|p|}} \text{if } p \in (-1)^{1} \times [0, \infty) \right\}
\]

Definition 6: The Sumudu transform of the Caputo fractional derivative is defined as
\[
S \left[ D_{\omega}^{n\beta} y(Y, \omega) \right] = v^{-n\beta} S[y(Y, \omega)] - \sum_{j=0}^{\beta-1} v^{-n\beta+j} y^{(j)}(0, \omega), j-1 < n\beta < j.
\]

The New Sumudu transform Iterative Method (NSTIM):

To illustrate this New Iterative Transform of Sumudu Method take into account a fractional partial differential equation with the initial conditions, which is both non-homogeneous and nonlinear:
\[
D_{\omega}^{n\beta} z(Y, \omega) + Lz(Y, \omega) + R(z(Y, \omega)) = g(Y, \omega),
\]
\(n-1 < n\beta \leq n, z(Y, 0) = h(Y)\)

where \(D_{\omega}^{n\beta}\) is the fractional Caputo derivative operator, \(D_{\omega}^{n\beta} = \frac{d^{n\beta}}{d\omega^{n\beta}}, L\)-operator(linear), \(R\)-operator(non-linear), \(g(Y, \omega)\) is continuous function.

Employing the Sumudu transform to Eq 12 obtain,
\[
S \left[ D_{\omega}^{n\beta} z(Y, \omega) \right] + S[L(z(Y, \omega))] + S[R(z(Y, \omega))] = S[g(Y, \omega)],
\]
employing the property of sumudu transformation, obtain,
\[
S[z(Y, \omega)] - v^{n\beta} \sum_{k=0}^{\beta-1} v^{-n\beta+k} z^{(k)}(Y, 0) + v^{n\beta} S[L(z(Y, \omega))] + S[R(z(Y, \omega))] - g(Y, \omega) = 0.
\]
employing the Sumudu transform of inverse to Eq 14,
\[
z(Y, \omega) = S^{-1} \left[ v^{n\beta} \sum_{k=0}^{\beta-1} v^{-n\beta+k} z^{(k)}(Y, 0) - v^{n\beta} S[L(z(Y, \omega))] + S[R(z(Y, \omega))] - g(Y, \omega) \right].
\]

Next, assume that,
\[ f(Y, \omega) = S^{-1} \left[ v^{-n\beta} \sum_{k=0}^{j-1} v^{-n\beta+k} z(k)(Y, 0) \right. \]
\[ \left. + v^{-n\beta} [g(Y, \omega)] \right] \]

\[ N(z(Y, \omega)) = -S^{-1} [v^{-n\beta} S[R(z(Y, \omega))]]. \]

\[ K(z(Y, \omega)) = -S^{-1} [v^{-n\beta} S[L(z(Y, \omega))]]. \]

Thus, Eq 15 will be reduced in the following form,
\[ z(Y, \omega) = f(Y, \omega) + K(z(Y, \omega)) + N(z(Y, \omega)). \]

The solution of the equation is given in the series form,
\[ z(Y, \omega) = \left( \sum_{m=0}^{\infty} z_m(Y, \omega) \right). \]

Obtaining
\[ K \left( \sum_{m=0}^{\infty} z_m(Y, \omega) \right) = \sum_{m=0}^{\infty} K(z_m(Y, \omega)). \]

Operator N (nonlinear) is decomposed as
\[ N(\sum_{m=0}^{\infty} z_m) = N(z_0) + \left\{ N(\sum_{j=0}^{m} z_j) - N(\sum_{j=0}^{m-1} z_j) \right\}. \]

Therefore, Eq 12 can be represented in the following form, Defining the recursive relation
\[ z_0(Y, \omega) = f(Y, \omega), \]
\[ z_1(Y, \omega) = K(z_0(Y, \omega)) + N(z_0(Y, \omega)), \]
\[ z_{r+1}(Y, \omega) = K(z_r(Y, \omega)) + \]
\[ \left\{ N \left( \sum_{j=0}^{r} z_j(Y, \omega) \right) - N \left( \sum_{j=0}^{r-1} z_j(Y, \omega) \right) \right\}, \]

for all \( r \geq 1 \)

Thus, \( z_1 + z_2 + \ldots + z_{m+1} = K(z_0 + \ldots + z_m) + N(z_0 + \ldots + z_m) \)

namely,

Convergence and Error Analysis:

**Theorem 1:** Let \( z_p(Y, \omega) \) and \( z_n(Y, \omega) \) be the members of Banach space H and the exact solution of Eq 1 be \( z(Y, \omega) \). The series solution \( \sum_{p=0}^{\infty} z_p(Y, \omega) \) converges to \( z(Y, \omega) \), if \( z_p(Y, \omega) \leq \lambda z_{p-1}(Y, \omega) \) for \( \lambda \in (0, 1) \), that is for any \( z > 0 \), \( \exists E \) such that \( |z_{p+n}(Y, \omega)| \leq z, \forall p, n > E \).

**Proof:** Let \( u_p(Y, \omega) = z_0(Y, \omega) + z_1(Y, \omega) + z_2(Y, \omega) + \ldots + z_p(Y, \omega) \) be the sequence of \( p \)-th partial sum of series \( \sum_{p=0}^{\infty} z_p(Y, \omega) \).

Now, consider
\[ ||u_{p+1}(Y, \omega) - u_p(Y, \omega)|| \]

\[ \leq \lambda ||u_{p}(Y, \omega)|| \]

\[ \leq \lambda^2 ||u_{p-1}(Y, \omega)|| \]

\[ \leq \lambda^3 ||u_{p-2}(Y, \omega)|| \]

\[ \vdots \]

\[ \leq \lambda^{p+1} ||u_0(Y, \omega)||. \]

for \( \forall n, p \in E \)

Consider,
\[ ||u_p(Y, \omega) - u_n(Y, \omega)|| \]
imples that there exists \( z_0(Y, \omega) \in H \) such that 
\[
\lim_{p \to \infty} z_p(Y, \omega) = z(Y, \omega).
\]
Hence, the proof has been completed.

**Theorem 2:** Let \( \sum_{p=0}^{\infty} z_p(Y, \omega) \) be the finite and approximate solution of \( z(Y, \omega) \). If \( ||z_{p+1}(Y, \omega)|| \leq \lambda ||z_0(Y, \omega)|| \) for \( \lambda \in (0, 1) \), then the maximum absolute error is
\[
||z(Y, \omega) - \sum_{p=0}^{q} z_p(Y, \omega)|| 
\leq \frac{\lambda^{q+1}}{1-\lambda} ||z_0(Y, \omega)||.
\]

**Proof:**

\[
||z(Y, \omega) - \sum_{p=0}^{q} z_p(Y, \omega)||
\leq \sum_{p=q+1}^{\infty} \frac{\lambda^{q+1}}{1-\lambda} ||z_0(Y, \omega)||
\leq \frac{\lambda^{q+1}}{1-\lambda} ||z_0(Y, \omega)||
\]

Thus, the proof has been completed.

\[
y(Y, \omega) = \frac{1}{(1+e^y)^2} + S^{-1}\left[\frac{1}{u-\beta}S^{\frac{\beta y}{\beta y^2}} + 6y(1-y)\right]
\]

Employing Sumudu transform on Eq. 29 and using the initial condition \( y(0) = 1 \) to obtain,

\[
S[y(Y, \omega)] = \frac{1}{(1+e^y)^2} + \frac{1}{u-\beta}S^{\frac{\beta y}{\beta y^2}} + 6y(1-y).
\]

By iteration, the following results are obtained

\[
y_0 = \frac{1}{(1+e^y)^2},
\]

\[
K[y(Y, \omega)] = S^{-1}\left[\frac{1}{u-\beta}S^{\frac{\beta y}{\beta y^2}} + 6y(1-y)\right].
\]
\[ y_0(Y, \omega) = \frac{1}{(1+e^Y)^2}, \]
\[ y_1(Y, \omega) = S^{-1}\left[ \frac{1}{u^{-\beta}} \Gamma(1+\beta) \right], \]
\[ y_2(Y, \omega) = y_1(Y, \omega) + y_1(Y, \omega) + \cdots \]
\[ y_3(Y, \omega) = \frac{1}{(1+e^Y)^2} + 10 \frac{e^Y}{(1+e^Y)^3} \Gamma(\beta + 1) \]
\[ + 50 \frac{e^Y(-1+2e^Y)}{(1+e^Y)^4} \frac{\omega^2}{\Gamma(2\beta + 1)} + \]
\[ 250e^Y \left[ 5 - 6e^Y - 15e^Y + 20e^Y \Gamma(2\beta+1) \right] \frac{\omega^2}{\Gamma(\beta+1)^2} + \cdots \]

Where \( E_\beta(\omega^\beta) \) is mittag leffer function defined by Eq 7.

putting \( \beta = 1 \), Eq 29 becomes the following equation,
\[ y(Y, \omega) = \frac{\partial^2 y}{\partial Y^2} - 6y - y \]

With accurate solution
\[ y(Y, \omega) = \frac{1}{(1+e^Y - 5\omega)^2}. \]

(See Fig. 1 and Table 1)

**Remark 1:** The linear time fractional Fisher equations are shown above. The estimated results of fractional Fisher equations of the time linear at values of \( \beta = 0.2, 0.4, 0.6, 0.8 \) and the accurate solution for \( \beta = 1 \) are shown below. Fig 1 (a), in 3-dimension view and in Fig 1 (b), in 2-dimension forms, respectively. The answer is so simple to discover that it is constantly dependent on the values of time-fractional derivatives.
Example 2: Suppose the nonlinear time fractional following Fisher’s eq. 54

\[
\frac{\partial^\beta}{\partial t^\beta} y(Y, \omega) = \frac{\partial^2 y(Y, \omega)}{\partial Y^2} + y(1 - y), 0 < \beta \leq 1 \quad 39
\]

The initial condition

\[
y(Y, 0) = \alpha, \quad 40
\]

employing Sumudu transform on the Eq 39 and using the initial condition of Eq 40 obtain,

\[
S[y(Y, \omega)] = \alpha + \frac{1}{u^\beta} S[\frac{\partial^2 y}{\partial Y^2} + y(1 - y)], \quad 41
\]

employing the Sumudu transform of the inverse formula,

\[
y(Y, \omega) = \alpha + S^{-1}[\frac{1}{u^\beta} S[\frac{\partial^2 y}{\partial Y^2} + y(1 - y)]], \quad 42
\]

\[
\text{namely,} \quad y(Y, \omega) = \alpha + S^{-1}[\frac{1}{u^\beta} S[\frac{\partial^2 y}{\partial Y^2} + y(1 - y)]]. \quad 43
\]

According to the NSTIM, result obtain

\[
y_0 = \alpha, \quad 44
\]

\[
K[y(Y, \omega)] = S^{-1}[\frac{1}{u^\beta} S[\frac{\partial^2 y}{\partial Y^2} + y(1 - y)]]. \quad 45
\]

By iteration, the following results are obtained

\[
y_0(Y, \omega) = \alpha, \quad 46
\]

\[
y_1(Y, \omega) = \alpha(1 - \alpha) \frac{(\omega^\beta)}{\Gamma(\beta+1)} \quad 47
\]

\[
y_2(Y, \omega) = \alpha(1 - \alpha)(1 - 2\alpha) \frac{(\omega^{2\beta})}{\Gamma(2\beta+1)} \quad 48
\]

\[
y_3(Y, \omega) = (\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4) \frac{(\omega^{3\beta})}{\Gamma(3\beta+1)} \quad 49
\]

\[
y_4(Y, \omega) = (1 - 2\alpha)(\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4) \quad 50
\]

\[
-(\alpha^2 - 2\alpha^3 + \alpha^4) \frac{(\omega^{4\beta})}{\Gamma(4\beta+1)} \quad 51
\]

\[
-2(\alpha - \alpha^2)(\alpha - 3\alpha^3 + 2\alpha^4) \frac{(\omega^{5\beta})}{\Gamma(5\beta+1)} \quad 52
\]

Therefore, the analytical estimated results of the problem in the series form can be found as,

\[
y(Y, \omega) = y_0(Y, \omega) + y_1(Y, \omega) + \cdots \quad 53
\]

\[
y(Y, \omega) = \frac{\alpha + (1 - \alpha) \frac{(\omega^\beta)}{\Gamma(\beta+1)} + (1 - \alpha)(1 - 2\alpha) \frac{(\omega^{2\beta})}{\Gamma(2\beta+1)}}{1 + (\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4) \frac{(\omega^{3\beta})}{\Gamma(3\beta+1)} + (1 - 2\alpha)(\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4) \frac{(\omega^{4\beta})}{\Gamma(4\beta+1)} - 2(\alpha - \alpha^2)(\alpha - 3\alpha^3 + 2\alpha^4) \frac{(\omega^{5\beta})}{\Gamma(5\beta+1)}} \quad 54
\]

Where - $E_\beta(\omega^\beta)$ is mittage leffer function defined by Eq 7.

Putting $\beta = 1$, Eq 39 becomes the following equation,

\[
y(Y, \omega) = \frac{\partial^2 y}{\partial Y^2} + y(1 - y), \quad 55
\]

with accurate solution

\[
y(Y, \omega) = \frac{\alpha e^{-Y}}{(1 - \alpha + \alpha e^{-Y})} \quad 56
\]

(See Fig. 2 and Table 2.)
Remark 2: The linear time fractional Fisher equations are shown above. The estimated results of the linear time fractional Fisher equations at values of $\beta=0.2,0.4,0.6,0.8$ and the accurate solution for $\beta=1$ are shown below in Fig. 2 (a), in 3-dimension view and in Fig. 2 (b), in 2-dimension forms respectively. The solution is so simple to discover that it is constantly dependent on the values of time-fractional derivatives.

![Figure 2. The estimated results of the linear time fractional Fisher equations at values of $\beta=0.2,0.4,0.6,0.8$ and 1](image)

The numerical solution obtained using the NSTIM of 5th order approximation of Example 2 is compared with the accurate solution for $\beta = 1$ in Table 2, which shows the efficiency and effectiveness of the method.

| $\beta$ | $\omega$ | $y$ (NSTIM) | $y$ (accurate) | $|y_{\text{NSTIM}} - y_{\text{accurate}}|$ |
|--------|---------|------------|---------------|-----------------|
| 0.2    | 0.3     | 0.109149   | 0.109032      | $1 \times 10^{-04}$ |
| 0.4    | 0.5     | 0.146032   | 0.143259      | $3 \times 10^{-03}$ |
| 0.6    | 0.7     | 0.159643   | 0.139239      | $2 \times 10^{-02}$ |
| 0.8    | 0.9     | 0.204733   | 0.118578      | $9 \times 10^{-02}$ |

Remark 3: Fig 3(a) and Fig 3(b) depict the absolute error between estimated and accurate solutions for $\beta=1$. By comparison, it is clear that by computing additional terms, the efficiency and accuracy of this method (NSTIM) can be significantly improved. The authors have used a few iterations in this post. However, the precision of the estimated solution could be substantially enhanced if they employed additional terms. As a result, the recommended method for solving the linear differential equation is both precise and efficient.
Conclusion

This research used the novel Sumudu transform iterative technique to solve linear time fractional Fisher equations. The novel Sumudu transform iterative approach (NSTIM) combines NIM and Sumudu to solve linear time fractional Fisher equations. The iterative Sumudu transform approach is more structured and accurate and requires less numerical calculation, according to the numbers. The approach reduces computational effort compared to conventional methods while maintaining good numerical precision. It can aid mathematicians and researchers working in the field of partial differential equations. The key advantage of this method is quick convergence and accuracy. NSTIM is an effective tool for discovering approximation and semi-analytical solutions.

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Authors’ Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for republication, which is attached to the manuscript.

Authors’ Contribution Statement

S. T. proposed the Concepts, ideas, method, and analysis. A. B. designed the manuscript. S. G. read the manuscript and revised it. K. K. Drafting of the manuscript with interpreted and plotted the graphs of the solution of examples using Mathematica 11.3. All authors read and approved the final manuscript.

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